

IAMP News Bulletin

January 2021

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MATHEMATICAL PHYSICS (ICMP)**

**August 2 - 7, 2021
(CICG) Geneva**

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- Thierry Bodineau (Ecole Polytechnique)
- Wojciech De Roeck (Leuven)
- Hugo Duminil-Copin (IHES, Geneva)
- Marco Gualtieri (Toronto)
- Peter Hintz (MIT)
- Antti Kupiainen (Helsinki)
- Juan Maldacena (IAS)
- Nikita Nekrasov (Simons Center)
- Yoshiko Ogata (Tokyo)
- Nicolai Reshetikhin (UC Berkeley, Amsterdam, St. Petersburg)
- Jan-Philip Solovej (Copenhagen)
- Jeremie Szeftel (CNRS, Sorbonne)
- Thomas Vidick (Caltech)
- Feng Xu (UC Riverside)
- Horng-Tzer Yau (Harvard)

PUBLIC LECTURE

- Michel Mayor (Geneva)

PRECEDED BY
**Young Researchers
Symposium (YRS)**

**July 29 - 31, 2021
University of Geneva**

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- Martin Hairer (Imperial College)
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- Stanislav Smirnov (Geneva)

THEMATIC SESSIONS

- Dynamical Systems
- Equilibrium Statistical Mechanics
- General Relativity
- Integrable Systems
- Many-body Quantum Systems and Condensed Matter Physics
- Nonequilibrium Statistical Mechanics
- Partial Differential Equations
- Probability & Random Structures
- Quantum Field Theory
- Quantum Information
- Quantum Mechanics & Spectral Theory
- String Theory & Quantum Gravity

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International Association of Mathematical Physics

International Association of Mathematical Physics News Bulletin, January 2021

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Bulletin Editor

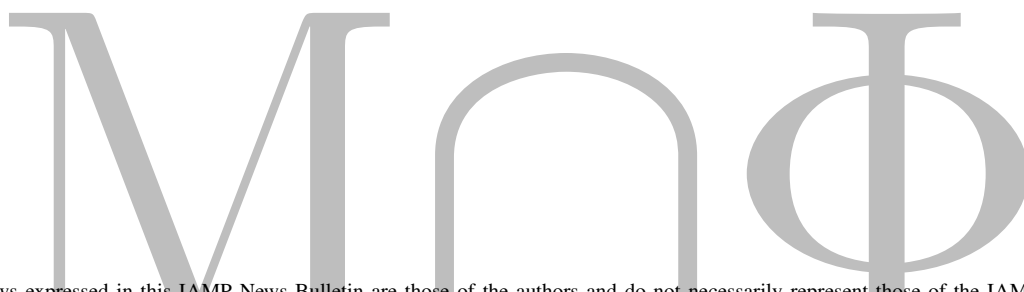
Evans Harrell

Editorial Board

Rafael Benguria, Yasuyuki Kawahigashi,
Manfred Salmhofer, Robert Sims

Contacts. <http://www.iamp.org> and e-mail: bulletin@iamp.org

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News Bulletin (International Association of Mathematical Physics)

Message from the President



Last October you elected the new IAMP Executive Committee (EC), which took office on January 1.

Of the previous twelve-member EC only three members continue for the new term: Sylvia Serfaty (Vice President), Herbert Spohn, and Daniel Ueltschi. They are joined by Dorothea Bahns (Treasurer), Percy Deift, Hugo Duminil-Copin, Michael Loss (Secretary), Bruno Nachtergaele (President), Stephane Nonnenmacher, Marcello Porta, Kasia Rejzner, and Jan Philip Solovej. As you probably know, IAMP limits continuous service on the EC to two terms. Had that not been the case, I would have happily voted in favor of a straight continuation of the previous EC for another term. I believe our predecessors did a terrific job indeed.

The association they handed over to the new team is in vibrant health. I wish to express my deep appreciation to the departing EC members for their dedicated service and wisdom. They are: Yosi Avron, Jan Dereziński, Alessandro Giuliani, Bernard Helffer, Vojkan Jaksic, Mathieu Lewin, Benjamin Schlein, Robert Seiringer, Simone Warzel.

The last three on this list served for two consecutive terms as Secretary, President, and Treasurer, respectively. When they first took office six years ago, I was delighted to see a star team of a new generation of mathematical physicists taking the lead. They did shine brightly and we owe a special thanks to them.

I strongly believe that the most important generation is always the next generation. Over the last decade or so, we saw a list of new IAMP members appear in almost every issue of this *News Bulletin*. Six years ago Robert encouraged all of us to recommend IAMP to junior and other colleagues working in this wide ranging and loosely connected field of ours. In response, the IAMP membership has continued to grow, become younger, and more representative of the full range of topics covered by mathematical physics. I hope this trend will continue and that you will join me in promoting further progress in this direction.

I know that many of you look forward to attending the International Congress on Mathematical Physics (ICMP) in person this summer. The congress will take place in Geneva, August 2-7, 2021. It is too early to tell to what extent the Covid-19 pandemic will have abated to make travel to Geneva feasible, but I am cautiously optimistic. To be on the safe side, the organizers are working to make remote attendance a good alternative possibility. The same holds for the Young Researchers Symposium (YRS) immediately preceding the conference (29-31 July, 2021). You can now register for the ICMP and the YRS, and find more details about the choice of in-person versus remote attendance and other information on the conference website, <https://www.icmp2021.com>.

You will hear more from us in these pages on topics of relevance for our community and IAMP. In the meantime, do not hesitate to contact me with concerns and ideas. I am looking forward to working with all of you and, hopefully, we will meet in Geneva this summer.

BRUNO NACHTERGAELE

Joel L. Lebowitz Honored With 2021 Dannie Heineman Prize for Mathematical Physics



Joel L. Lebowitz, winner of the 2021 Dannie Heineman Prize for Mathematical Physics

WASHINGTON, October 7, 2020 – The American Institute of Physics and the American Physical Society announce Joel Lebowitz, director of the Center for Mathematical Sciences Research at Rutgers University, as the recipient of the 2021 Dannie Heineman Prize for Mathematical Physics. The award is given annually to recognize significant contributions to the field.

“It gives me pleasure to receive the Heineman Prize, and I hope it encourages others to pursue this field,” said Lebowitz, who cites his Brooklyn College teacher, Melba Phillips, and his Syracuse University thesis advisor, Peter Bergmann, as early influencers in his career.

“We are excited to present this year’s Heineman Prize for Mathematical Physics to Dr. Joel L. Lebowitz,” said Michael Moloney, CEO of AIP. “His distinguished international career for six decades made a tremendous impact in the field of statistical mechanics and mathematical physics. We at AIP congratulate him on this win.”

The citation on the award reads: “for seminal contributions to nonequilibrium and equilibrium statistical mechanics, in particular, studies of large deviations in nonequilibrium steady states and rigorous analysis of Gibbs equilibrium ensembles.”

A native of Taceva in the former Czechoslovakia (now in Ukraine), Lebowitz was deported

with his family to the Auschwitz concentration camp in May 1944. He was liberated one year later and moved to the United States, where he began his high school education at Yeshiva Torah Vodath in Brooklyn, New York.

“[Phillips and Bergmann] were both humanist scientists, caring about people and social justice,” Lebowitz said. “As a college senior, I took a reading course with Phillips based on a book on statistical mechanics that was written by Bergmann, which led to my going to Syracuse to do my thesis with him.”

After earning a bachelor’s degree at Brooklyn College and master’s and doctorate degrees from Syracuse University, Lebowitz became a National Science Foundation postdoctoral fellow at Yale University, where his mentor was Lars Onsager.

He worked at the Stevens Institute of Technology, in Hoboken, New Jersey, and Yeshiva University, in New York City, before joining Rutgers University in 1977, a position he currently holds.

Lebowitz was nominated for the Heineman Award by Elliott Lieb, an American mathematical physicist and professor of mathematics and physics at Princeton University who specializes in statistical mechanics, condensed matter theory, and functional analysis.

Lebowitz and Lieb worked together to prove “the existence of thermodynamics for ordinary matter with Coulomb interactions,” the force between two electrically charged particles. It is an accomplishment that Lebowitz said was the most exciting moment of his career.

His work in nonequilibrium statistical mechanics investigates “how macroscopic systems, ranging in size and nature from living cells to galaxies, behave dynamically,” he said.

“Everything alive or active is in a nonequilibrium state. Statistical mechanics tries to understand the properties of such systems in terms of the properties of their microscopic constituents. This leads to difficult mathematical problems, most of which are still unresolved at the present time.”

Lebowitz became the editor in chief of the *Journal of Statistical Physics* in 1975, a position he held for 43 years. He also was editor/co-editor of the *Annals of The New York Academy of Sciences*, *Collective Phenomena*, for 12 years. He has received many well-deserved awards during his career, including, most recently, the “Grande Médaille” from the French Academy of Sciences in 2014.

In addition to his accomplished career as a researcher and published author of work in mathematical physics, Lebowitz is also involved in human rights work. Citing his time as an inmate in a Nazi concentration camp, he believes scientists “can and should strive to have a positive influence.”

“By actively protesting against violations of human rights, in particular those of scientists, students, scholars, etc.” he said. “They should join organizations like the Committee of Concerned Scientists, devoted to the defense of such people. I am a co-chair of that organization.”

His current interests are in problems of nonequilibrium statistical mechanics, a field mentioned in the citation for the Heineman Award.

“I am working on several problems in nonequilibrium involving heat conduction and entropy production in systems in contact with several heat reservoirs at different temperatures,” Lebowitz said. “Problems quite close to what I worked on in my thesis about 65 years ago.”

Once a mathematical physics researcher, always a mathematical physics researcher.

ABOUT THE HEINEMAN PRIZE

The Heineman Prize is named after Dannie N. Heineman, an engineer, business executive, and philanthropic sponsor of the sciences. The prize was established in 1959 by the Heineman Foundation for Research, Education, Charitable and Scientific Purposes, Inc. The prize will be presented by AIP and APS on behalf of the Heineman Foundation at a future APS meeting. A special ceremonial session will be held during the meeting, when Lebowitz will receive the \$10,000 prize. <http://www.aps.org/programs/honors/prizes/heineman.cfm>

ABOUT AMERICAN INSTITUTE OF PHYSICS

The American Institute of Physics advances, promotes and serves the physical sciences for the benefit of humanity. AIP offers authoritative information, services, and expertise in physics education and student programs, science communication, government relations, career services for science and engineering professionals, statistical research in physics employment and education, industrial outreach, and the history of physics and allied fields. <http://www.aip.org>

ABOUT AMERICAN PHYSICAL SOCIETY

The American Physical Society (www.aps.org) is a non-profit membership organization working to advance and diffuse the knowledge of physics through its outstanding research journals, scientific meetings, and education, outreach, advocacy, and international activities. APS represents over 53,000 members, including physicists in academia, national laboratories and industry in the United States and throughout the world. Society offices are in College Park, Maryland (Headquarters), Ridge, New York, and Washington, DC.

For more information, please contact
Larry Frum, AIP Media
American Institute of Physics
+1 301-209-3090
media@aip.org

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Image courtesy of Joel Lebowitz.

2021 AMS-SIAM Birkhoff Prize to Gunther Uhlmann



Gunther Uhlmann, winner of the 2021 George Birkhoff Prize in Applied Mathematics

December 4, 2020: The 2021 AMS-SIAM George David Birkhoff Prize in Applied Mathematics is awarded to Gunther Uhlmann for his fundamental and insightful contributions to inverse problems and partial differential equations, as well as for his incisive work on boundary rigidity, microlocal analysis and cloaking. Uhlmann's work is distinguished by its mathematical beauty and relevance to many significant applications, especially in medical imaging, seismic prospecting and general inverse problems.

Response of Gunther Uhlmann

I would like to thank the AMS and the Society for Industrial and Applied Mathematics (SIAM) for the great honor of being named the recipient of the 2021 George Birkhoff Prize in Applied Mathematics. Several of the previous recipients of the award are some of my mathematical heroes. I would like to also thank my collaborators, graduate students, and postdocs, who have enriched my life both professionally and personally.

Many people were very influential early in my career and I can mention only a few. Nicolas Yus was my undergraduate mentor in Chile and I owe him my thanks for his great support and teaching. Warren Ambrose made it possible for me to go to graduate school at Massachusetts Institute of Technology (MIT), and he was a continuous source of support and encouragement, especially in my early years in the United States. Herbert Clemens also helped me to come to the United States, and he has been an example to emulate in my life. My PhD advisor Victor Guillemin taught me so much - he has a contagious enthusiasm for mathematics. Richard Melrose shared with me many times his great insight, and he has been a true friend. I met Alberto Calderón during my graduate studies at MIT; he is one of my mathematical heroes, such an original mathematician. Norberto Kerzman was also very supportive and encouraging during my graduate studies and we became friends. The year I was at Courant, I had the great fortune of meeting Louis Nirenberg. He taught me many things in mathematics and was one

of the kindest people I have ever met – a wonderful role model for anybody to follow. I also treasured the friendship I started with Cathleen Morawetz during my stay at Courant.

Most of all I have had the unwavering support of my family, my late wife Carolina, my daughter Anita, and my son Eric. Without them this would not have been possible. Carolina would have been so proud. This prize is dedicated to our five grandchildren Thomas, Eli, Louis, Charlie and little Carolina. They are my joy.

Biographical Sketch of Gunther Uhlmann

Gunther Uhlmann was born in Quillota, Chile, in 1952. He studied mathematics as an undergraduate at the Universidad de Chile in Santiago, gaining his Licenciatura degree in 1973. He continued his studies at MIT, where he received a PhD in 1976 under the direction of Victor Guillemin. He held postdoctoral positions at MIT, Harvard, and the Courant Institute. In 1980 he became assistant professor at MIT and then moved in 1985 to the University of Washington, where he was appointed Walker Family Endowed Professor in 2006. From 2010-2012 he also held the Endowed Excellence in Teaching Chair at the University of California and was appointed the Si-Yuan Professor at Institute for Advanced Study, Hong Kong University of Science and Technology in 2014. Uhlmann received a Sloan Research Fellowship in 1984 and a Guggenheim Fellowship in 2001. Also, in 2001 he was elected a corresponding member of the Chilean Academy of Sciences and in 2013 a Foreign Member of the Finnish Academy of Sciences. He was elected to the American Academy of Arts and Sciences in 2009 and as SIAM Fellow in 2010. He was an invited speaker at International Congress of Mathematicians in Berlin in 1998 and a plenary speaker at International Council for Industrial and Applied Mathematics in Zurich in 2007. He gave the AMS Einstein Lecture in 2012, a plenary lecture at the International Congress of Mathematical Physics in 2015 and a plenary lecture at the 2016 Latin American Congress of Mathematics. He was Clay Senior Scholar at Mathematical Sciences Research Institute in 2010 and 2019 and Chancellor Professor at University of California, Berkeley in 2010. Uhlmann was awarded the AMS Bôcher Prize in 2011, the Kleinman Prize by SIAM in 2011 and the Solomon Lefschetz Medal by the Mathematical Council of the Americas in 2017.

Background of the prize

Given every three years, the Birkhoff Prize is awarded jointly by the AMS and the Society for Industrial and Applied Mathematics (SIAM) for an outstanding contribution to applied mathematics in the highest and broadest sense. The prize will be recognized during the 2021 Virtual Joint Mathematics Meetings in January. [Read more and see list of past recipients.](#) Contact AMS Communications at com-staff@ams.org.

The American Mathematical Society is dedicated to advancing research and connecting the diverse global mathematical community through our publications, meetings and conferences, MathSciNet, professional services, advocacy, and awareness programs.

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Lost Luminaries

Mathematical physics lost several luminaries in 2020, and we plan to publish retrospectives of some of their scientific legacies in the pages of the *News Bulletin* in due course, including Freeman Dyson, Jean Ginibre, and Andrew Lenard. (Readers, please inform the Editor of others deserving of similar remembrance.)

In this issue we recall Andrew Lenard, beginning with some memories by Elliott Lieb followed by an old article about one of Lenard's notable contributions, which he never published himself, which gives a sense of his character. We hope for a discussion of Lenard's other scientific contributions in a future issue.



Andrew Lenard (1927-2020)

Photo credit: Chris Howell, Herald-Times

In Memoriam: Memories of Andrew Lenard

by ELLIOTT H. LIEB (Princeton)

It was with great sadness that I learned of Andrew Lenard's death on March 17, and thus the loss of a friend and an important contributor to mathematical physics. Andrew played a large role in my life. I can't recall when or how we met, but it was no later than the early 1960s. We took a quick and a long-term liking to each other. He was a cultured intellectual and spoke Hungarian, German, and French, in addition to polished English. K. Strambach, in his review [1] of Andrew's German paper on the Miquelian Möbius plane, stated that "the style of the paper gives it the rank of a piece of German literature." This, I might say, characterizes his papers in English as well. It was only many years later I learned that Andrew had been on a cattle car to Auschwitz that, luckily, got detoured to a labor camp in Austria.

Andrew's publications cover an unusually broad area and are exceptionally profound and insightful. Although the core of his work is probably the quantum-mechanical many-body problem and statistical mechanics, he went as far afield as differential geometry. He started his career in nuclear physics and scattering theory, but around 1960 he caught the bug of "one-dimensional physics" and wrote many important papers on this subject. To me the most impressive one was the highly non-trivial 1964 proof [2] that the one-dimensional hard core Bose gas has no Bose-Einstein condensation. No one else could find a proof.

It was in 1967, however, that we first impacted each other scientifically. I had calculated the number of "ice" configurations of a square lattice of size N (which turns out to be $(4/3)^{3N/2}$). This is the number of ways arrows can be placed on the lines of a large checkerboard so that out of every vertex exactly two arrows point inward and two point outward. Andrew immediately recognized that this is also the number of ways to color the squares of a checkerboard with three colors in such a way that adjacent squares never have the same color. (For two colors, there are only two ways.) Thus, Andrew solved one of the few non-trivial coloring problems. Being his usual, exceptionally modest self, he left it to others to announce this publicly.

The really big scientific impact, however, came from his two beautiful (and happily, well known) papers in 1967–68 with Dyson [3, 4]. In brief, they broke new ground by proving ordinary matter is stable and this stability occurs only because electrons are fermions and not bosons. While elementary quantum mechanics, in the form of the uncertainty principle (really Sobolev's inequality), accounts for the fact that the negative electrons in an atom do not fall into the positive nucleus, it does not imply that the size of a solid composed of many atoms grows larger rather than smaller in size as the number of atoms increases. Indeed, size decrease would happen if electrons were bosons, which means that their quantum wave-function is required to be symmetric instead of anti-symmetric under permutation of the particle coordinates—as is the case for fermions. The relatively few physicists who thought about the matter somehow intuited this fact, but no one before Dyson and Lenard tried to prove it from first principles. This work opened a new horizon in mathematical physics, conceptually and mathematically.

Andrew's work had a huge effect on me because it set the course for a good part of what I did with the rest of my scientific life—for which I am ever grateful.

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Andrew Lenard: A Mystery Unraveled

by J. PRAUGHT and R.G. SMIRNOV (Halifax)

The theory of bi-Hamiltonian systems has its roots in what is commonly referred to as the “Lenard recursion formula”. The story about the discovery of the formula told by Andrew Lenard is the subject of this article.

1 Introduction

This aim of this review article is to present the untold story about the use of the name Lenard in many concepts that form the backbone of bi-Hamiltonian (multi-Hamiltonian) theory. Originally, the theory came to prominence with the fundamental 1978 paper by Franco Magri [17], followed almost immediately by the 1979 paper due to Israel Gel’fand and Irina Dorfman [13] that developed and extended the results presented in [16] and [17]. Since then, many scientists have been working on the development of the theory of bi-Hamiltonian systems, making it one of the most active areas of research in the field of mathematical physics (see, for example, [1, 2, 3, 4, 5, 6, 8, 9, 13, 14, 17, 18, 19, 20, 24, 25, 26], as well as the relevant references therein).

The majority of the hundreds of papers written to date on the subject are invariably based on the use of such concepts as the “Lenard bicomplex”, “Lenard chain”, “Lenard recursion operator”, “Lenard scheme”, and so forth (see, for instance, [1, 14, 18, 20]). This leads one to believe that Andrew Lenard must have made a fundamental contribution to the theory, yet, as everybody working in the area knows, *no paper on the subject under his name has ever been written*. Although the papers by Clifford Gardner *et al* [12] and Peter Lax [16] contain short paragraphs that strongly allude to Andrew Lenard’s contribution, the whole story told by him (see below) appears to be as fascinating as the result itself. Furthermore, it must be said that the discovery of Lenard had fundamental consequences beyond the theory of bi-Hamiltonian systems that it originated. The notion of a *recursion operator*, introduced by Peter Olver in [23], is not, in general, a byproduct of the existence of two or more Hamiltonian structures. Although the Lenard recursion operator for the Korteweg–de Vries equation comes from the recursion relation (15), more generally, a recursion operator is a property of a symmetry group nature (see [23, 24] for more details), rather than the existence of a bi-Hamiltonian structure.

In what follows, we reproduce¹ the story obtained by one of us (JP) in full, preceded by a brief review of the mathematical background involved. We believe that the results of this historical investigation will be of interest to the scientists working in the area as well as anyone interested in the history of 20th century mathematics.

¹With A. Lenard’s permission.

2 The emergence of the theory

In the last forty years or so, the Korteweg–de Vries (KdV) equation has received much attention in the mathematical physics literature following the pioneering work of Kruskal and Zabusky [28] in the mid-sixties. As is well-known, in this work the authors have reported numerical observations demonstrating that the KdV solitary waves pass through each other with no change in shape or speed. The results presented in a series of papers by Gardner, Green, Kruskal, Miura and those that followed [10, 22, 21, 27, 11, 7, 15, 12], gave rise to the new *theory of solitons* and indicated applications to many areas of mathematical physics that are still being actively advanced today. Thus, for example, soon after the breakthrough of 1965, Gardner, Green, Kruskal, and Miura [10] enriched the theory with another fundamental development; it was a new method later called the *inverse scattering method* (ISM). In a nutshell, the method allows one to find the solution to the nonlinear problem of solving the KdV equation via a series of linear computations. Moreover, by using the ISM, as well as the new technique later called the *Miura transform*, the authors demonstrated the existence of an infinite number of conservation laws for solutions of the KdV equation and explicitly derived several of them [21, 22]. In turn, the existence of an infinite number of conserved quantities for the KdV equation was another important step in advancing the new theory.

Importantly, this discovery provided a framework for the introduction of a new Hamiltonian formalism, suitable not only for studying the KdV equation but also other nonlinear partial differential equations that exhibited similar properties (see [6] and the references therein). It was first shown that the Hamiltonian formalism of classical mechanics could be naturally incorporated into the study of the KdV equation. The consequences of this discovery, reported in 1971, independently and almost simultaneously by Gardner [11] and Faddeev & Zakharov [7], eventually reached far beyond the study of the KdV equation. In what follows we reproduce the main features of the Hamiltonian formalism for the KdV equation and then show how the Lenard recursion formula fits naturally within the general theory.

Consider the KdV equation of the following form:

$$u_t = 6uu_x + u_{xxx}. \quad (1)$$

Gardner [11] and Faddeev & Zakharov [7] observed that the right hand side of equation (1) can be rewritten as follows:

$$u_t = P_0 \frac{\delta H_0}{\delta u(x)}, \quad (2)$$

where H_0 is given by

$$H_0 = \int \left(u^3 - \frac{1}{2} u_x^2 \right) dx, \quad (3)$$

and

$$P_0 = \frac{\partial}{\partial x}, \quad (4)$$

while the expression $\frac{\delta}{\delta u(x)}$ denotes the *gradient* (or, the *Frèchet derivative*). The gradient δ_u acts on any functional $F = F(u, u_x, u_{xx}, \dots)$ as follows:

$$\frac{\delta F}{\delta u} = \frac{\partial f}{\partial u} - \frac{\partial}{\partial x} \left(\frac{\partial f}{\partial u_x} \right) + \frac{\partial^2}{\partial x^2} \left(\frac{\partial f}{\partial u_{xx}} \right) - \dots, \quad (5)$$

where $F(u, u_x, u_{xx}, \dots) = \int f(u, u_x, u_{xx}, \dots) dx$.

One immediately observes that formula (2) bears a striking resemblance to the corresponding formula for a Hamiltonian vector field X_{H_0} in classical mechanics, namely

$$X_H = P_0 dH_0 \quad (6)$$

or, alternatively, $X_H = [P_0, H_0]$, where $[\cdot, \cdot]$ denotes the Schouten bracket. The quantities P_0 and H_0 are the corresponding Poisson bi-vector and Hamiltonian function, respectively, defined on a finite-dimensional manifold M . The triple (M, P_0, X_{H_0}) is said to be a *Hamiltonian system*. The most important feature of the Poisson bi-vector that appears in formula (6) is that it can be used to define the corresponding Poisson bracket, which is the mapping $\{\cdot, \cdot\}_0 : \mathcal{F}(M) \times \mathcal{F}(M) \rightarrow \mathcal{F}(M)$ given by

$$\{f_1, f_2\}_0 := P_0 df dg, \quad (7)$$

where $f_1, f_2 \in \mathcal{F}(M)$ and $\mathcal{F}(M)$ denotes the space of smooth functions on M . The \mathbb{R} -bilinear mapping (7) is skew-symmetric and satisfies the Jacobi identity. As is well-known, these key properties are central to the Hamiltonian formalism of classical mechanics developed for finite-dimensional systems. The most remarkable observation that has been made independently in [11] and [7] is that the differential operator P_0 appearing in formula (2) can be used to define a Poisson bracket as well. Thus, in complete analogy with (7) one defines:

$$\{F_1, F_2\}_0 := \int \frac{\delta F_1}{\delta u(x)} \frac{\partial}{\partial x} \frac{\delta F_2}{\delta u(x)} dx. \quad (8)$$

Remarkably, the bracket defined by (8) is also skew-symmetric and satisfies the Jacobi identity (see [16] for the proof). Equation (1) is rich in conserved quantities, three of which are classical:

$$\begin{aligned} F_0 &= \int \frac{u}{2} dx, \\ F_1 &= \int \frac{u^2}{2} dx, \\ H_0 = F_2 &= \int \left(u^3 - \frac{u_x^2}{2} \right) dx. \end{aligned} \quad (9)$$

In view of formula (5), their corresponding gradients are given by

$$\begin{aligned} G_0 &= \frac{\delta F_0}{\delta u(x)} = \frac{1}{2}, \\ G_1 &= \frac{\delta F_1}{\delta u(x)} = u, \\ G_2 &= \frac{\delta H_0}{\delta u(x)} = \frac{\delta F_2}{\delta u(x)} = 3u^2 + u_{xx}. \end{aligned} \quad (10)$$

The following few “simple” formulas had a paramount impact on the development of a theory which had ramifications and echoes in many areas of mathematics, including differential geometry, the theory of Lie groups, and Hamiltonian mechanics to name a few. The first observation is that the representation of type (2) for the KdV equation (1) is not unique. For example, the operator

$$P_1 = \frac{\partial^3}{\partial x^3} + 4u \frac{\partial}{\partial x} + 2u_x \quad (11)$$

can be used to define *another* Poisson bracket in much the same way as in (8):

$$\{F_1, F_2\}_1 := \int \frac{\delta F_1}{\delta u(x)} P_1 \frac{\delta F_2}{\delta u(x)} dx. \quad (12)$$

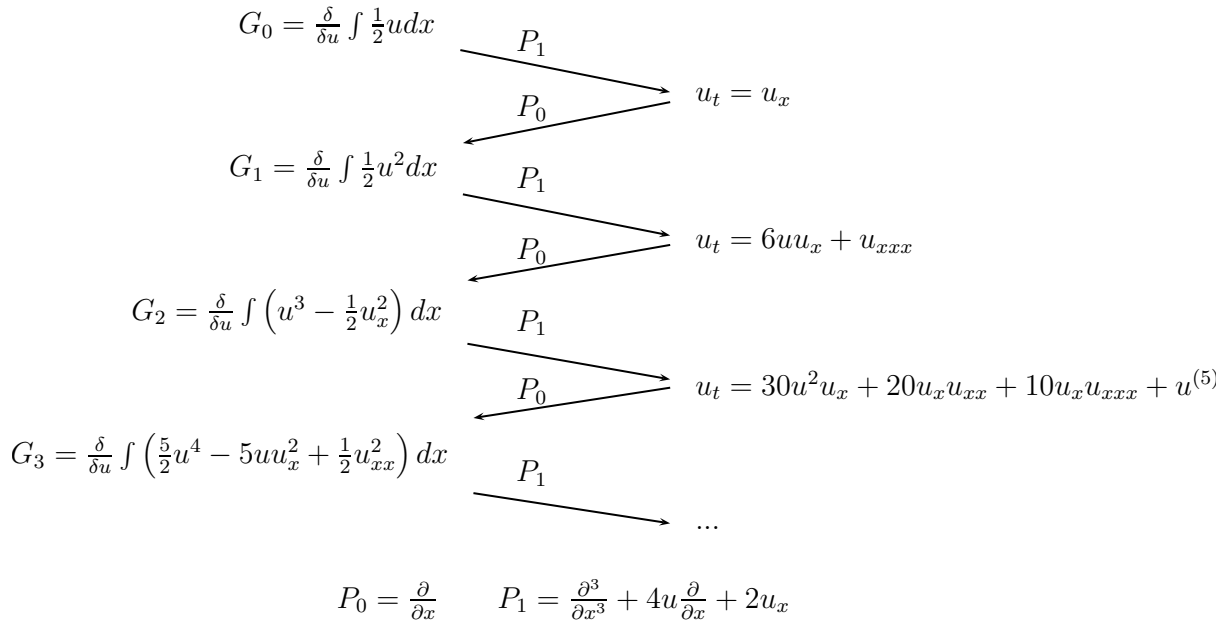


Figure 1: The Lenard recursion formula.

Remarkably, the Poisson bracket $\{\cdot, \cdot\}_1$ is also skew-symmetric and satisfies the Jacobi identity (see [24] for more details and proofs). Moreover, it can be matched with the corresponding Hamiltonian H_1 , that together with (11) yields a formula analogous to (2):

$$u_t = P_1 \frac{\delta H_1}{\delta u(x)}, \quad (13)$$

where H_1 is given by

$$H_1 = F_1 = \int \frac{u^2}{2} dx \quad (14)$$

and the right hand side of (13) is the same as the right hand side of (1). Combining the formulas (2) and (13), one easily arrives at the following important formula which can be viewed as a precursor to the *Lenard recursion formula*:

$$P_1 G_1 = \frac{\partial}{\partial x} G_2 = P_0 G_2. \quad (15)$$

Note that in view of (8), (12), and (15), the functionals F_1 and F_2 (having the gradients G_1 and G_2 respectively) are in involution with respect to both $\{\cdot, \cdot\}_0$ and $\{\cdot, \cdot\}_1$:

$$\{F_1, F_2\}_0 = \{F_1, F_2\}_1 = 0. \quad (16)$$

Furthermore, Lax proved in [16] the existence of more of these conserved functionals F_n , $n = 0, 1, 2, \dots$ exhibiting the property (16). His proof is based on a generalization of the relation (15) which is nothing but the celebrated *Lenard recursion formula*:

$$P_1 G_n = \frac{\partial}{\partial x} G_{n+1} = P_0 G_{n+1}, \quad (17)$$

where the G_n 's are the gradients of the conserved functionals F_n 's. It is easy to check that G_0 , G_1 and G_2 given by (10) satisfy the relation (17). Furthermore, it easily follows from (17) that the functionals F_0, F_1, F_2, \dots corresponding to the gradients G_0, G_1, G_2, \dots generated via (17) are mutually in involution with respect to both Poisson brackets $\{\cdot, \cdot\}_0$ and $\{\cdot, \cdot\}_1$. In addition to the above, the Lenard recursion formula (17) has a number of important consequences, among which we single out the following two:

- The existence of two Hamiltonian representations for the KdV equation (1) gives rise to an infinite sequence of conserved functionals of (1).
- For every $n \geq 1$, the Lenard recursion formula (17) defines a higher order KdV equation, which has the same conserved quantities as the basic KdV equation (1). Therefore, the Lenard recursion formula leads to the *KdV hierarchy*.

We proceed with the following diagram (see Figure 1), which illustrates the properties and consequences of the Lenard recursion formula.

Shortly after this discovery concerning the KdV equation, it was shown by Magri [17] that the property of having two Hamiltonian representations was not a specific feature of the KdV equation alone, but rather a general property that could be found for other nonlinear PDEs with the same remarkable consequences. In his celebrated 1978 paper [17], Magri studied from this viewpoint, the Harry Dym and the modified KdV equations, thus developing a general scheme for studying these soliton equations as bi-Hamiltonian systems. A year later, in another fundamental paper, Gel'fand and Dorfman [13] extended these ideas to the field of finite-dimensional Hamiltonian systems. One cannot help noticing that Hamiltonian formalism originated in classical mechanics and was then applied to the study of soliton equations, while the bi-Hamiltonian formalism travelled in the opposite direction.

And the story began ...

3 Andrew Lenard's story

In this section we reproduce the story told by Andrew Lenard describing the events preceding and following the discovery of the Lenard recursion formula. In order to make the exposition clearer, we refer throughout the story to the corresponding references or the formulas presented in the previous section. Here is the story.

* * *

“Thank you for your communication. It is quite appropriate in the connection of your work, and I shall try to reply as best as I can.

In the earlier part of the 1960s, I was a scientific staff member of the Plasma Physics Laboratory (PPL) operated by Princeton University in conjunction with the Atomic Energy Commission. There, Martin Kruskal was a friend and colleague. He, together with his co-worker Norman Zabusky, discovered an astonishing phenomenon of the KdV differential equation, not until then noticed; namely, that in spite of its non-linear nature, certain wave solutions maintained their shapes unchanged after passing through a time interval of intense non-linear interaction². This was followed by the discovery of a type of “linearization” of the problem by a functional transformation relating it to the 1-dimensional Schrödinger Equation³. In addition, first one and then several simply expressible constants of motion⁴ were found for the KdV evolution equation. Due to the combined work of Clifford Gardner, John Greene, and others, soon an infinite hierarchy of such constants of motion were generated⁵.

I left the PPL at this point to come to Indiana University. However, on a visit back to Princeton during the summer of 1967 (I believe) I went back to the PPL to see my old friends. It was there that something remarkable happened.

I arrived at coffee time in the afternoon. In the common room there were some blackboards. In front of one a crowd was gathered, centered around Kruskal, excitedly discussing something. I went up to ask what it was. They explained that another differential equation, similar to KdV but of higher order, was found, showing all those features of KdV I just described⁶. Someone wanted to know how one could *systematically* discover it, rather than just by lucky hit and miss. I heard Martin Kruskal shout at me: “There must be a method to generate many more, probably infinitely many such higher and higher order DE’s, don’t you think, Andrew?”

I asked for a yellow pad and pen, and went to sit down in a quiet corner to gather my thoughts. I was at that point particularly expert on generating functions as a means of summarizing information about infinite sequences in one mathematical construct. So naturally I tried this idea on the problem at hand, and it worked! It took me only *fifteen minutes or so*, and I could explain to the gathered friends how by means of a generating function an infinite hierarchy of KdV-like DE’s could be generated, all of them having the same kind of behavior⁷.

This was greeted with admiration and satisfaction. I had my coffee and left.

Later, I saw that an article in the Comm. Appl. Math. (Courant Institute, NYU) by Gardner,

²The author refers to the results presented in [28].

³That is, the inverse scattering method [10].

⁴See (9).

⁵See the references [10, 21, 22].

⁶See Figure 1.

⁷See Figure 1.

Greene and Miura and perhaps others, had an appendix on my discovery⁸. I myself *never published anything, nor concerned myself with the subject, then or since*.

Several times during the intervening years I was surprised to hear my name being mentioned in connection with this, but actually much of it in connection with mathematics too high for me to appreciate. For instance, someone once told me that what I discovered was a dynamical system on a symplectic manifold with *two* different Hamiltonian structures⁹. And someone mentioned the “Lenard-Recursion Operator”¹⁰, and asked whether that was the same person as I.

Naturally, I am satisfied that I could make a contribution, even in such a fortuitous and judicious manner as I told you.

I hope this story will be satisfactory for you and answer your questions. By all means, feel free to share it with any like minded person if you care to. I don’t mind it at all if the history of how “Lenard” became a concept in this area will be generally known.

Much good luck for your own studies, and sincerely yours:

Andrew Lenard

P.S. I recall that a mathematician at Dalhousie University (probably retired by now) whom I knew as a friend and colleague when he was at Indiana University during the early 1970s, is Peter Fillmore. Say hello to him for me if you see him.”

* * *

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⁸See the reference [12].

⁹See the formulas (2) and (13) as well as, for example, the references [13, 19, 24] for more details.

¹⁰See [24] for more details.

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IUPAP Conference Sponsorship: Applications deadline: 1 June 2021

The **1st of June 2021** is the due date for the submission of IUPAP sponsorship application forms for conferences that will be held anytime in **2022** (i.e. anytime between 1 January and 31 December 2022). The conference sponsorship rules and guidelines are accessible on the IUPAP website:

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ICMP registration

Dear member of the IAMP,

It is our pleasure to inform you that registration for the International Congress on Mathematical Physics (ICMP Geneva, 2-7 August 2021) is now open.

Due to the uncertainty related to the COVID 19 pandemic, we are currently planning to hold ICMP in physical presence and also to stream talks for participants that are unable to travel to Geneva. This is reflected in the flexible registration options (one for full on-site participation, one for remote participation, with a reduced fee and with the possibility to switch from one to the other).

We would be very grateful if you could promote the Congress amongst your students and network. Please find here the ICMP poster & website links:

https://www.nccr-swissmap.ch/application/files/1516/0510/8694/Poster-A4_Flexible_web.pdf

<https://www.icmp2021.com/>

We hope that you will be able to participate and contribute to the success of this XX International Congress on Mathematical Physics, which will be a major event, where new results and future challenges will be discussed, illustrating the richness and vitality of Mathematical Physics.

Note that the deadline for contributed talks/poster submission and for applications by junior participant for financial support is February 15, 2021.

Thank you,

ANTON ALEKSEEV (University of Geneva)
BENJAMIN SCHLEIN (University of Zurich)
Chairs of the Local Organizing Committee

News from the IAMP Executive Committee

New individual members

IAMP welcomes the following new members

1. PROF. OWEN GWILLIAM, University of Massachusetts at Amherst.
2. DR. MICHELE SCHIAVINA, ETH Zurich, Switzerland.
3. MR. BEREND VISSER, University of York, UK.
4. PROF. CHRIS BOURNE, Tohoku University, Japan.
5. MR. BEN WOOTTON, Taringa, Australia.

Open positions

Postdoctoral Fellowship in Mathematical Physics in Prague

The Center of Advanced Applied Sciences of the Czech Technical University in Prague announces a postdoctoral position in mathematical physics in the research team “Geometry and spectral properties of quantum systems”.

The position is for one year, available immediately, with the possibility of extension for a second year upon mutual agreement. The gross salary is approximately 45 000 CZK monthly before tax. There are no teaching duties associated with the position.

Applicants should have a PhD in mathematics or theoretical physics (or equivalent) obtained preferably after January 1, 2017. They must show strong research promise in at least one of the following research domains: operators; Jacobi and other structured matrices, orthogonal polynomials; spectral theory; partial differential equations; geometric analysis.

An experience in the project topic area is an advantage but not necessary.

The applications including

1. curriculum vitae (including list of publications);
2. brief research statement (past, current and future interests);
3. two letters of recommendation;

should be sent by e-mail to Pavel Exner (exner@ujf.cas.cz) and Pavel Stovicek (stovicek@fjfi.cvut.cz). All documents should be submitted as pdf files. The letters of recommendation should be sent directly by the persons providing the reference.

Complete application packages should arrive before February 28, 2021; the application will be evaluated and the decision taken in the order of arrival. For any further information about the position please contact Pavel Exner and Pavel Stovicek at the e-mail addresses above.

Postdoctoral Fellowship in Mathematical Physics in Göttingen

A three-year postdoc position “Emmy Noether postdoc” in Mathematics or Mathematical Physics is available at the Mathematical Institute in Göttingen, Germany. The full description of the position and information on how to apply can be found here:

<https://www.uni-goettingen.de/de/305402.html?cid=100799>

(English version towards the end of the page). The deadline for applications is February 5, 2021. For further information please contact Dorothea.Bahns@mathematik.uni-goettingen.de.

For an updated list of academic job announcements in mathematical physics and related fields visit

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Call for nominations for the 2021 IAMP Early Career Award

The IAMP Executive Committee calls for nominations for the 2021 IAMP Early Career Award. The prize was instituted in 2008 and will be awarded for the fourth time at the ICMP in Geneva in July 2021. The prize is sponsored by Springer. The Early Career Award is given in recognition of a single achievement in mathematical physics. The total prize value amounts to 3000 Euro and is reserved for scientists whose age in years since birth on July 31 of the year of the Congress is less than 35. The nomination should include the name of the candidate accompanied by a brief characterization of the work singled out for the prize. Members of the IAMP should send their nomination or nominations to the President (president@iamp.org) and to the Secretary (secretary@iamp.org). A list of previous winners and the details of the award selection process can be found at: <http://www.iamp.org>.

Nominations should be made not later than on January 31, 2021.

Michael Loss (IAMP Secretary)

ELLIOTT H. LIEB

Departments of Mathematics and Physics
Princeton University
Princeton, NJ 08544, USA
lieb@math.princeton.edu

ROMAN G. SMIRNOV

Department of Mathematics and Statistics
Dalhousie University
Halifax, Nova Scotia, Canada, B3H 3J5
smirnov@mathstat.dal.ca

ANTON ALEKSEEV

Section de mathématiques
24, rue du Général Dufour
Case postale 64
1211 Geneva 4, Switzerland
secretariat-math@unige.ch

BENJAMIN SCHLEIN

Institut für Mathematik
Universität Zürich
Winterthurerstrasse 190
8057 Zürich, Switzerland
benjamin.schlein@math.uzh.ch

MICHAEL LOSS

School of Mathematics
Georgia Institute of Technology
Atlanta, GA 30332-0160, USA
secretary@iamp.org

EVANS HARRELL

School of Mathematics
Georgia Institute of Technology
Atlanta, GA 30332-0160, USA
bulletin@iamp.org

BRUNO NACHTERGAELE

Department of Mathematics
University of California
Davis, CA 95616-8633, USA
president@iamp.org