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Cover photo: Waves (and wavelets) in the Mediterranean Sea in Marseille.

Know thy neighbor

by Pavel Exner (IAMP President)

One of many attempts, usually futile, to define a human says that it is an intersection of relations. Even in a narrow scientific sense hardly any of us can define himself or herself as a "pure" mathematical physicist. And a plant grows best if we take care of all its root system.

Looking back at the IAMP history we see various forms of collaboration with our scientific cousins which kept changing over the time. We had our partner delegates to our congresses, we had a society which became our Associate Member being much larger than we are. An important factor was always a personal relation to the partners and the relation usually persisted as long as this link remained strong.

This all we had in mind when we started talking to the European Mathematical Society about a cooperation agreement trying to determine common interests. We found them in exchanging and promoting information and in supporting professional development of our members. This was formulated in a document which the Executive Committees on both sides approved and I had the pleasure to sign together with Marta Sanz-Solé, the EMS president.

One of our declared common intentions is to hold schools on subjects of common interest, say every second year. As an optimal environment for such meetings we see institutes which are our Associate Members and belong at the same time to ERCOM, the family of European research centers in mathematics associated with the EMS. At present we are discussing the possibility of having the first such school next year.

The information exchange means in the minimal sense announcements swaps. For instance, the EMS Newsletter will publicize our next congress in Aalborg and in turn, in a later issue of our Bulletin we will inform our members about the next European Congress of Mathematics which will convene next June in Cracow. However, the exchange will not be limited to announcements. We also made an agreement about exchange of articles between the respective publications, and in the present Bulletin issue you will find the first example, an interview with Yves Meyer which our editorial board picked from the June issue of the EMS Newsletter.

The main importance of the concluded cooperation agreement I see in the possibility to use it as a model to strengthen our relations with other partner societies, both topically and geographically. We will keep exploring these paths, even if it is a long run and the continuation of the effort will be in hands of the next Executive Committee.

An EMS interview with Yves Meyer, part I

The IAMP News Bulletin wishes to extend to the IAMP membership this reprint made earlier for EMS members. We publish here (and in the next issue) a complete version of an interview, originally taken by Ulf Persson (Göteborg, Sweden) for the European Mathematical Society, whose excerpt has appeared in the EMS Newsletter, June 2011, issue 80. The IAMP News Bulletin is very thankful to Yves Meyer for the complete original file of his interview as well as to Ulf Persson for authorisation to reproduce all his questions.

The French elitist educational system, especially as when it comes to mathematics. Its pros and cons.

Since Fields medals exist all the French Fields medallists (excepting Alexander Grothendieck) have been alumni of the "École Normale Supérieure de la rue d'Ulm" (which I will abbreviate as ENS-Ulm). This is a striking fact. There are a few other Écoles Normales. The one where I am teaching (ENS-Cachan) is less prestigious than ENS-Ulm and more oriented to applied mathematics. In France applied mathematics has been less prestigious than pure mathematics and technology has been less prestigious than science. This theme will be present all along this essay. The main difference between ENS-Ulm and say Harvard or Princeton is the following. You have already spent two years of intense training in maths and physics when you take the entrance exam. There are other options in ENS-Ulm, as biology and humanities, with completely different entrance exams. That is the main difference with American Universities. You have already chosen a field of knowledge when you enter. If you enter ENS-Ulm, you know that you shall give up money and power definitely. It is a choice of life. Your life will be devoted to acquiring and transmitting knowledge. I entered ENS-Ulm in 1957 and only 40 students in Sciences and 37 in Humanities were admitted. The entrance exam to ENS-Ulm was quite selective. Many among my schoolmates were expecting to become high school teachers. I will later explain why in those times being a high school teacher was so rewarding. The brightest among us undertook some research work.

The French elitist educational system was designed soon after 1789 to fight against the extravagant privileges of the French nobility. This was the goal of the French Revolution. But before describing our elitist system, let us stress that it only applies to a tiny part of higher education. High school is not selective in France. High school ends with an exam called the "baccalauréat". This exam is quite easy since eighty per cent of the candidates will succeed. Moreover every teenager who obtains a "baccalauréat" is accepted in the university of his choice. The best among them do not go to University but instead take medicine or the training classes for elite schools. Tuition does not exist. But our open admission system is hypocritical since half of the undergraduate students drop out during the first year of college. This is typical of France, a country in which noble ideals come along with a poor management and no attention is paid to the details. In contrast with this loose system, medicine and engineering are highly selective in France. There are more than two hundred engineering schools, and most of them are not affiliated to any university. Medicine is a cursus which is given inside the university but follows completely distinct rules. There is a strict numerus clausus. The number of students admitted in Medicine after the first year's exam ranges from 4,000 to 8,000 depending on the job market. The list of the French elite schools needs to be completed with École Nationale d'Administration and business and management schools, like *Hautes Études Commerciales* (HEC). The number of students admitted to one of these elite schools is about ten per cent of these who are entering a University. You cannot enter one of the elite schools unless you accept to endure two or three years of intense training after the *baccalauréat*. This training is given in what we call "Classes de Préparation aux Grandes Écoles". At the end of this training you shall take one of the exams and if you succeed you are allowed to enter the corresponding elite school. In sciences ninety per cent of the students from *Classes Préparatoires* will enter one of the engineering schools. The students who fail move to the University and are admitted directly in the second or third year of College.

The French elitist system is a democratic system, based upon strictly anonymous written exams. Every candidate is labeled by a number. The professors who are rating the written exams do not know the names of the candidates. The choice of the most talented candidates is completely fair and no biases are introduced. However this system is much criticized nowadays. It is accused to reproduce an elite. The criticism is grounded on the following fact. The family of the candidate is often playing an important psychological role in helping the candidate to accept the intense training which is needed to enter an elite school. But people which are making this criticism against the French elitism are confusing economical status and intellectual level. When they are defining an elite they claim that a primary school teacher with a rather low income belongs to the elite, which is simply ridiculous. If some parents are very demanding to their children, such a family is viewed as being favored.

I am returning now to the situation in French high schools. Let me begin with figures. In 1957 only 9,500 boys and girls got a *baccalauréat* in sciences. *Baccalauréat* is the exam you take when you leave high school. *Baccalauréat* is also the ticket you need for entering a university. These figures did not meet the needs of a developed country, and in 1959 Général de Gaulle ordered a reorganization of French high schools. De Gaulle's reform worked, since nowadays the figures are raising to 146,300 which is very good, even if the level of *baccalauréat* in sciences is much lower than it was in 1957. Before 1959 children at the age of eleven had to take an exam for entering high school and the family played a key role in the success. Nowadays high school begins later on, at the age of fifteen, and is almost open admission which explains the rise from 9,500 to 146,300 in sciences. It is a success but a price had to be paid. Here comes the bad news. Before entering high school all the French children between eleven and fifteen shall receive exactly the same education in a structure named "Collège d'Enseignement Secondaire". Unlike Germany and the United States, France has always been willing to build a completely egalitarian educational system. These "Collèges d'Enseignement

Secondaire" were created by Jean Berthoin in 1959 (De Gaulle happened to be President of France at the end of 1958) and modified in 1963 and 1975. The goal is to offer all the children some basic education so that they can decide what they want to be at the output. After attending "Collège d'Enseignement Secondaire" the teenagers either enter a traditional high school (general education), a technical high school, or begin professional training. But "Collège d'Enseignement Secondaire" is the place where our educational system is facing the most serious problems. The children who are receiving exactly the same teaching up to the age of fifteen have already developed extremely different interests and skills. Many children are just bored by what is being taught and are becoming violent. Technology is not taken seriously in our Collèges d'Enseignement Secondaire. Technology is not given in French education the importance it deserves.

Let me mention here an extraordinary endeavor to improve the interest of children in science and technology. Georges Charpak, Nobel prize in physics, launched "la main à la pâte" ("hands in the dough") to reform the teaching of natural sciences in French primary schools. This program is supported by the French Academy of Sciences. "La main à la pâte" is defined by Yves Quéré in the following terms:

> *The strategy was founded on a simple priority: to use science to support the child's mental development. This meant inculcating a taste for questioning, a sense of observation, intellectual rigor, practise with reasoning, modesty in the face of facts, an ability to distinguish between true and false, and an attachment to logical and precise language.…The Swedish Academy, and a number of others, followed parallel paths.*

Let me stress that "la main à la pâte" is not an elitist approach to education. Quite the opposite. Indeed it was successfully initiated in a black ghetto of Chicago. Mathematics is not taken into account by "la main à la pâte", and the official explanation is that mathematics is taught with care and efficiency in France.

The Fields medalist Laurent Lafforgue carefully analyzed the flaws of education in France and concluded that they are due to a lack of courage and ambition of the French government. Laurent Lafforgue proved that a century ago the French school systems achieved more with children who were not prepared for primary school since most of them could not speak French. Indeed many dialects or other languages were spoken in the country in the early nineteenth century. Following Lafforgue's views, French education is seriously ill. A cheap trick which is used now in France is to tune the educational level to what can be acquired without much pain by the average children during these critical years at Collège d'Enseignement Secondaire. The teachers are not very demanding. An obvious consequence is the deterioration of education in French high schools. The disease is propagating from Collège d'Enseignement Secondaire to high school. Our "Collèges d'Enseignement Secondaire" are failing to achieve their ambitious goals and every four year a new Minister of Education comes with a brillant idea to fix these problems. In our fully centralized organization it is not permitted to adapt what is taught in a given school to the needs of the children attending that school.

The lowering of education in French high schools implies that the intellectual level which is required for entering an elite school depends more on what is learned inside the family than was the case fifty years ago. The criticism made by the French sociologist Pierre Bourdieu against the French educational system as leading to a "reproduction of an elite" is absolutely fair. But the disciples of Pierre Bourdieu who are making these criticisms are exactly the same who are in favor of our "Collège d'Enseignement Secondaire". It is a vicious circle.

The problems I am mentioning do not affect France only. A UNESCO program is aimed at understanding the role of "shadow education" on the achievements of children at school, as far as mathematics is concerned. Shadow education means the intellectual training a child is receiving from his family. A specialist of these issues is Georges Haddad (g.haddad@unesco.org).

These difficulties which are serious cannot be solved by the suppression of ENS-Ulm for the sake of egalitarism. It would be the same disaster as the abolition of Harvard or Princeton. Let us observe that gender or ethnic quotas are unlawful in France. For a long time there were two separate Ecoles Normales Supérieures, one for boys and one for girls. Now they are fused together and the number of girls entering has been divided by four, as compared to the numbers fifty years ago. It is a disaster.

Education in France relies on some intellectual and moral traditions, as is the case in every country. France cannot be fully understood if you do not accept the skepticism of Montaigne, if you do not know the philosophy of Descartes or the fights of Voltaire against the exorbitant power of religion in the eighteenth century. These moral and intellectual values which have shaped France are becoming meaningless to many teenagers of the 21st century. Moreover these values are questioned by some of our new immigrants who are not willing to accept what they view as a cultural imperialism. A new humanism should be built and combine the moral and intellectual values of France together with the Jewish and the Muslim inheritances. I have never heard of Rachi of Troyes (1040-1105) at school. At Tunis where I was attending high school I have never heard of Ibn Khaldoun, which is a shame since he was born at Tunis in 1332. Ibn Khaldoun was one of the major intellectual figures in medieval Islam. Is France ready for recognizing our debt to Rachi de Troyes and to Ibn Khaldoun ? Is Europe ready to acknowledge that we inherited from a brillant Islamic culture ? This reconciliation is urgently needed for reshaping France.

There are some serious drawbacks in the French elitist educational system. I will focus on one of them. From 1976 to 1986 I was teaching at École Polytechnique, which is the prototype of a French elite school. Most of the students of École Polytechnique want to become bosses of some important companies. In 1981, during one of my classes, I evoked the scientific and technological challenges that France was going to meet. I was delivering this passionate message to the full class consisting of 460 students. They laughed loudly. I interrupted my speech and remained silent and embarrassed. One student sitting in the first row stood up and told me the following: "If we ever had a technological problem to be solved during our professional life, we would order an engineer from École Centrale (a less prestigious engineering school) to do it. We are born to command." We are back to the absurd privileges of the French nobility. The hierarchy between the French engineering schools is extremely rigid, and explains the arrogant attitude of the students of École Polytechnique. But the same hierarchy exists between the top American universities. It exists nowadays between universities all over the world, since the Shanghaï ranking was introduced. But in the States the elitist attitude is compensated by the fact that you will soon be evaluated on your own achievements and not on your university's degree. In France your career is governed throughout your life by what you did when you were twenty years old. In France you are rarely been given a second chance in your life. Some people try to change this rigid attitude. Nowadays the traditional ranking of the scientific elite schools is challenged by new schools oriented to management and finance. It implies that École Polytechnique might eventually lose its privileges and be forced to adapt to a new environment.

As I understand you started out as a teacher, was that in personal terms an advantage? Not starting out with research inhibited by high expectations ?

To understand my decision to begin my life as a teacher, I need to evoke what was my country in the late fifties. A terrible war was raging in Algeria. The French army answered the legitimate demands of Algerian nationalists with torture and napalm. All young French men were drafted unless they were graduate students. In other words beginning a Ph.D. was a cheap trick for avoiding to be drafted. Otherwise you were forced to fight a cruel and unjust war. Could I claim that my research, which did not exist at that time, was so important to my country that I should be exempted ? No, certainly not ! Beginning a Ph.D. to avoid being drafted would be like marrying a woman for her money. I wanted to fall in love with research, not to use it as a clever way to obtain a privilege. I confessed to the Army I was not preparing a thesis. Taking this decision, I felt I was showing some solidarity with my age cohort. This looks childish today. I was drafted. Then I asked to be a teacher in a military school, a decision which was encouraged by the Army. It was a way of doing your military service. I was sent to the Prytanée Militaire de La Flèche. La Flèche is a tiny city located at two hundred miles southwest of Paris. This school was formerly a college run by the Jesuits. It is the place where the French philosopher Descartes studied. Nowadays this high school belongs to the Army. It was dedicated to the education of male children of the officers serving in Algeria or in Germany. Germany was at that time divided into four occupation zones. One zone was occupied by France. I taught there three years, which included two years of military service. Of course I was not paid, which was the reason the Army liked this arrangement. I was twenty one years old and my students were seventeen or eighteen. The students were separated from their family and were receptive to my sympathy. Many of my students became professionnal mathematicians and are now my friends. I was eventually sent to Algeria during a short period, from June 1962 to September 1962. I arrived at a place where fortunately the fighting was over. This happened during the summer and my students did not suffer too much from the absence of their teacher. My way of teaching was evaluated two times by truly experienced specialists (Inspecteurs Généraux). hese experts told me that I was not a good teacher. Indeed a good teacher at the high school level needs to be much more methodic and organized than I was. These Inspecteurs Généraux advised me to apply to the University. I had other problems with my teaching. I eventually felt guilty to be the only one who is always right while kids are wrong most of the time. In some sense I needed to feel the pain of learning that my students endured. To do research means to be ignorant most of the time and to often make the many mistakes I criticized in correcting my student's home works. Socrates makes very clear that he needs a discussion with his friends to discover the truth. Truth is never given to him as a gift from God; truth needs to be elaborated through a collective work. It was not the way I was teaching. It was to be the way I would later to work with my research students. In 1963 I applied for a position at Strasbourg as a teaching assistant. I was 24 years old. I wrote my Ph.D. there.

My experience of teaching in a high school shaped my entire life. I understood that I was more happy to share than to possess. If I read a beautiful novel, I want to share my pleasure with someone. Supervising a thesis has always been a most rewarding experience. It means giving my Ph.D. student the best of myself. I always hoped that she/he eventually would become a better mathematician than I am. This happened to quite a few of my fifty graduate students. My first research student was Aline Bonami. I keep very strong ties with my former students, as in a family.

Beginning a career as a high school teacher should not be proposed as a model. The labor market was much more open in the early sixties. When I decided to switch to the university, I immediately found a position as an instructor.

You have moved freely between different disciplines in mathematics. Is there a danger of being trapped in a narrow subfield and a loss of inspiration that too many professional mathematicians fall prey too ?

Have I ever said: "Whenever you feel competent about a theory, just abandon it!" as quoted by Professor Ramachandran? It looks so arrogant. My changes of orientation need to be explained more seriously. I am not smarter than my colleagues who have found their home inside mathematics. I changed several times. I have always been a nomad. I cannot stand more than ten years in a department of mathematics. I moved form Strasbourg to Orsay, then to École Polytechnique, then to the University of Paris-Dauphine and finally to ENS-Cachan. I also spent two years at the University of Nantes. I do not move because I have problems with my chairman or I am offered a better job. I am moving because I cannot prevent myself from moving. It is a kind of disease. I do not belong to a University and I suffer from this inability to stay. I am from Tunis (Tunisia). Tunis in the fifties could be compared to Trieste, the Trieste of Italo Svevo, Umberto Saba, or Claudio Magris, the Trieste of Eduardo Weiss, a city where the Austrian influence was feeding the Italian literature. The Tunis of my childhood was a melting pot where people from all over the Mediterranean sea had found a peaceful exile. Italians, Maltese, Sephardim, Berbers, Arabs and French were living together. As a child I was obsessed with the desire of crossing the frontiers between these distinct ethnic groups. But I was limited by my ignorance of the languages which were spoken in the streets of Tunis. I came back to France when I was seventeen. I am not rooted in France. My sister who is one year older could not stand the France of the war in Algeria and moved to Morocco after graduating. The natural choice would have been Tunis but it was the time when Habib Bourguiba and Général de Gaulle initiated an absurd and bloody fight about Bizerte, a major naval basis in Tunisia. Cooperation agreements were cancelled, and that is why my sister went to Morocco. She spent her whole life teaching there. She speaks Arabic. When I am visiting

Spain, I feel like a Spaniard. I know the language, the literature and the terrible history of the Spanish Civil War. When I am in Madrid I read *El País*. I read *The New York Times* in the States. In my professional life I obsessively tried to cross frontiers. I am excited by the move to a new math department. When I move I know I am going to meet new colleagues. We will work on a new subject. For instance when I was hired at Ecole Polytechnique, I was encouraged by Charles Goulaouic to switch from harmonic analysis to partial differential equations. The byproduct of my influence on PDEs was the theory of para-differential operators developed by Jean-Michel Bony (see the blog of Terry Tao: http://terrytao.wordpress.com/2010/08/20/spielman-meyer-nirenberg/). When I was hired at Paris-Dauphine, I began an exciting collaboration with Pierre-Louis Lions. We proved a "div-curl lemma". Let me say how this happened. Pierre-Louis entered my office and asked me if the following was true: if both the vector fields $B(x)$ and $E(x)$ belong to $L^2(\mathbb{R}^n)$, if $E(x)$ is divergence free and if the curl of $B(x)$ vanishes identically, then the pointwise inner product $E(x) \cdot B(x)$ belongs to the Hardy space $\mathcal{H}^1$. I replied to Pierre-Louis: "No, it cannot be true, otherwise I would have known it." Pierre-Louis was surprised by my arrogance. I spent the night working on it and the following morning I had a proof. Pierre-Louis had a vision, to which I brought my technical skills. I had similar experiences with Raphy Coifman and Alberto Calderón. They had splendid visions. I was fascinated as a child. I was guided by these stars shining in my night sky. I then worked hard and could prove what they were dreaming about. These stories contradict the way I wrote my Ph.D. I did not have a supervisor. I am full of contradictions.

Here are some examples of moves I remember with nostalgia. In 1972 I was visiting the Hebrew University at Jerusalem. I wanted to work with Yitzhak Katznelson. But Katznelson was suffering some terrible headaches and I discussed with Benjamin Weiss instead. We followed a line of research pioneered by D. Ornstein and proved that Riesz products are Bernoulli shifts. A Riesz product, when constructed with powers of 3, is a specific probability measure on the circle group which is ergodic with respect to the mapping $x \mapsto 3x$. Ergodicity is easily checked. Benjamin and I proved that this ergodic system is isomorphic to a Bernoulli shift. I was rather ignorant of ergodic theory when I began that work. We adapted the method used by Katznelson for automorphisms of the n-dimensional torus. Benjamin and I never published this paper, which I regret today. This example illustrates my way of doing research: dive into deep waters and do not be scared to swim against the stream. But you can only do it if you are working with someone who is an expert of the field you are entering. You should never do it alone! Otherwise the best you will achieve is "to rediscover the wheel". In this work with Benjamin I brought my experience in harmonic analysis and he provided us with his deep understanding of ergodic theory.

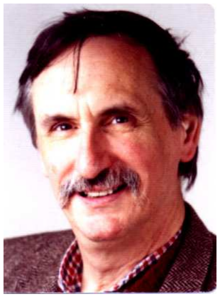
A similar experience happened in June 1974. I was visiting Washington University, at Saint Louis, Missouri. I was invited there by Guido Weiss and I expected to work with him. But Guido was busy with difficult decisions to be taken by Washington University. Raphy Coifman entered my office the first day I was there and told me we should attack Calderón's conjectures. At that time I did not know what a singular integral operator might be. I was completely unaware of Calderón's program, but I accepted. Working day and night during the two months of my visit, Raphy and I were eventually able to prove the boundedness of the "second commutator". This spectacular progress was immediately announced by Charles Fefferman in his one hour address at the ICM at Vancouver (August 1974). It took us seven more years to prove the full set of Calderón's conjectures. Later on in this essay I will return to our solution.

Concerning wavelets, the story is quite strange. …

(… *to be continued in the next issue of the bulletin* …)

A mathematical theory of quantum effects and phase transitions in solids

by Sergio Albeverio & Yuri Kozitsky
(Bonn, Germany & Lublin, Poland)




Sergio Albeverio is Emeritus Professor and member of the Hausdorff Center of Mathematics at the University of Bonn, Germany. Yuri Kozitsky is a professor of the Institute of Mathematics at Maria Curie-Sklodowska University in Lublin, Poland. Sergio Albeverio received his PhD, directed by R. Jost and M. Fierz, in 1967 at ETH, Zürich. Since then he worked at various universities, including Imperial College (London), Princeton, and the Universities of Marseille, Naples and Oslo. From 1977 on he had permanent professor positions in Bielefeld and Bochum, and since 1997 in Bonn. He had also research and teaching positions at many Universities in Europe, China, Japan, Mexico, Russia and U.S.A. In 1993 S. Albeverio received the Max- Planck - Research Prize (with Z.M.Ma and M. Röckner), and he was named Doctor honoris causa of the University of Oslo (2002, on the occasion of the Bicentennial of N. H. Abel). He has made contributions to most areas of mathematics, from probability theory to analysis, mathematical physics, geometry, algebra and number theory. Also worked on applications in biology, economics, engineering, physics, architecture. He has over 750 publications in scientific journals, 9 research monographs and 30 books of proceedings, and is among the world wide highly cited scientists.

Yuri Kozitsky received his PhD in theoretical and mathematical physics, directed by I. R. Yukhnovskii, in 1981 at the Institute for Theoretical Physics in Kiev, Ukraine. Since then he held various university positions in Lviv, Ukraine. In 1996 moved to Lublin, Poland to take a professor position at Maria Curie-Sklodowska University. His research areas include mathematical problems of quantum statistical mechanics, operator theory, dynamical systems. Their collaboration goes back to 1997, and resulted in a joint monograph (with Y. Kondratiev and M. Röckner) published in 2009 by the European Mathematical Society.

Quantum Fluctuations and Collective Phenomena

Understanding what differentiates the collective behavior of a large *quantum* system from that of its *classical* counterpart is a challenging problem of mathematical physics. Intuitively, it is clear that intrinsic randomness of quantum systems should enhance thermal fluctuations in their work against ordering. However, in many cases, the mathematical mechanism of such quantum effects remains unclear. It turns out that for a certain infinite particle system, this mechanism can be described in detail, qualitatively agreeing with relevant experimental data. We call it *quantum stabilization* of quantum anharmonic crystals.

An anharmonic oscillator is a mathematical model of a point particle moving in a potential field with multiple minima and sufficient growth at infinity. In the simplest case, the particle motion is one-dimensional and the potential has two minima – wells, separated by a potential barrier. If the motion is governed by the laws of classical mechanics, in the low energy states the particle is confined to one of the wells, producing a degeneracy – the multiplicity of states with the same energy. Imagine now that an infinite system of such particles is arranged into a crystal. That is, each particle is localized in the vicinity of its own crystal site. Suppose also that the particles interact with each other. The corresponding model, called an *anharmonic crystal*, is often used in solid state physics, e.g., in the theory of ferroelectrics or apex high-temperature superconductors. For this model, the low-temperature equilibrium thermodynamic states (phases) can also be multiple. If this is the case, as the temperature changes the anharmonic crystal undergoes a phase transition – a collective phenomenon caused by the interaction and by the degeneracy mentioned above. If the particles move according to the laws of quantum mechanics, they can tunnel through the potential barriers. This tunneling motion eliminates the degeneracy, which might affect the ability of the phase transition to occur or even suppress it completely. Such quantum effects were first discussed in [10]. Later on, a number of publications on this topic appeared, see [8, 11, 12, 13] and the literature quoted therein. The key conclusion of those works is that the quantum effects become strong whenever the particle mass $m_{\rm ph}$ gets small. This agrees with the experimental data on the isotopic effect in hydrogen bound ferroelectrics [4] and in YBaCuO-type high-temperature superconductors [9]. At the same time, experiments show that high hydrostatic pressure applied to a ferroelectric crystal diminishes the phase transition temperature, see e.g. the table on page 11 in [4]. Hence, the reduction of the distance between the wells amplifies quantum effects. From this one concludes that also the shape of the localizing potential plays an important role here.

In the mathematical theory of classical anharmonic crystals, a thermodynamic phase is a Gibbs measure defined by means of the *Dobrushin-Lanford-Ruelle (DLR) equation*. In view of the absence of explicit definitions of such phases for quantum crystals, the results mentioned above were based on rather indirect indications, e.g., the lack of symmetry breaking [12] or of an order parameter [8]. It soon had become clear that a mathematical theory, which describes phase transitions and quantum effects in a unified way, as well as takes into account also the role of the localizing potential, ought to be based on a precise definition of thermodynamic phases. Such a theory was elaborated in a series of works, see e.g., [1, 2, 5, 7] and the citations therein. The key aspect of this theory is the use of path integral techniques, in which the phases in question are constructed as probability measures on path spaces that solve the corresponding DLR equations. Thereafter, necessary and sufficient conditions are derived for a phase transition to occur, containing the particle mass $m_{\rm ph}$, the interaction strength $\hat{J}_0$, and a localizing potential parameter υ_*. Roughly speaking, both conditions can be expressed through one and the same parameter $Q = 4(m_{\rm ph}/\hbar^2)\upsilon_*^2\hat{J}_0$, and have the following surprisingly simple form: (a) $Q > d\theta(d)$ (sufficient); (b) $Q \geq 1$ (necessary). That is, if $Q < 1$, at all temperatures the crystal is stable against ordering. Here $\theta(d)$ is a certain function of the lattice dimension d, such that $d\theta(d) > 1$ and $d\theta(d) \to 1$ as $d \to +\infty$.

A self-contained presentation of path integral methods in the statistical mechanics of systems of interacting quantum anharmonic oscillators is given in the monograph [3], addressed to both communities – mathematicians and physicists. The reader can find here a collection of facts, concepts, and tools relevant to the application of functional-analytic and measure-theoretical methods to problems of quantum statistical mechanics. In particular, a complete description of the theory mentioned above, as well as of its far-going extensions and implications, is given. In the remaining part of this article, based on [3] and written in cooperation with Y. Kondratiev and M. Röckner, we outline some of its details.

Phase Transition and Quantum Stabilization in More Detail

For simplicity, we will speak about the model described by the following Hamiltonian:

$$H = -J \sum_{x \sim y} q_x q_y + \sum_x H_x, \quad J > 0, \tag{1}$$

where the first sum is taken over all pairs of nearest neighbors x, y in the 'hypercubic' crystal lattice $\mathbb{Z}^d$, whereas the second sum runs over all $x \in \mathbb{Z}^d$. The first term describes the interaction between the particles which is taken to be of dipole-dipole type with intensity J, typical for protons which form hydrogen bounds in the KDP-type ferroelectric compounds, see [4]. We recall that the dipole moment is proportional to the particle displacement q_x. The single-particle Hamiltonian is taken to be of the form

$$H_x = H_x^{\rm har} + V(q_x) := \frac{1}{2m} p_x^2 + \frac{a}{2} q_x^2 + V(q_x). \tag{2}$$

Here $m = m_{\rm ph}/\hbar^2$, $H_x^{\rm har}$ stands for the Hamiltonian of a harmonic oscillator of rigidity $a > 0$, whereas V is an anharmonic potential, which is assumed to grow at infinity faster than q^2. In the simplest case, one takes

$$V(q) = -b_1 q^2 + b_2 q^4, \qquad b_1, b_2 > 0. \tag{3}$$

The momentum p_x and displacement q_x are scalar. In the units used here, they obey the canonical commutation relation in the form

$$p_x q_y - q_y p_x = \delta_{xy}/\mathrm{i}, \qquad \mathrm{i} = \sqrt{-1}.$$

The Hamiltonian (1) has no direct mathematical meaning and serves as a form for 'local' Hamiltonians H_Λ, indexed by finite subsets of $\mathbb{Z}^d$, which one obtains from (1) by restricting the summations to $x, y \in \Lambda$. For each Λ, by the assumptions made above H_Λ is a lower bounded and self-adjoint operator in the 'physical' Hilbert space $\mathcal{H}_\Lambda = L^2(\mathbb{R}^{|\Lambda|})$, where $|\cdot|$ stands for cardinality. Moreover, for any $\beta > 0$,

$$Z_\Lambda := \mathrm{trace} \exp\left(-\beta H_\Lambda\right) < \infty. \tag{4}$$

Then the local Gibbs state ϱ_Λ at temperature $T = 1/\beta k_B$, k_B being Boltzmann's constant, is set to be

$$\varrho_\Lambda(A) = \mathrm{trace}\left\{A \exp\left(-\beta H_\Lambda\right)\right\}/Z_\Lambda. \tag{5}$$

It is a positive linear functional on the algebra $\mathfrak{C}_\Lambda$ of all bounded linear operators $A : \mathcal{H}_\Lambda \to \mathcal{H}_\Lambda$. By Høegh-Krohn's theorem, see page 72 in [3], ϱ_Λ can be recovered from its values on the linear span of products of the form

$$\mathfrak{a}^\Lambda_{t_1}(F_1)\cdots\mathfrak{a}^\Lambda_{t_n}(F_n), \qquad n\in\mathbb{N}, \quad F_1,\ldots,F_n\in\mathfrak{F}_\Lambda, \quad t_1,\ldots,t_n\in\mathbb{R}, \tag{6}$$

where $\mathfrak{F}_\Lambda$ is a 'rich enough' family of multiplication operators by bounded measurable functions $F : \mathbb{R}^{|\Lambda|} \to \mathbb{C}$, whereas $\mathfrak{a}^\Lambda_t(A) = \exp(\mathrm{i}tH_\Lambda)A\exp(-\mathrm{i}tH_\Lambda)$, $A\in\mathfrak{C}_\Lambda$, is the time automorphism $\mathfrak{a}^\Lambda_t : \mathfrak{C}_\Lambda \to \mathfrak{C}_\Lambda$ which describes the local dynamics. Thus, the Green functions

$$G_{F_1,\ldots,F_n}(t_1,\ldots,t_n) := \varrho\left[\mathfrak{a}^\Lambda_{t_1}(F_1)\cdots\mathfrak{a}^\Lambda_{t_n}(F_n)\right] \tag{7}$$

with $F_1,\ldots,F_n\in\mathfrak{F}_\Lambda$ determine the state ϱ_Λ. Each Green function admits an analytic continuation, also denoted by $G_{F_1,\ldots,F_n}$, to the domain

$$\mathcal{D}_n := \{(\zeta_1,\ldots,\zeta_n)\in\mathbb{C}^n : 0 < \mathrm{Im}(\zeta_1) < \cdots < \mathrm{Im}(\zeta_n) < \beta\}.$$

Furthermore, see Theorem 1.2.32 on page 78 in [3], $G_{F_1,\ldots,F_n}$ is continuous on the closure $\overline{\mathcal{D}}_n$ and can uniquely be recovered from its restriction to the set

$$\mathcal{D}^{(0)}_n := \{(\zeta_1,\ldots,\zeta_n)\in\overline{\mathcal{D}}_n : \mathrm{Re}(\zeta_1) = \cdots = \mathrm{Re}(\zeta_n) = 0\}.$$

Thus, the Matsubara functions

$$\Gamma_{F_1,\ldots,F_n}(\tau_1,\ldots,\tau_n) := G_{F_1,\ldots,F_n}(\mathrm{i}\tau_1,\ldots,\mathrm{i}\tau_n) \tag{8}$$

uniquely determine the state (5). The main ingredient of our technique is the following representation, see Theorem 1.4.5 in [3],

$$\Gamma_{F_1,\ldots,F_n}(\tau_1,\ldots,\tau_n) = \int_{\Omega_\Lambda} F_1(\omega_\Lambda(\tau_1))\cdots F_n(\omega_\Lambda(\tau_n))\nu_\Lambda(d\omega_\Lambda), \tag{9}$$

where ν_Λ is a 'local Euclidean Gibbs measure', which is a probability measure on the measurable space $(\Omega_\Lambda, \mathcal{B}(\Omega_\Lambda))$. Here

$$\Omega_\Lambda = \{\omega_\Lambda = (\omega_x)_{x\in\Lambda} : \omega_x \in C_\beta\}, \quad C_\beta := \{\phi\in C([0,\beta]\to\mathbb{R}) : \phi(0) = \phi(\beta)\}.$$

That is, C_β is the Banach space of 'continuous temperature loops', Ω_Λ is equipped with the corresponding product topology and with the Borel σ-field $\mathcal{B}(\Omega_\Lambda)$. The measure ν_Λ has the following Feynman-Kac representation

$$\nu_\Lambda(d\omega_\Lambda) = \exp\left[-I_\Lambda(\omega_\Lambda)\right]\chi_\Lambda(d\omega_\Lambda)/N_\Lambda, \tag{10}$$

where

$$I_\Lambda(\omega_\Lambda) = -J\sum_{x\sim y,\ x,y\in\Lambda}\int_0^\beta \omega_x(\tau)\omega_y(\tau)d\tau + \sum_{x\in\Lambda}\int_0^\beta V(\omega_x(\tau))d\tau \tag{11}$$

is the energy functional, and the measure χ_Λ has the form

$$\chi_\Lambda(d\omega_\Lambda) = \prod_{x\in\Lambda} \chi(d\omega_x). \tag{12}$$

Here χ is the Høegh-Krohn measure – a probability measure on the Banach space C_β, constructed by means of the harmonic part of the Hamiltonian (2), c.f. pp. 99 and 125 in [3]. Thus, the representation (9) leads to the description of the states (5) in terms of Gibbs measures, similar to those describing classical anharmonic crystals. Here, however, each real-valued q_x is replaced by a continuous path ω_x, which is an infinite dimensional vector. Going further in this direction, one can define 'global' Gibbs states of the model (1) as the probability measures on the space of 'tempered configurations' satisfying the DLR equation, see Chapter 3 in [3]. It can be shown that the set of all such measure $\mathcal{G}$ is a nonempty compact simplex with a nonempty extreme boundary $\mathrm{ex}(\mathcal{G})$, the elements of which correspond to the thermodynamic phases. Therefore, the considered model has a phase transition if $|\mathrm{ex}(\mathcal{G})| > 1$ for big enough β.

From now on, for the lattice dimension d we suppose $d \geq 3$. For $p = (p_1, \ldots, p_d)$ with $p_i \in (-\pi, \pi]$, $i = 1, \ldots, d$, we set

$$\varepsilon(p) = \sum_{i=1}^{d} (1 - \cos p_i), \qquad \theta(d) = \frac{1}{(2\pi)^d} \int_{(-\pi,\pi]^d} \frac{dp}{\varepsilon(p)}. \tag{13}$$

Let also $f : [0, +\infty) \to (0, 1]$, be defined implicitly by $f(u \tanh u) = u^{-1} \tanh u$, $u > 0$, and $f(0) = 1$. Then, for a fixed $\alpha > 0$, the function

$$\phi(t) := t\alpha f(t/\alpha), \quad t > 0 \tag{14}$$

is differentiable and monotone increasing to α^2 as $t \to +\infty$. Finally, for a, b_1, and b_2 as in (2) and (3), we set

$$\upsilon_* = \frac{2b_1 - a}{12b_2}. \tag{15}$$

Observe that $\upsilon_* > 0$ whenever $b_1 > a/2$, and thereby the potential energy in (2) has two wells. By means of the infrared estimates, see [6] or Section 6.2 in [3], we obtain the following result, c.f. Theorem 3.1 in [5] or Theorem 6.3.6 in [3].

Theorem 1 *Let the parameters introduced above obey the following condition*

$$8m\upsilon_*^2 J > \theta(d). \tag{16}$$

Then $|\mathrm{ex}(\mathcal{G})| > 1$ *for* $\beta > \beta_*$*, where the latter is the unique solution of the equation*

$$2\beta J \upsilon_* f(\beta/4m\upsilon_*) = \theta(d). \tag{17}$$

Such β_* exists since LHS(17) $= (J/2m)\phi(\beta)$, where ϕ is as in (14) with $\alpha = 4m\upsilon_*$, which is an increasing function of β tending to $8m\upsilon_*^2 J$ as $\beta \to +\infty$. Thus, for our model (1) -

(3), a necessary condition for the phase transitions to be suppressed at all temperatures can be written in the form

$$4m\upsilon_*^2 \hat{J}_0 \leq d\theta(d). \tag{18}$$

Here $\hat{J}_0 := 2dJ$ stands for the interaction of a given oscillator with all its neighbors. In order to get the corresponding sufficient condition, we turn to the spectral properties of the Hamiltonian (2), which acts in $\mathcal{H}_x = L^2(\mathbb{R})$. By Proposition 4.1 in [5] or Theorem 1.1.60 in [3], the spectrum of H_x entirely consists of simple eigenvalues E_n, $n \in \mathbb{N}_0$, such that $n^{-\gamma}E_n \to +\infty$, as $n \to +\infty$, for some $\gamma > 1$. The simplicity means that each E_n corresponds to exactly one state, contrary to the classical case where the degeneracy discussed above might occur. Since $E_n \sim n^\gamma$, the splitting $E_n - E_{n-1}$ between the energy levels increases with n. Hence its minimal value

$$\Delta = \min_{n \in \mathbb{N}} (E_n - E_{n-1}) \tag{19}$$

can be calculated from $E_0, E_1, \ldots, E_k$ for some finite $k \in \mathbb{N}$. By means of the analytic perturbation theory for linear operators and the mentioned asymptotics, one obtains, see Theorem 4.1 in [5], that Δ is a continuous function of $m \in (0, +\infty)$, such that $m^{2/3}\Delta \to \Delta_0$, as $m \to 0$, for some $\Delta_0 \in (0, +\infty)$. Then

$$R_m := m\Delta^2 \tag{20}$$

is a continuous function of $m \in (0, +\infty)$, such that $R_m \sim m^{-1/3}\Delta_0^2$ as $m \to 0$. The sufficient condition in question has the following form, see Theorem 4.6 in [5] or Theorem 7.3.1 in [3].

Theorem 2 *Let the parameters introduced above obey the following condition*

$$\hat{J}_0 < R_m. \tag{21}$$

Then $|\mathcal{G}| = 1$ for all $\beta > 0$, i.e., in this case there is no phase transitions at all temperatures.

In the harmonic case where $V \equiv 0$, one has $R_m^{\text{har}} = a$; hence, in this case (21) is merely a stability condition. By analogy, we call R_m the *quantum rigidity* and (21) the *quantum stability* condition. According to Theorem 2, a *quantum stabilization* occurs if (21) holds, see also [1]. Note that the rigidity R_m can be made arbitrarily big either by making m small or Δ big (e.g. by making the wells closer to each other). For big R_m, the particle 'forgets' about the details of the potential energy in the vicinity of the origin (including instability) and oscillates as if its equilibrium at zero were stable, as in the harmonic case.

Of course, it would be instructive to compare (21) with (18). This can be done by means of the estimate $R_m \leq 1/4m\upsilon_*^2$, $m > 0$, obtained in Theorem 4.2 in [5] or in Theorem 7.1.1 in [3]. Therefore, if (21) holds, then $4m\upsilon_*^2\hat{J}_0 < 1$. On the other hand, for $d \geq 4$, the function $\theta(d)$ obeys the following estimate $\theta(d) < (d-1)^{-1}$ c.f. page 308 in [3]. Then, for $d \geq 4$, by (18) we have $4m\upsilon_*^2\hat{J}_0 \leq d/(d-1)$. Comparing the latter with (18) we see that mathematically strict conditions for a phase transition in a quantum anharmonic crystal to occur at some temperature, or to be suppressed at all temperatures, can be formulated in terms of just one parameter, $4mv_*^2\hat{J}_0$, comprising the particle mass, the potential energy parameter (15), and the interaction parameter $\hat{J}_0$. A similar theory has been developed also for more general models of quantum anharmonic crystals, see [5, 7] and Section 6.3 in [3]. All of them are based on the representation (9) and on the study of the properties of the measures (10) and (12) performed in [3] by means of various methods of functional analysis and operator theory.

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On superconductivity, BCS theory and mathematical physics

by Walter F. Wreszinski (São Paulo, Brazil)

Walter F. Wreszinski is a native of Rio de Janeiro (Brazil) and obtained his Ph.D. in 1973 at the Seminar für Theoretische Physik der ETH, Zürich, in the field of mathematical physics, having Prof. Dr. K. Hepp as thesis advisor. Since 1990 he has been full professor at the Departamento de Física Matemática, Instituto de Física, USP (University of São Paulo). His main research interests are mathematical statistical mechanics and quantum field theory.

In this note we should like to revisit some "conventional wisdom" , which complements the very nice review with the same title by J.-B. Bru and W. de S. Pedra (IAMP News Bulletin, april 2011), but does not seem to be widespread and may, therefore, be useful, specially to young researchers who are not aware of the (very) old history of the subject.

As is well known, a relevant part of mathematical physics is devoted to the rigorous study of models which describe different levels of approximation to the properties of real physical systems, the approximations being frequently heuristic , i.e., not under mathematical control. Just to give one example, the Heisenberg model of (anti-, ferri- or ferro-) magnetism is a heuristic approximation to to other models of itinerant electrons. In spite of that, a theory of ferromagnetism based on the Heisenberg model would be universally praised as extremely valuable and important. One necessary requirement for this is that the model in question does not violate fundamental physical principles, e.g., symmetry principles. For many-body systems, two such principles are local gauge covariance (or gauge invariance of the second kind) and Galilean covariance. It is, however, not pointed out even in the very best textbooks [1] that the BCS theory violates the covariance of the dynamics with respect to the gauge transformations of the second kind

$$\vec{A} \to \vec{A} + \nabla\chi \text{ together with } \psi \to \psi \exp(\frac{-ie\chi}{\hbar c}) \,, \tag{1}$$

where $\vec{A}$ denotes the vector potential and ψ the second quantized Fermi field (see [2], p. 214, for a comprehensive formulation). It is, of course, the invariance of the Hamiltonian under (1) that leads to the local charge conservation and well-known explicit formulas for the charge, electric current and magnetic polarization in terms of these fundamental fields (see, e.g., [2]). The well-known expression for the interacting pair potential in many-body theory, as well as the electron-phonon interaction Hamiltonian, is trivially invariant under (1), but the point is that the BCS theory is based on a **truncation** of such a fully gauge invariant model: this alarming illness was noticed in the early days ([3], [4]), and all attempts to cure it ([5], [6], [7]) were based on uncontrolled approximation schemes - see the discussion on pg. 208 of Sewell's book [2]).

The failure of the BCS theory to yield a local current density also explains its inability to explain the surprising Meissner effect [1]: when a superconductor is cooled to a temperature T below the critical temperature in the presence of a static magnetic field (which does not surpass a critical value $H_c(T)$), a spontaneous surface current develops, which exactly screens the magnetic field at all points in the bulk, sufficiently far from the surface. This is, perhaps, the most fundamental property of superconductors and it is, therefore, a very important question whether any proofs of the Meissner effect exist.

In the presence of a magnetic field the symmetry (1) may be combined with translation invariance to yield covariance of the theory under the so-called magnetic translation group of J. Zak [8]. This was used by Sewell [9] to provide a proof of the Meissner effect in the bulk of a superconductor under the hypothesis of off-diagonal long-range order (ODLRO) (see the review by Bru and Pedra or [2] for remarks on this concept and references). Incidentally, it is ODLRO which yields a microscopic justification of the Leggett macroscopic wave function [10], a function which varies appreciably only over distances which are characteristic of variations of the electromagnetic fields themselves. The derivation of superconductor electrodynamics in ([1], p. 169) uses a current vector field defined in terms of this macroscopic wave function, instead of the would-be current of the microscopic theory. Of course, several open problems remain for a complete understanding of the Meissner effect: a proof of ODLRO for the would-be gauge invariant model Hamiltonian, and, for the same, a microscopic derivation of the surface currents, which are, presumably, physically responsible for the effect.

In close analogy to the above shortcoming of the BCS model, the approximations of the Bogoliubov type to the weakly interacting Bose gas (WIBG) (see the excellent review [11]) are not Galilei covariant. For the usual Bogoliubov approximation, this fact, which is also due to a (different) truncation of the original many-body Hamiltonian, was also not mentioned in [1]. In contrast to BCS, however, the Bogoliubov theory forms the basis of a general mathematically controlled scheme for quantities such as the pressure, see the remarks and references in Bru-Pedra. Bogoliubov's c-number substitution for the $\vec{k} = \vec{0}$ mode operator $a_{\vec{0}}$ has also been rigorusly proved, see [12] and references given there.

In spite of their obvious successes - in the BCS case, the concept of Cooper pairs, of macroscopic phase coherence, the existence of an energy gap (the latter leading to the observed behaviour of the specific heat at low temperature [1]), in the WIBG as a model of liquid Helium II the qualitative properties of the quasi-particle spectrum - the failure to fulfill basic physical principles means that these theories cannot be qualified as fundamental. I n fact, it is ironic that that Galilei invariance - an important element in the phenomenological Landau quasiparticle theory - is violated by the WIBG, which, however, "explains" the Landau theory, and that the BCS theory fails to provide the supercurrent, the basic observable of the theory, whose stationarity in rings represents one of the most spectacular examples of non-equilibrium stationary states (NESS) [13]. In [2] one finds a theory of the metastability of supercurrents in terms of a superselection rule, but, again, in the absence of a concrete realisation, the phenomenon continues to defy a microscopic explanation.

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ICMP12 News Bulletin No. 1

The preparations for ICMP12 are underway. These news bulletins will keep you informed about developments. Check also www.icmp12.com regularly.

ICMP12 will take place at the Aalborg Congress and Culture Center, Aalborg, Denmark, August 6–11, 2012. The congress will be preceded by a **Young Researcher Symposium**, August 3–4, 2012.

Plenary speakers As of July 17, 2011, confirmed speakers are: Dmitry Dolgopyat, University of Maryland, Philippe Di Francesco, CEA Saclay, Shu Nakamura, University of Tokyo, Jeremy Quastel, University of Toronto, Renato Renner, ETH Zürich, Wilhelm Schlag, University of Chicago, Benjamin Schlein, University of Bonn, Mu-Tao Wang, Columbia University, Simone Warzel, Technical University Munich, Avi Wigderson, Institute for Advanced Study, Princeton.

Topical Sessions Confirmed sessions organizers also given, as of July 17, 2011.
Dynamical systems, classical and quantum; Kening Lu, Brigham Young University, and Rafael de la Llave, Georgia Institute of Technology.
Equilibrium and non-equilibrium statistical mechanics; Horia Cornean, Aalborg University, and Antti Kupiainen, University of Helsinki.
PDE and general relativity; Chang-Shou Lin, National Taiwan University. and Hans Ringström, KTH Stockholm.
Stochastic models and probability; Laszlo Erdős, LMU München, and Ofer Zeitouni, Weizmann Institute.
Operator algebras, exactly solvable models.
Quantum mechanics and spectral theory; Rafael Benguria, P. Universidad Católica de Chile, and Jacob Schach Møller, University of Aarhus.
Quantum information and computation; Barbara Terhal, RWTH Aachen, and Michael Wolf, TU München.
Quantum many-body theory and condensed matter physics; Mathieu Lewin, Université de Cergy-Pontoise, and Marcel Griesemer, Universität Stuttgart.

Quantum field theory.
String theory and quantum gravity; Volker Schomerus, DESY, and Laurent Freidel, Perimeter Institute.
Reaching beyond: Mathematical physics in other fields.

Young Researcher Symposium: Confirmed plenary speakers as of July 17, 2011: Robert Seiringer, McGill University, and Thomas Spencer, Princeton University.

Deadlines: Deadlines for contributed talks will be announced in the next news bulletin. Registration will open in January 2012.

Arne Jensen (Congress Convenor)

Call for prize nominations for prizes to be awarded at ICMP 2012

The Henri Poincaré Prize

The Henri Poincaré Prize, sponsored by the Daniel Iagolnitzer Foundation, was created in 1997 to recognize outstanding contributions in mathematical physics, and contributions which lay the groundwork for novel developments in this broad field. The Prize is also created to recognize and support young people of exceptional promise who have already made outstanding contributions to the field of mathematical physics. The prize is awarded every three years at the International Congress of Mathematical Physics (ICMP), and in each case is awarded usually to three individuals. A list of previous winners can be found here: http://www.iamp.org/page.php?page=page_prize_poincare. Nominations for the 2012 Prize, which is to be awarded at the ICMP in Aalborg, can be made to the President (president@iamp.org) or Secretary (secretary@iamp.org). There is no prescribed format for a nomination but the following information will help the Prize Committee with its task:

- listing or description of the scientific work in support of the nomination
- a recent c.v. of the nominee
- a proposed citation should the nominee be selected for an award

Please use email for your nomination; you will receive an acknowledgment that your nomination has been received.

To ensure full consideration please submit your nominations by November 30, 2011.

The IAMP early career award

The IAMP Early Career Award was instituted in 2008 and was awarded for the first time at the ICMP in Prague in 2009. It will next be awarded to at the ICMP in Aalborg in August 2012. The prize rules:

- The prize will be awarded in recognition of a single achievement in mathematical physics. The total prize value amounts to 3000 Euro and is reserved for scientists whose age in years since birth on July 31 of the year of the Congress is less than 35
- A nomination for the prize should include the name of the candidate accompanied by a brief characterization of the work singled out for the prize. Members of the IAMP should send their nomination or nominations to the President (president@iamp.org) and to the Secretary (secretary@iamp.org) by e-mail, and a fax or a hard copy should follow the e-mail

- The recipient of the prize will be selected by a committee of 5 members determined by votes within the Executive Committee of the IAMP, which shall decide the membership of the committee within one month of the expiration of the deadline for nominations

- The Committee may consider candidates that have not been nominated by the members. The names of committee members will remain confidential within the EC until the prize ceremony. The IAMP will make every reasonable effort to make attendance at the ceremony possible for the selected candidate

Nominations should be made not later than on February 6, 2012.

Pavel Exner
Jan Philip Solovej

The IUPAP Young Scientist Prize in Mathematical Physics

The IUPAP prize (`http://www.iupap.org`) is awarded triennially to at the most three young scientists satisfying the following criteria:

- The recipients of the awards in a given year should have a maximum of 8 years of research experience (excluding career interruptions) following their PhD on January 1 of that year

- The recipients should have performed original work of outstanding scientific quality in mathematical physics

- Preference might be given to young mathematical physicists from developing countries

The awards will be given for the second time at the ICMP 2012 in Aalborg; the deadline for nominations is August 2, 2011. Please submit your nomination to Ana Bela Cruzeiro (abcruz@math.ist.utl.pt) and to Herbert Spohn (spohn@ma.tum.de) as officers of the IUPAP C18 commission for mathematical physics.

Pavel Exner
Jan Philip Solovej
Herbert Spohn
Ana Bela Cruzeiro

News from the IAMP Executive Committee

New individual members

IAMP welcomes the following new members

1. Sven Bachmann, Department Mathematics, UC Davis, Davis, CA, USA
2. Anuj Jhankal, Department of Applied Mathematics, Birla Institute of Technology, Jaipur, India
3. Muhammad Kaurangini, Department Mathematical Sciences, Kano University of Science and Technology, Wudil, Nigeria
4. Rakesh Kumar, Department of Basic and Applied Science, University College of Engineering, Punjabi University, Patiala, India
5. Andreas Leiser, Dept. of Mathematics, ETH Zurich, Zürich, Switzerland
6. Pieter Naaijkens, Institute for Mathematics, Astrophysics and Particle Physics, Radboud Universiteit Nijmegen, Nijmegen, Netherlands
7. Rachna Rani, Department of Basic and Applied Science, Jasdev Singh Sandhu Institute of Engineering & Technology, Patiala, India

Open positions

- A PhD position is available at the Mathematics Department of the University of Utrecht (The Netherlands) to work on rigorous statistical mechanics, specifically on the theory of cluster and virial expansions. The position carries a pocket salary of the order of 1400 euros per month (12 payments) plus two slightly smaller extra payments in December and July. No tuition is due, but the student must pay for health insurance. Students in The Netherlands are considered public servants, hence the position carries retirement benefits.

  Interested people should send me a message, including a brief CV. Roberto Fernandez (r.fernandez1@uu.nl).

Recent conference announcements

- Sep 6– 9, 2011, Workshop on Quantum Field Theory aspects of Condensed Matter Physics, Rome
  LINK: http://cmtp.uniroma2.it/11QFTCM/

- Aug 16– 24, 2011, SUMMER SCHOOL on current topics in Mathematical Physics, Erwin Schrödinger Institute, Vienna
  LINK: http://www.maphy.uni-tuebingen.de/~chha/Home.html

Other announcement from the executive committee:

The Henri Poincaré Prizes sponsored by the Daniel Iagolnitzer Foundation and the IAMP early career award will be awarded at the ICMP 12 in Aalborg August 2012. A detailed call for nominations can be found earlier in the bulletin.

Jan Philip Solovej (IAMP Secretary)