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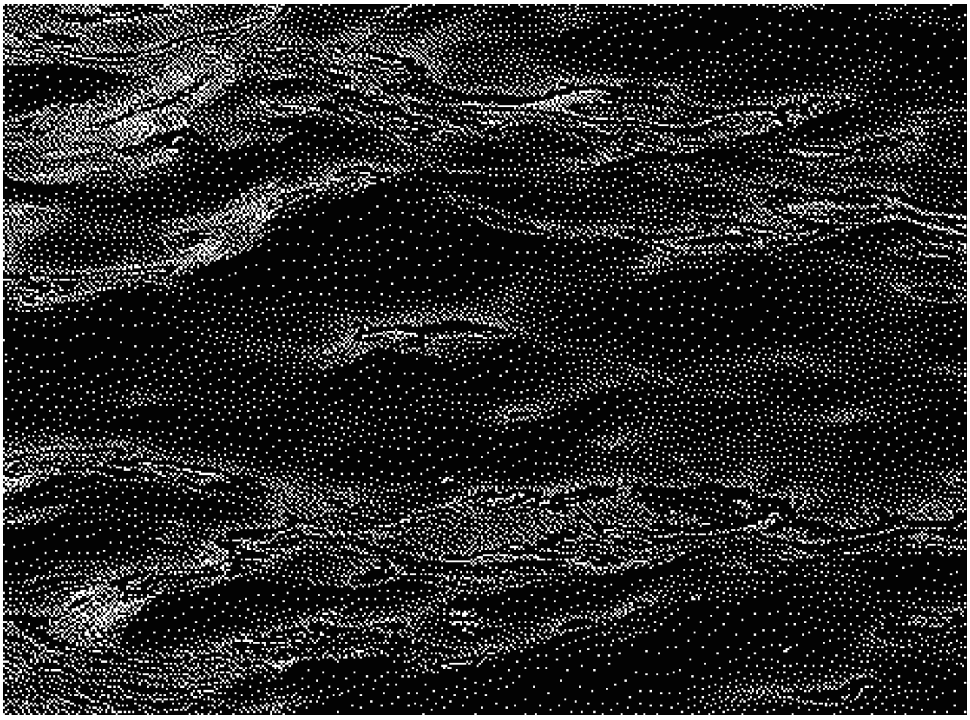

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Contacts. http://www.iamp.org and e-mail: bulletin@iamp.org

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The problem of asymptotic completeness for massive AQFT and gapped quantum spin systems

by WOJCIECH DYBALSKI (Munich)

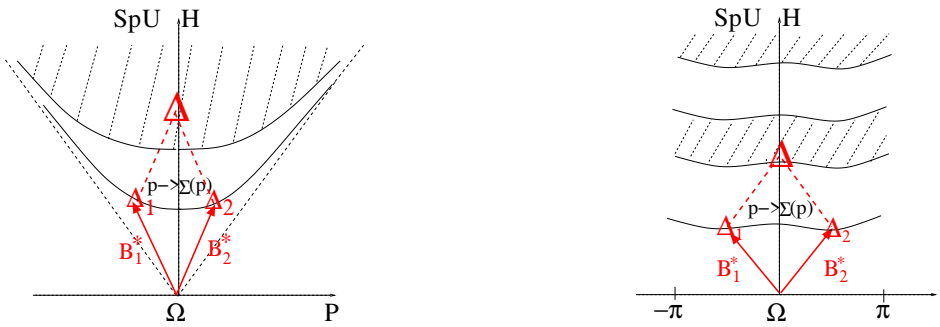
Scattering theory and asymptotic completeness of quantum systems with infinitely many degrees of freedom is still rather poorly understood. Non-quadratic dispersion relations of particles and a possible presence of 'charged sectors' hinder the application of standard arguments from N-body quantum mechanics. In this note we discuss recent progress on this subject, partially based on the author's collaborations with C. Gérard, S. Bachmann and P. Naaijkens. In a broad framework, which encompasses both models of algebraic QFT and quantum spin systems, we establish a generalized variant of asymptotic completeness, which is compatible with the presence of charged sectors. In the last part of the article we suggest possible sufficient conditions for conventional asymptotic completeness for such systems.

1 Introduction

Since the early days of quantum mechanics, scattering theory has been a central tool for comparison of theory with experiment. Mathematical foundations of scattering theory were laid by giants of mathematical physics such as L.D. Faddeev and T. Kato, as discussed in recent issues of this *Bulletin* [Si18, De18]. The problem of asymptotic completeness, i.e., the question of particle interpretation of all states in the physical Hilbert space of the theory, emerged as one of the main problems of mathematical scattering theory. Its solution in N-body quantum mechanics for particles with quadratic dispersion relations is an impressive chapter of the 20th century mathematical physics whose milestones are [En78, SiSo87, Gr90, De93]. However, if the assumption of quadratic dispersion relations is dropped, even in quantum mechanics the problem of complete particle interpretation is largely open beyond the two-body scattering. It is therefore not a surprise that in relativistic algebraic QFT (AQFT) or for quantum spin systems, where basic excitations typically have non-quadratic dispersion relations, asymptotic completeness is rather poorly understood. An additional problem for quantum systems with infinitely many degrees of freedom is a possible breakdown of the Stone-von Neumann uniqueness theorem and the resulting multitude of 'charged sectors'. This aspect undermines the conventional property of asymptotic completeness, inherited from quantum mechanics, and calls for more suitable concepts.

In this article we outline some recent progress on scattering theory and the problem of asymptotic completeness for massive AQFT and gapped quantum spin systems, made in [DG14, DG14.1, BDN16, Dy18]. In order to treat AQFT and quantum spin systems in parallel in this review, we introduce the following general setting which collects the assumptions of direct relevance to scattering theory. From the outset, we restrict attention to AQFT in vacuum representations and quantum spin systems in representations of translation invariant ground states.

1. We denote by Γ the abelian group of space translations and by $\widehat{\Gamma}$ its Pontryagin dual, i.e., the momentum space. In AQFT we have $\Gamma = \mathbb{R}^d$ and $\widehat{\Gamma} = \mathbb{R}^d$, whereas for quantum spin systems $\Gamma = \mathbb{Z}^d$ and $\widehat{\Gamma} = S_1^d$, where the latter denotes the d-dimensional torus.



(a) A spectrum in AQFT.

(b) A spectrum in a quantum spin system.

Figure 1. Schematic shapes of the energy-momentum spectra (a) in massive AQFT and (b) in a gapped quantum spin system. Neighbourhoods Δ_1, Δ_2 of some points on the isolated mass shell $p \mapsto \Sigma(p)$ and the region Δ of the multiparticle spectrum illustrate the geometric situation from assumption (a) of Theorem 2. The operators B_1^*, B_2^* create from the vacuum single-particle states living in the regions Δ_1, Δ_2.

2. We consider a C^*-dynamical system $(\mathfrak{A}, \alpha)$, where $\mathfrak{A}$ is a C^*-algebra of observables and $\mathbb{R} \times \Gamma \ni (t, x) \mapsto \alpha_{(t,x)}$ is a group of automorphisms which describes translations of observables in space and time.

3. We assume that there is a norm-dense subalgebra $\mathfrak{B} \subset \mathfrak{A}$ of *almost-local operators*. We refrain from giving the formal definition here, merely state that for $v \in \mathbb{R}^d$ with sufficiently large norm, we have

$$\|[B_1, \alpha_{(s,vs)}(B_2)]\| = O(|s|^{-\infty}), \quad B_1, B_2 \in \mathfrak{B}, \tag{1.1}$$

that is, the commutator decays faster than any inverse power of s. In AQFT this bound follows from locality and $|v|$ should be larger than the velocity of light, whereas in quantum spin systems it is a consequence of the Lieb-Robinson bounds [LR72] and $|v|$ should be larger than the Lieb-Robinson velocity.

4. We suppose that $\mathfrak{A}$ acts irreducibly on a Hilbert space $\mathcal{H}$ and that α is unitarily implemented on $\mathcal{H}$. That is, there is a unitary representation $\mathbb{R} \times \Gamma \ni (t, x) \mapsto U(t, x)$ s.t.

$$\alpha_{(t,x)}(A) = U(t, x) A U(t, x)^*, \quad A \in \mathfrak{A}. \tag{1.2}$$

The spectrum of U, denoted $\operatorname{Sp} U$, is the support of its inverse Fourier transform[1]. It is a subset of $\mathbb{R} \times \widehat{\Gamma}$ which consists of all the possible values of the total energy and momentum of the system. The spectrum should contain a simple eigenvalue at $\{0\}$, corresponding to the vacuum vector Ω and a smooth mass shell $\widehat{\Gamma} \ni p \mapsto \Sigma(p)$ carrying single-particle states. Instead of stating all the assumptions on the spectrum here, we refer to Figure 1 for schematic shapes and remark that Σ should not be a constant function and should be isolated from the rest of the spectrum.

[1] We follow here the conventions from [BDN16].

This is a rather abstract setting, so let us give some examples: On the relativistic side we mention the ϕ^4 models of constructive QFT in two and three spacetime dimensions for small values of the coupling constant [GJS73, SZ76, Bur77]. On the side of spin systems, the Ising model in strong transverse magnetic fields in any space dimensions satisfies all the above assumptions [BDN16, Ya04, Ya05, Po93]. We stress that the single-particle states introduced above are complicated collective excitations of the basic degrees of freedom in these models

2 Scattering states

The problem of construction of scattering states is the following[2]: Given a collection of single-particle states $\Psi_1, \dots, \Psi_N \in \mathcal{H}$, living on the mass shell $p \mapsto \Sigma(p)$ in $\operatorname{Sp} U$, we would like to define a state $\Psi^{\mathrm{out}} \in \mathcal{H}$ which describes the corresponding configuration of N particles. Anticipating that asymptotically these particles should be non-interacting bosons, the state Ψ^{out} should have all the properties of the symmetrized tensor product of the constituent single-particle states Ψ_i, $i = 1, \dots, N$. Nevertheless, it should be an element of $\mathcal{H}$ and not of $\otimes_{\mathrm{s}}^N \mathcal{H}$. The first step towards the solution of this problem is suggested by the theory of Fock spaces: we pick almost-local operators $B_i^* \in \mathfrak{B}$ which create these single-particle states from the vacuum, i.e., $B_i^* \Omega = \Psi_i$. In order to select such generalized creation operators, we compute the Arveson spectrum $\operatorname{Sp}_{B_i^*} \alpha$, which is the support of the inverse Fourier transform of $(t, x) \mapsto \alpha_{(t,x)}(B_i^*)$. The key property of the Arveson spectrum is the energy-momentum transfer relation, which says that for any Borel subset $\Delta \subset \operatorname{Sp} U$,

$$B_i^* \, E(\Delta) \mathcal{H} \subset E(\overline{\Delta + \operatorname{Sp}_{B_i^*} \alpha}) \mathcal{H}, \tag{2.1}$$

where $E(\,\cdot\,)$ denotes the spectral measure of U. In particular, if $\operatorname{Sp}_{B_i^*} \alpha$ is contained in a small neighbourhood Δ_i of a point of the mass shell (cf. Figure 1) then by choosing $\Delta = \{0\}$ in (2.1) we obtain that $B_i^* \Omega$ is a single-particle state. As operators with Arveson spectrum in prescribed subsets are in abundance, it is easy to obtain such generalized creation operators for a dense family of single-particle vectors.

The next step is to define the Haag-Ruelle creation operators by smearing the above generalized creation operators with wave packets of the particles involved:

$$B_{i,t}^*(g_{i,t}) := \int_\Gamma d\mu(x) \alpha_{(t,x)}(B_i^*) g_{i,t}(x), \quad g_{i,t}(x) := \int_{\widehat{\Gamma}} dp \, \mathrm{e}^{-\mathrm{i}\Sigma(p)t + \mathrm{i}p\cdot x} \hat{g}_i(p), \tag{2.2}$$

where $d\mu$, resp. dp, is the Haar measure on Γ, resp. $\widehat{\Gamma}$, the function $p \mapsto \Sigma(p)$ is the mass shell appearing in the spectrum (cf. Figure 1) and t is the time parameter. The role of the Haag-Ruelle creation operators is to compare the interacting evolution appearing in $(t, x) \mapsto \alpha_{(t,x)}$ and the free evolution of the wave packet at asymptotic times. This is the content of the Haag-Ruelle theorem, whose relativistic variant dates back to [Ha58, Ru62], with various later simplifications [He65, Ar]. It was adapted to quantum spin systems by S. Bachmann, P. Naaijkens and the present author in the work [BDN16] which received a prize of the journal *Annales Henri Poincaré* in 2016. While a similar enterprise had been accomplished in Euclidean lattice field

[2] We discuss the outgoing scattering states here. The incoming case is analogous.

theory [BF91], earlier works on scattering in quantum spin systems relied more heavily on properties of particular models [Ya04.1, GS97, Ma83].

Theorem 1. The following limits exist and are called the outgoing scattering states

$$\Psi^{\mathrm{out}} := \lim_{t\to\infty} B^*_{1,t}(g_{1,t})\dots B^*_{N,t}(g_{N,t})\Omega. \tag{2.3}$$

The velocity supports $V(g_i) := \{\, \nabla\Sigma(p) \,|\, p \in \operatorname{supp} \hat{g}_i \,\}$ of the wave packets g_i are assumed to be disjoint. The incoming scattering states Ψ^{in} are constructed analogously by taking the limit $t \to -\infty$.

Let us recall the main steps of the proof in order to indicate how the above general assumptions ensure the existence of multi-particle scattering states. The argument relies on Cook's method, that is, we try to make sense of the formula

$$\Psi^{\mathrm{out}} = \Psi_{t_0} + \int_{t_0}^{\infty} (\partial_\tau \Psi_\tau) d\tau, \tag{2.4}$$

where $t \mapsto \Psi_t$ is the approximating sequence of Ψ^{out} and $t_0 \geq 0$ is arbitrary. For this purpose we check that $t \mapsto \|\partial_t \Psi_t\|$ is an integrable function. This is easy to see for $N = 1$ in which case there is an exact cancellation of the interacting and the free dynamics for arbitrary t:

$$\partial_t (B^*_{1,t}(g_{1,t})) = 0. \tag{2.5}$$

The assumption that the mass shell is isolated from the rest of the spectrum enters crucially here. Now the case $N = 2$ is treated using the Leibniz rule and (2.5)

$$\begin{aligned} \partial_t \Psi_t &= \partial_t (B^*_{1,t}(g_{1,t}))\, B^*_{2,t}(g_{2,t})\Omega + B^*_{1,t}(g_{1,t})\, \underbrace{\partial_t (B^*_{2,t}(g_{2,t}))\Omega}_{=0} \\ &= [\partial_t (B^*_{1,t}(g_{1,t})), B^*_{2,t}(g_{2,t})]\Omega = O(t^{-\infty}), \end{aligned} \tag{2.6}$$

where in the last step we used the assumption (1.1) about the decay of commutators at large spacelike separation and the disjointness of velocity supports of the two wave packets. This argument easily generalizes to arbitrary N, which completes this outline of the proof.

Given the scattering states, one can construct the wave operators in a standard manner. Let $\mathcal{H}_1$ be the single-particle subspace, i.e., the spectral subspace of the mass shell $p \mapsto \Sigma(p)$, and let $\Gamma(\mathcal{H}_1)$ be the corresponding symmetric Fock space. The outgoing wave operator $W^{\mathrm{out}} : \Gamma(\mathcal{H}_1) \to \mathcal{H}$ is defined by the relation

$$W^{\mathrm{out}}(a^*(\Psi_1)\dots a^*(\Psi_N)\Omega) = \lim_{t\to\infty} B^*_{1,t}(g_{1,t})\dots B^*_{N,t}(g_{N,t})\Omega, \tag{2.7}$$

where $\Psi_i := B^*_{i,t}(g_{i,t})\Omega$ and $a^{(*)}$ are the creation/annihilation operators on $\Gamma(\mathcal{H}_1)$. By computing the scalar products of the scattering states it is easy to check that W^{out} is an isometry. The same is true for the analogously defined incoming wave operator W^{in} which enters into the definition of the scattering matrix

$$S := (W^{\mathrm{in}})^* W^{\mathrm{out}}. \tag{2.8}$$

(a) Conventional asymptotic completeness. (b) Generalized asymptotic completeness.

Figure 2. (a) Conventional asymptotic completeness requires that every vector $\Psi \in \mathcal{H}$ is a configuration of charge-zero particles from $\mathcal{H}^{\mathrm{out}}$. (b) Generalized asymptotic completeness requires that every vector Ψ gives rise to a configuration of charge-zero particles after filtering it through a suitable measurement apparatus. Moreover, every configuration of charge-zero particles can be obtained by this procedure.

If S is different from the identity, we say that the theory is interacting. To our knowledge, the interaction has been proven only in some two-dimensional relativistic systems [OS76, Le08, Ta14]. It is one of the central problems of AQFT to exhibit an interacting model in four-dimensional spacetime. In the context of quantum spin systems there are candidates for interacting theories in an arbitrary dimension, for example the Ising model mentioned above, but we are not aware of a proof.

3 Generalized asymptotic completeness

The conventional property of asymptotic completeness requires that the subspace $\mathcal{H}^{\mathrm{out}} = \operatorname{Ran} W^{\mathrm{out}}$, spanned by the scattering states (2.3), is in fact the full Hilbert space. That is

$$\mathcal{H}^{\mathrm{out}} = \mathcal{H}, \tag{3.1}$$

so that every vector $\Psi \in \mathcal{H}$ has an interpretation in terms of particles (cf. Figure 2(a)). In the setting from the Introduction, the only known examples which are interacting and asymptotically complete are certain two-dimensional relativistic models [Le08, Ta14][3].

A reason for this scarcity of examples may be the following special feature of quantum systems with infinitely many degrees of freedom: The algebra of observables $\mathfrak{A}$ may have many inequivalent representations labelled by some quantum number which we call 'charge'. The vacuum representation we are interested in, has the charge equal to zero. Let us now consider the particle content of the region Δ of the multiparticle spectrum in Figure 1. Apart from the pairs of the charge-zero particles living on the mass shell in the spectrum of U there may also be, e.g., pairs of oppositely charged particles whose single-particle constituents live in different representations. States describing such oppositely charged pairs live in the vacuum representation (as their total charge is zero), but are orthogonal to all the scattering states (2.3) of the charge-zero particles. In this case the asymptotic completeness property (3.1) fails.

As we typically do not have access to all the charged representations, it is reasonable to generalize the concept of asymptotic completeness so that it is compatible with the presence

[3] We also mention partial results on $(\phi^4)_2$ [SZ76, CD82] and in certain lattice systems which do not quite fit into our framework [AB01, GS97]. There is also progress on asymptotic completeness in wedge-local QFT [DT11, Du18].

of charged pairs. The idea is illustrated on Figure 2(b): Any state $\Psi \in \mathcal{H}$ should give rise to a configuration of charge-zero particles after filtering it through a suitable particle detector. Moreover, every configuration of charge-zero particles should be obtainable in this way. Arguably, such a concept should suffice to interpret physical experiments, since in the experimental reality there is always some intervening apparatus. It turns out that such generalized asymptotic completeness can be formulated and proven under the general assumptions stated in the Introduction.

The first question is how to identify particle detectors in such a general mathematical formalism. This question was first posed by Araki and Haag in the setting of AQFT [AH67]. They came up with the time-dependent families of observables of the form

$$C_t := \int_\Gamma d\mu(x)\alpha_{(t,x)}(B^*B)h\left(\tfrac{x}{t}\right), \tag{3.2}$$

where $B^* \in \mathfrak{B}$ is a generalized creation operator as above and $h \in C_0^\infty(\mathbb{R}^d)$. Their limits as $t \to \infty$ can be computed on scattering states $\Psi_1^{\text{out}}, \Psi_2^{\text{out}} \in \mathcal{H}^{\text{out}}$ of bounded energy and, schematically, have the following form [AH67, Bu90]:

$$\lim_{t\to\infty}\langle \Psi_1^{\text{out}}, C_t\Psi_2^{\text{out}}\rangle = \int_{\widehat{\Gamma}} dp \underbrace{\langle p|B^*B|p\rangle h(\nabla\Sigma(p))}_{\text{sensitivity of the detector}} \underbrace{\langle \Psi_1^{\text{out}}, a^*_{\text{out}}(p)a_{\text{out}}(p)\Psi_2^{\text{out}}\rangle}_{\text{particle density}}. \tag{3.3}$$

Here $a^*_{\text{out}}(p)a_{\text{out}}(p) := W^{\text{out}}a^*(p)a(p)(W^{\text{out}})^*$ is the asymptotic particle density in momentum space, and the remaining part of the integrand above can be interpreted as the sensitivity of the detector. It should be stressed, however, that the above results do not say anything about the convergence of $\{C_t\}_{t\in\mathbb{R}}$ on states which are not in $\mathcal{H}^{\text{out}}$. This question, which is essential for asymptotic completeness, has been a long-standing open problem in AQFT (see [Ha, Section VI.2.3]). First results of this sort were obtained by C. Gérard and the present author in [DG14, DG14.1] in the relativistic setting and then generalized to gapped quantum spin systems in [Dy18]. These results rely only on the general assumptions outlined in the Introduction and can be summarized as follows:

Theorem 2. Fix a small subset Δ of the multiparticle spectrum as in Figure 1. Then the following limits exist:

$$A^{\text{out}} := \text{s-}\lim_{t\to\infty} C_{1,t}\dots C_{N,t}E(\Delta), \text{ where } C_{i,t} := \int_\Gamma d\mu(x)\alpha_{(t,x)}(B_i^*B_i)h_i\left(\tfrac{x}{t}\right), \tag{3.4}$$

provided that

(a) B_i^* are generalized creation operators of single-particle states living in subsets Δ_i of $\operatorname{Sp} U$ s.t. $\Delta_1 + \dots + \Delta_N \subset \Delta$ (cf. Figure 1).

(b) h_i have mutually disjoint supports[4].

[4] We skip here some more technical restrictions on these functions.

Denote by $[A^{\mathrm{out}}\mathcal{H}]$ the subspace of $\mathcal{H}$ spanned by the ranges of all the operators A^{out} constructed as above for different choices of Δ. Then

$$\mathcal{H}^{\mathrm{out}} = [A^{\mathrm{out}}\mathcal{H}] \oplus \mathbb{C}\Omega, \tag{3.5}$$

that is, generalized asymptotic completeness holds (cf. Figure 2(b)).

We remark that assumption (a) of this theorem ensures that the detectors A^{out} annihilate configurations involving charged particles (if any) as seen a posteriori from relation (3.5). They also annihilate possible 'bound states', corresponding to embedded mass shells passing through Δ, which we did not exclude by assumption. The technically most challenging part of the proof of Theorem 2 is the existence of the limit in (3.4) on arbitrary vectors from $\mathcal{H}$. For more contrived choices of the detectors $C_{i,t}$ this had been shown in [DG14.1] by adapting the quantum-mechanical method of propagation estimates [SiSo87]. In [Dy10] a different technique was found which applies to the usual Araki-Haag detectors, as stated above. We outline the argument from [Dy10] in the remaining part of this section.

The strategy is to approximate A^{out} by linear combinations of rank-one operators $|\Psi^{\mathrm{out}}\rangle\langle\tilde{\Psi}^{\mathrm{out}}|$ at the level of the respective approximating sequences. In addition to the convergence, which then follows from Theorem 1, this also gives relation (3.5). For the purpose of this approximation argument, we define the mapping $a_{B_1,\ldots,B_N} : \mathcal{H}_{\mathrm{c}} \to \mathcal{H} \otimes L^2(\Gamma^N)$ given by

$$(a_{B_1,\ldots,B_N}\Psi)(x_1,\ldots,x_N) := \alpha_{x_1}(B_1)\ldots\alpha_{x_N}(B_N)\Psi, \tag{3.6}$$

which was introduced in [DG14, DG14.1]. Here $B_1,\ldots B_N \in \mathfrak{B}$ are as in Theorem 2, $\mathcal{H}_{\mathrm{c}} \subset \mathcal{H}$ is the dense domain of vectors of bounded energy and the fact that the $\mathcal{H}$-valued function on the r.h.s. of (3.6) is square-integrable follows from [Bu90]. In terms of these maps, we can write for scattering states living in Δ

$$|\Psi^{\mathrm{out}}\rangle\langle\tilde{\Psi}^{\mathrm{out}}| = \lim_{t\to\infty} a^*_{\alpha_t(\underline{B})}\big(|\Omega\rangle\langle\Omega| \otimes \mathrm{e}^{-\mathrm{i}t\underline{\Sigma}(-\mathrm{i}\nabla_{\underline{x}})}|\underline{g}\rangle\langle\underline{\tilde{g}}|\mathrm{e}^{\mathrm{i}t\underline{\Sigma}(-\mathrm{i}\nabla_{\underline{x}})}\big)a_{\alpha_t(\underline{\tilde{B}})}E(\Delta), \tag{3.7}$$

where we introduced the short-hand notation

$$\alpha_t(\underline{B}) := (\alpha_t(B_1),\ldots\alpha_t(B_N)), \tag{3.8}$$

$$\underline{\Sigma}(-\mathrm{i}\nabla_{\underline{x}}) := \Sigma(-\mathrm{i}\nabla_{x_1}) + \cdots + \Sigma(-\mathrm{i}\nabla_{x_N}), \tag{3.9}$$

$$\underline{g} := (g_1,\ldots,g_N), \tag{3.10}$$

and $g_i := g_{i,t=0}$ denotes the initial data of the wave packets in (2.2). Now the approximating sequence of A^{out} can be expressed as follows:

$$C_{1,t}\ldots C_{n,t}E(\Delta) = a^*_{\alpha_t(\underline{B})}\big(\mathbb{1}_{\mathcal{H}} \otimes \underline{h}(\underline{x}/t)\big)a_{\alpha_t(\underline{B})}E(\Delta) + O(t^{-\infty}), \tag{3.11}$$

where $\underline{h}(\underline{x}/t) := h_1(x_1/t)\ldots h_n(x_n/t)$ and the disjointness of the supports of h_i, together with (1.1), ensure the rapid decay of the error term above. Let us now compare the r.h.s. of (3.11) and (3.7). First, exploiting the presence of the projection $E(\Delta)$, assumption (a), and the energy-momentum transfer relation (2.1), we can replace $\mathbb{1}_{\mathcal{H}}$ with $|\Omega\rangle\langle\Omega|$ in (3.11). Next, since $E(\Delta)$ restricts the total momentum of the system to a compact set, we can replace $\underline{h}(\underline{x}/t)$ with

$\underline{h}(\underline{x}/t)\chi(-\mathrm{i}\nabla_{\underline{x}})$, where χ is an approximate characteristic function of a sufficiently large ball. Using the trivial identity

$$\underline{h}(\underline{x}/t)\chi(-\mathrm{i}\nabla_{\underline{x}}) = \mathrm{e}^{-\mathrm{i}t\underline{\Sigma}(-\mathrm{i}\nabla_{\underline{x}})}\underline{h}(\underline{x}/t + \nabla\underline{\Sigma}(-\mathrm{i}\nabla_{\underline{x}}))\chi(-\mathrm{i}\nabla_{\underline{x}})\mathrm{e}^{\mathrm{i}t\underline{\Sigma}(-\mathrm{i}\nabla_{\underline{x}})}, \tag{3.12}$$

we incorporate the free time evolution. Finally, by approximating the compact operator $\underline{h}(\underline{x}/t+\nabla\underline{\Sigma}(-\mathrm{i}\nabla_{\underline{x}}))\chi(-\mathrm{i}\nabla_{\underline{x}})$ with finite-rank projections, we can indeed approximate the detector (3.11) by finite linear combinations of terms of the form (3.7). A careful reader may notice that the latter limit has to be exchanged with the limit $t \to \infty$. It turns out that this is not trivial and relies on assumption (b) about the disjointness of supports of h_i. This technical point, which we do not discuss further here, will play a role in the next section.

4 Towards conventional asymptotic completeness

While the conventional asymptotic completeness relation (3.1) does not follow from the general assumptions stated in the Introduction, it may well hold in a more restrictive framework. Let us try to identify here additional conditions which imply this property. For concreteness, we focus on the relativistic case.

As suggested in [Bu94],[Ha, Section VI.1.2] and shown under certain assumptions in [Dy09, Dy10], the presence of an energy density T^{00} among the quantum fields of the theory is relevant for particle aspects. Namely, the relation $H = \int T^{00}(x)d\mu(x)$ allows the Hamiltonian with the Araki-Haag detectors (3.2) to be approximated, and ensures their non-vanishing limit points as $t \to \infty$ on states of non-zero energy. Referring to [Dy09, Dy10] for more details on such approximation procedure, let us state it here schematically as

$$H \simeq \int d\mu(x)\, \alpha_x(B^*B) \tag{4.1}$$

and try to combine it with the observations from the previous section. Suppose, by contradiction, that there is a vector Ψ in the region Δ of $\mathrm{Sp}\, U$ (cf. Figure 1) which is orthogonal to all the scattering states of particles living on the isolated mass shell[5]. Since energy is strictly positive in Δ, we can write, using (4.1) and setting $\Psi_t := U(t)\Psi$, $B(x) := \alpha_x(B)$,

$$\begin{aligned} 0 \;&<\; \langle\Psi, H\Psi\rangle = \langle\Psi_t, H\Psi_t\rangle \simeq \int d\mu(x)\, \langle\Psi_t, (B^*B)(x)\Psi_t\rangle \\ &\leq\; c\int d\mu(x)\, \langle\Psi_t, B^*(x)HB(x)\Psi_t\rangle, \end{aligned} \tag{4.2}$$

where in the last step we used that $B\Psi_t$ lives on the isolated mass shell (cf. Figure 1) on which H is bounded from below by a positive constant. By applying (4.1) again, we obtain

$$\begin{aligned} &0 < \langle\Psi, H\Psi\rangle \lesssim \\ &c\int d\mu(x_1)d\mu(x_2)\, \langle\Psi_t, B^*(x_1)B^*(x_2)B(x_2)B(x_1)\Psi_t\rangle h_1\left(\tfrac{x_1}{t}\right) h_2\left(\tfrac{x_2}{t}\right). \end{aligned} \tag{4.3}$$

[5]For example, Ψ could be a 'bound state' from an embedded mass shell passing through Δ. We will rule out this possibility only in (4.4) below.

Here we inserted the approximate characteristic functions $h_1, h_2 \in C_0^\infty(\mathbb{R}^d)$ of sufficiently large balls, since the resulting error terms involve detectors of particles moving faster than light which vanish as $t \to \infty$. The expression on the r.h.s. of (4.4) can be put in a similar form as the r.h.s. of (3.11), which in turn was approximated by linear combinations of $|\Psi^{\rm out}\rangle\langle\tilde{\Psi}^{\rm out}|$. Since Ψ is orthogonal to all such scattering states, one might (erroneously) suspect that the r.h.s. of (4.4) tends to zero as $t \to \infty$ and gives the desired contradiction. This overlooks the fact that h_1, h_2 in (4.4) have overlapping supports, in conflict with assumption (b) of Theorem 2. As a consequence, the last step of the comparison of (3.11) with $|\Psi^{\rm out}\rangle\langle\tilde{\Psi}^{\rm out}|$ (the exchange of two limits mentioned at the end of Section 3) requires additional assumptions. Without going into details, we merely state here that a sufficient condition has the schematic form

$$\int d\mu(x)\langle\Psi_t, (C^*C)(x)\Psi_t\rangle = O(t^{-\varepsilon}), \tag{4.4}$$

for suitable $\varepsilon > 0$ and all $C \in \mathfrak{B}$ s.t. $\mathrm{Sp}_{C^*}\alpha \subset \Delta$. Such C^* create from the vacuum hypothetical particles living in the region Δ of the multiparticle spectrum. If our theory contained such particles, e.g. corresponding to additional mass shells embedded in the multiparticle spectrum, then the conventional asymptotic completeness relation (3.1), taking only the isolated mass shell $p \mapsto \Sigma(p)$ into account, would fail. Accordingly, the Araki-Haag detector on the l.h.s. of (4.4) would have a non-zero limit for Ψ from such embedded mass shells. This follows by a simple modification of formula (3.3) and is in conflict with condition (4.4). Thus the role of this latter condition is to ensure sufficient regularity of the spectrum inside Δ, in particular the absence of embedded mass shells. We recall that the Mourre estimate plays a similar role in quantum-mechanical proofs of asymptotic completeness.

Summing up, we expect that the properties schematically stated in (4.1) and (4.4) are the key additional ingredients needed for conventional asymptotic completeness. The problem of their proper mathematical formulation and verification in concrete models is left for future research.

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Welcome to Jan Philip Solovej, new Editor-in-Chief of JMP !

The IAMP is pleased to welcome Jan Philip Solovej as the new Editor-in-Chief of the *Journal of Mathematical Physics*. The appointment was announced by the American Institute of Physics in February with the following profile:

Prof. Solovej is currently Professor of Mathematics at the University of Copenhagen, and the Centre Leader of the Villum Centre of Excellence for the Mathematics of Quantum Theory (QMATH). His research focuses on mathematical understanding of quantum physics, specifically in the stability and structure of atoms, Bose and Fermi gases, and thermodynamic stability.

In addition to his faculty and QMATH positions, Prof. Solovej currently sits on the Board of the Independent Research Fund in Denmark and was elected to the Royal Danish Academy of Sciences and Letters in 2000. He was previously a faculty member at Princeton University, where he also earned his Ph.D. in 1989, and held various positions at the Erwin Schrödinger Institute, Université Paris (6 and Dauphine), Aarhus University, University of Toronto, and University of Michigan. He has served on the editorial boards of the *Journal of European Mathematical Society* and the *Journal of Spectral Theory*.

Herbert Spohn wins the 2019 Boltzmann Medal

Congratulations to Herbert Spohn of the Technical University Munich, who will receive the 2019 Boltzmann Medal at the StatPhys 27 Meeting in Buenos Aires in July! Spohn is being honored for "his wide-ranging and highly influential work in non-equilibrium statistical physics." The *News Bulletin* anticipates a full announcement in due course.

News from the IAMP Executive Committee

New individual members

IAMP welcomes the following new members

1. Dr. Ivan Contreras, Amherst College, USA
2. Dr. Niels Benedikter, IST Austria
3. Dr. Domenico Monaco, University of Roma 3, Italy
4. Prof. Madalin Guta, University of Nottingham, UK
5. Dr. Giacomo De Palma, University of Copenhagen, Denmark
6. Dr. George Jaroszkiewicz
7. Dr. Mark Scrowston, Dublin, Ireland

Recent conference announcements

Operators, Functions, and Systems of Mathematical Physics (OFSMP 2019).
June 10-14, 2019. Baku, Azerbaijan.
This conference is partially supported by IAMP.
http://inspirehep.net/record/1719156

GLaMP 2019: the fourth Great Lakes Mathematical Physics Meeting
June 28-30, 2019. Oberlin College, Ohio USA.
This conference is partially supported by IAMP.
https://sites.google.com/msu.edu/glamp2019/home

3rd IMA Conference on Nonlinearity and Coherent Structures
July 10-12, 2019. Northumbria University, Newcastle upon Tyne (UK).
https://ima.org.uk/10959/3rd-ima-conference-on-nonlinearity-and-coherent-structures/

7th Bremen Dynamics Summer School and Symposium on Dynamical systems - pure and applied

Aug. 5-9, 2019. University of Bremen (Germany).

http://sos.math.uni-bremen.de/

QMath 14: Mathematical Results in Quantum Physics

Aug. 12-16, 2019. Aarhus University (Denmark).

This conference is partially supported by IAMP.

http://conferences.au.dk/qmath14/

Quantissima in the Serenissima III

This conference is partially supported by IAMP.

Aug. 19-23, 2019. Venice (Italy).

http://www.mathphys.org/Venice19/

Summer school and Workshop on Quantum Transport and Universality

Sept. 16-25, 2019. University Roma 3 (School: Sept. 16-19) and Accademia Nazionale dei Lincei, Roma (Workshop: Sept. 23-25).

The Analysis of Complex Quantum Systems: Large Coulomb Systems and Related Matters

Oct. 21-25, 2019. CIRM, Luminy (France)

This conference is partially supported by IAMP.

https://conferences.cirm-math.fr/2066.html

Open positions

Professor of Mathematics and Physics, ETH Zurich (Switzerland)

The Department of Mathematics (http://www.math.ethz.ch) and the Department of Physics (http://www.phys.ethz.ch) at ETH Zurich invite applications for the above-mentioned position. The new professor will be based in the Department of Mathematics and associated to the Department of Physics.

Applicants should demonstrate an outstanding research record and a proven ability to direct research work of high quality. The successful candidate should have a strong background and a worldwide reputation in mathematical physics as well as excellent teaching skills. Teaching responsibilities will mainly involve undergraduate (German or English) and graduate courses (English) for students in mathematics, physics and engineering.

Please apply online: http://www.facultyaffairs.ethz.ch

Applications should include a curriculum vitae, a list of publications, a statement of future research and teaching interests, and a description of the three most important achievements. The letter of application should be addressed to the President of ETH Zurich, Prof. Dr. Joël Mesot. The closing date for applications is 15 September 2019. ETH Zurich is an equal opportunity and family friendly employer and is responsive to the needs of dual career couples. We specifically encourage women to apply.

Junior Research Start Fellowships, Munich (Germany)

The Munich Center for Quantum Science and Technology (MCQST) is a new Cluster of Excellence funded by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) under Germany's Excellence Strategy. MCQST is covering all areas of quantum science and technology from basic research to applications.

To support young talents on their transition from postdocs to independent researchers, MCQST is offering Junior Researcher START Fellowships. With a budget of 300.000 EUR for a period of two years, the fellow has the opportunity to start her/his own scientific project. The funding can be used for instrumentation, staff, consumables or travels, in accordance with DFG rules for the research project. If necessary, the fellow can use part of the funds to finance her/his own position. Laboratory and office space have to be provided by one of the MCQST Principle Investigators (PIs) or participating institutions.

Applications for the Junior Researcher START Fellowships should include: CV, list of publications and talks, two-page research proposal, support letter from MCQST PI or institution, committing to offer laboratory and office space, letter of recommendation from a third person.

Please send all documents as single PDF-file to tatjana.wilk@lmu.de until 15 May 2019.

Together with the International Advisory Board, the MCQST Executive Committee will select one Junior Research START Fellow in 2019 (with subsequent calls in following years).

Selection criteria are scientific excellence and an original research proposal. The start of the funding period will be defined together with the MCQST office, but should not start later than 01 January 2020.

PhD position at University of Basel, Switzerland

A 4 year PhD position is available in the Dept. of Mathematics and Computer Sciences of the University of Basel, Switzerland.

The position is funded by the Swiss National Science Foundation through the project "From Newton and Schrödinger many-body dynamics to the Boltzmann equation", coordinated by Chiara Saffirio.

Candidates are expected to have a strong background in analysis and to be interested in kinetic theory and/or quantum mechanics. The position has limited teaching duties (courses can be offered in English or German).

Interested candidates should send a CV and should arrange for two recommendation letters to be sent to chiara.saffirio@math.uzh.ch. Deadline for application is May 31, 2019. For additional information, contact Chiara Saffirio at the address above.

Postdoc position at University of Basel

A 2 year postdoc position is available in the Dept. of Mathematics and Computer Sciences of the University of Basel, Switzerland.

The position is funded by the Swiss National Science Foundation through the project "From Newton and Schrödinger many-body dynamics to the Boltzmann equation", coordinated by Chiara Saffirio.

Candidates are expected to have a strong background in analysis and to be interested in kinetic theory and/or quantum mechanics (preferably with focus on derivation of effective equations). The position has limited teaching duties (courses can be offered in English or German).

Interested candidates should send a CV and should arrange for two recommendation letters to be sent to chiara.saffirio@math.uzh.ch. Deadline for application is May 31, 2019. For additional information, contact Chiara Saffirio at the address above.

For more information on these positions and for an updated list of academic job announcements in mathematical physics and related fields visit

http://www.iamp.org/page.php?page=page_positions

Benjamin Schlein (IAMP Secretary)

WOJCIECH DYBALSKI
Zentrum Mathematik
Technische Universität München
Boltzmannstr. 3
85748 Garching bei München
dybalski@ma.tum.de

BENJAMIN SCHLEIN
Institut für Mathematik
Universität Zürich
Winterthurerstrasse 190
8057 Zürich, Switzerland
secretary@iamp.org

EVANS HARRELL
School of Mathematics
Georgia Institute of Technology
Atlanta, GA 30332-0160, USA
bulletin@iamp.org