IAMP News Bulletin
January 2016

International Association of Mathematical Physics

International Association of Mathematical Physics News Bulletin, January 2016

Contents

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Cover picture: Winter at the Mittag-Leffler Institute

ISSN 2304-7348
News Bulletin (International Association of Mathematical Physics)

Renormalizable and asymptotically free tensor field theories

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The author received the International Union of Pure and Applied Physics Young Scientist Prize in Mathematical physics in 2015 for his pioneering work on the renormalization of tensor field theories and his discovery of their generic asymptotic freedom.

To all the students of the developing world

The main question of these notes conveys sundry facets that, I must confess, I was not primarily interested in when I initially decided to dive into the subject called quantum gravity. I always loved mathematics and have always been fascinated by math beyond physical phenomena. Along the way, however, pressing questions about the physics of the abstract objects and graphs that I drew, integrals and sums that I computed, started to take over, simply because they become natural questions of the framework. Like a curious creature that always wants to surprise me, the intriguing interplay between mathematics and physics that I cannot anymore distinguish in my scientific life, coalesces today, I guess, in the award of the IUPAP Young Scientist Prize in Mathematical physics. I deeply thank the International Union of Pure and Applied Physics and its selection committee in mathematical physics for judging my work worthy of this international prize, which I wish to dedicate to all the students of the developing world, among whom I was once numbered.

1 Introduction

The research question and its context

Nowadays, one of the most investigated questions in physics, is formulated as: how does Einstein's theory of general relativity, which explains the geometry of spacetime and relates it to its matter content, extend to a sensible microscopic theory in regimes when the geometry of spacetime must itself fluctuate? The regime in question is that of very small distances typically of the order of the Planck length, or else that of very high energy, including the physics of the early universe. There is a wide consensus among physicists that the laws of physics as we know them will inevitably fail at such a scale, involving colossal energies compared to those of our experiments today. Consensus refers to no experimental evidence available at this scale from which we could conclusively confirm this hypothesis. A putative reason of the breaking of these laws would be that, at extremely high energy, our spacetime would no longer be smooth but would acquire new properties, making it a quantum spacetime. The previous question consequently relates to another: the physics that we know happens in a smooth spacetime, but what does physics become in a quantum spacetime? Some progress has been made in the last decades to understand this small-scale regime [1]-[6], although today no theory provides a complete picture, and, more to the point, no scenario available today can be experimentally tested.

One of greatest challenges of working on a quantum spacetime is to describe physical phenomena at the Planck length scale without experimental guidance. The Planck length is about 10^{-35} meters and, naturally, it might be difficult to apprehend how small that is. The electroweak length (10^{-18} meters) is the smallest size detected today by particle colliders. This is where our cutting-edge technology stops. But comparing an object that is 10^{-18} meters to an object of the Planck length is like comparing the earth to an atom! What is remarkable is that theoretical and mathematical physics allow one to investigate the laws of physics even on that tiny scale.

Our approach and goal

Several motivations make us adopt the point of view that our spacetime could be actually built from discrete geometrical structures. These discrete structures are like building blocks of Planck length scale, which are glued to form the "fabric" of our spacetime. We have a proposal for: what is a quantum spacetime? Precisely, our goal is to define a theory which models the building blocks of our spacetime and their evolution, together with a robust mechanism which will allow us to predict the formation of our spacetime and to justify why it then appears to us in the way that we perceive it.

Overview of tensor field theories

A central question is how to make the foregoing proposal mathematically and physically consistent, and this entails choosing a framework. The framework must be regular enough to perform calculations and fertile enough to lead to interesting predictions.

Among the few schemes which turn out to be successful for addressing the reconstruction of a spacetime and recovering the laws of gravity on it is the so-called framework of matrix models [7]. A matrix model generates Feynman ribbon graphs mapped to triangulations (polygonizations more generally) of surfaces. From random discrete triangulations to continuum 2D geometry, one proves generically that matrix models undergo a phase transition at the limit when the building blocks proliferate while the area of each block becomes null. The theory of random matrices then became a success story for quantum gravity. Soon after, matrix models were generalized to tensor models [8] in view of extending the previous success to any $D > 2$. However, as topology and geometry in higher dimensions remain difficult subjects, tensor models faced dramatic issues. We emphasize that the main tool for addressing analytically the partition function of matrix models named 't Hooft $1/N$ expansion [9] was crucially missing for tensor models. The partition function of a rank D tensor model generates simplicial manifolds in dimension D which could not be sorted out at that time. In last resort, only numerical results have been achieved in ranks higher than 2. Ambjørn et al. inferred from their numerics that the type of geometry obtained at the continuum was very singular and related to branched polymer geometries. Hence, because of their intractable behavior and their singular continuum limit, tensor models were quite abandoned or deeply reformulated.

An enriched version of tensor models has been stated by Boulatov in [10]. Tensors become tensor fields over several copies of an abstract Lie group as a more convenient scheme to address a lattice gauge theory version of 3D Euclidean gravity. Depending on the dimension, the tensor-field interactions describe vertices which provide fusion rules and exchanges of momenta as in ordinary quantum field theory (QFT). Thus was born a new line of investigation for quantum gravity and geometry, known as group field theory (GFT) [11]. While the problem of obtaining an emergent spacetime at some proper continuum limit was to be understood, the GFT framework appeared very appealing for performing quantum field theory computations and, naturally, the question of its renormalizability was systematically addressed. Several power-counting theorems were formulated without reaching full renormalizability of the most prominent models. One of the most crucial aspects of Boulatov/GFT is their nonlocal interactions: fields interact in a region of the background which does not reduce to a point. This feature radically differs from other types of QFTs and enhances the complexity to identify the correct generalized locality principle [12] for this class of models. In a different setting, Grosse and Wulkenhaar (GW) in [13] discovered a nonlocal and renormalizable field theory derived from noncommutative geometry [3]. The GW model translates to a nontrivial matrix model and therefore it was clear that a reduced rank 2 tensor model was renormalizable. Nevertheless, in that time, none of the techniques developed for the GW model had been exported, whether to tensor models or to GFTs.

In 2009, Gurau introduced a simple but powerful idea [15] which will change the story of tensor models. In Gurau's theory, each tensor has a supplementary index called color with a special rule for gluing the colored simplexes. For the first time, the type of topology generated at the level of the partition function of tensors was understandable. Colored tensor models generate Feynman graphs associated with simplicial pseudo-manifolds in any dimension [15]. One year later, Gurau revealed an appropriate notion of $1/N$-expansion [16] for the very same class of models. It did not take long to analytically prove that colored tensor models undergo a phase transition [17] and to conclude that the transition leads to branched polymers as found by Ambjørn et al. twenty years ago by numerics. Branched polymers, being singular, colored tensor models still have a way to go for extracting, via a mechanism like phase transition, a geometry similar to that of our present spacetime. However, it was clear that more was about to be revealed from this type of models.

Colored tensor models were also at the basis of the discovery by Ben Geloun and Rivasseau of the first type of renormalizable nonlocal field theories using tensors of rank $D \geq 3$ [18]. The model, called tensor field theory (TFT), generates 4D simplicial geometries and was built with fields over $U(1)^4$. Endowed with a Laplacian dynamics like usual QFT, TFT interactions were chosen among a set of unitary invariants obtained from colored models [19]. Written in the momentum space, TFT extends the GW model to higher rank but also modifies its dynamics. In the last four years, the renormalization program for TFTs has achieved many results and uncovered renormalizable actions which could not have been guessed just a few years ago. Several follow-up studies have been performed on that framework [20]-[22] (see [22] and [21] for reviews) including the solution of the GFT renormalization issue.

Why is renormalizability important for TFT, or for any tensor model? Renormalizability for any quantum field theory is a very desirable feature because it mainly ensures that the theory is consistent and survives after several energy scales. All known interactions of the standard model are renormalizable. Renormalizability also gives a mathematical sense to a system dealing with infinitely many degrees of freedom by subtracting divergences entailed by those in physical quantities. Quantum field theory predictability relies on the fact that, from the Wilsonian point of view [23, 24], these infinities should not be ignored but should locally (from one scale to the other) reflect a change in the form of the theory [12]. In particular, if TFTs are to describe at low energy any physical reality like our spacetime, the renormalization group (RG) analysis for TFTs offers a natural mechanism to flow from a certain model describing a given simplicial geometry at some scale to another with another geometry at another scale while dealing consistently with these infinities.

TFTs possess also another interesting feature. Several models turn out to be UV asymptotically free [20]-[22][25]. In general, a model is called UV asymptotically free if it makes sense at arbitrary high energy scales and possesses a trivial UV fixed point defined by the free theory. Quantum chromodynamics (QCD), the theory of strong interactions, is a typical example of this kind. From the UV going in the IR direction, the renormalized coupling constants grow up to some critical value, at which one reaches a new phase described in terms of new degrees of freedom (quark confinement in QCD). If tensor models are generically asymptotically free, this could be a nice feature because it would mean that, (1) in the case that these models actually describe a theory of gravity, this theory would be sensible at arbitrary high energy, and (2) in the IR, the models likely experience a phase transition after which, hopefully, the final degrees of freedom may encode more geometrical data than the initial ones.

In the next section, we will present a generic TFT model and explain few considerations about it. Then, a streamlined analysis of the amplitudes will be given as well as its associated power counting theorem. We explain that the analysis of the divergence degree yields a list of renormalizable models. Section 3 gives a summary of the results and perspectives of TFTs.

2 Renormalizable TFTs

TFT Models

We now present TFT as a quantum field theory with tensor fields. In the way that a Dirac field describes a spin-$\frac{1}{2}$ particle like an electron or a quark and a gauge field describes a spin-1 particle like a photon, the *tensor* field would describe an elementary particle of spacetime itself. Adapted to the present situation, our ultimate goal is to make consistent the scenario that the tensor fields evolve under a certain dynamics to form a large and continuous universe in which the principles of general relativity must be valid. Our results are encouraging for the realization of this goal although it will take striving efforts to get there. We present in the following the simplest class of TFT models which have been found renormalizable.

Consider a rank d complex tensor $\phi_{\mathbf{P}}$, with $\mathbf{P} = (p_1, p_2, \dots, p_d)$ a multi-index, and denote $\bar\phi_{\mathbf{P}}$ its complex conjugate. The nature of indices p_k can be chosen of several types but, for simplicity, we consider here that these belong to integers: $p_k \in \mathbb{Z}$. This choice corresponds, for instance, to a field theory over d copies of $U(1)$ and consequently $\phi_{\mathbf{P}}$ denote nothing but the Fourier components of a tensor field $\phi : U(1)^d \to \mathbb{C}$. As a physical input, we consider $\phi_{\mathbf{P}}$ as a $(d-1)$ simplex. We refer to such a correspondence, between tensor fields and simplexes, as *duality* in the following.

An action S of a tensorial model is built by convoluting replicas of $\phi_{\mathbf{P}}$ and $\bar\phi_{\mathbf{P}}$ using kernels. S is of the general form

$$
\begin{aligned}
&S[\bar\phi, \phi] = \mathrm{Tr}_2(\bar\phi \cdot K \cdot \phi) + \mu\, \mathrm{Tr}_2(\bar\phi \cdot \phi) + S^{\mathrm{int}}[\bar\phi, \phi]\,, \\
&\mathrm{Tr}_2(\bar\phi \cdot K \cdot \phi) = \sum_{\mathbf{P},\mathbf{P}'} \bar\phi_{\mathbf{P}}\, K(\mathbf{P};\mathbf{P}')\, \phi_{\mathbf{P}'}\,, \qquad \mathrm{Tr}_2(\bar\phi \cdot \phi) = \sum_{\mathbf{P}} \bar\phi_{\mathbf{P}} \phi_{\mathbf{P}} = \sum_{p_i} |\phi_{p_1,p_2,\dots p_d}|^2\,, \\
&S^{\mathrm{int}}[\bar\phi, \phi] = \sum_{n_b} \lambda_{n_b} \mathrm{Tr}_{n_b}(\bar\phi^{n_b} \cdot \mathcal{V}_{n_b} \cdot \phi^{n_b})\,,
\end{aligned} \tag{1}
$$

where, like a generalized trace, Tr_n denotes a summation over indices of the n couples of tensors $\phi_{\mathbf{P}}$ and $\bar\phi_{\mathbf{P}}$, K and $\mathcal{V}_{n_b}$ are kernels to be specified, and μ (mass) and λ_{n_b} are coupling constants. The index b describes a particular type of summation or convolution pattern. Setting $\mathcal{V}_{n_b}$ to unit weight and restricting the range of the tensor indices p_k to a finite integer N_k, Tr_{n_b} generates a unitary invariant [19]. Specifically, we characterize the kinetic term K in rank d by giving $K(\mathbf{P};\mathbf{P}') = \boldsymbol{\delta}_{p_i,p'_i}(\sum_{i=1}^d p_i^{2a})$, with $\boldsymbol{\delta}_{p_i,p'_i} := \prod_{i=1}^d \delta_{p_i,p'_i}$. For a field $\phi : U(1)^d \to \mathbb{C}$, the kernel K is the sum of $2a$-power of the eigenvalues of d Laplacian operators over the d copies of $U(1)$. The real parameter a is free at this point but we will choose it positive and such that $0 < a \le 1$. The kinetic term $\mathrm{Tr}_2(\bar\phi \cdot K \cdot \phi) + \mu\, \mathrm{Tr}_2(\bar\phi \cdot \phi)$, at the quantum level, is associated with a Gaussian field measure

$$
d\nu_C(\bar\phi, \phi) = \prod_{\mathbf{P}} d\bar\phi_{\mathbf{P}} d\phi_{\mathbf{P}}\, e^{-[\mathrm{Tr}_2(\bar\phi \cdot K \cdot \phi) + \mu\, \mathrm{Tr}_2(\bar\phi \cdot \phi)]} \tag{2}
$$

of covariance $C = 1/(K+\mu)$ for a positive definite kernel $K+\mu$. Usually called the propagator in the language of field theory, $C \equiv C(\mathbf{P};\mathbf{P}')$ is graphically represented by a collection of d segments called a stranded line (see Figure 1). We give now examples of interactions. At fixed $d = 3$, we can write a ϕ^4-type interaction using a convolution as

$$
\mathrm{Tr}_{4;1}(\phi^4) = \sum_{p_i,p'_i} \phi_{123}\, \bar\phi_{1'23}\, \phi_{1'2'3'}\, \bar\phi_{12'3'}\,, \quad \phi_{123} := \phi_{p_1,p_2,p_3}\,. \tag{3}
$$

In any rank d, this contraction pattern of the 4 tensors easily extends using the same cyclic pattern, see Figure 1. It should be clear that the interaction (3) is not symmetric in its indices (the index 1 breaks the color symmetry). We will always consider a symmetric interaction $S^{\rm int}$ by always including all symmetric partners associated with a given convolution pattern. A general interaction is also graphically represented by a stranded vertex

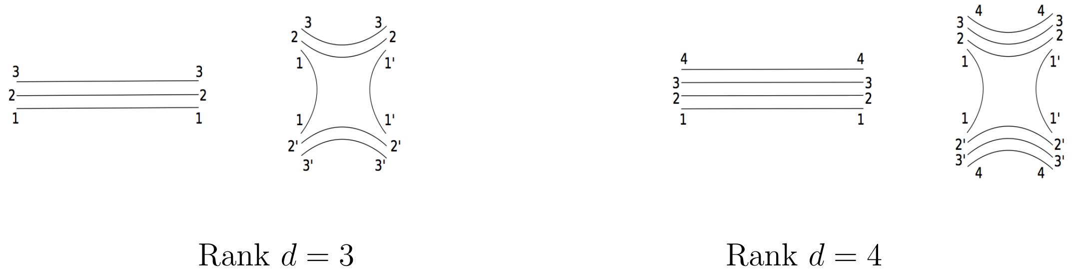

Figure 1: Examples of rank $d = 3$ and 4 propagators (stranded lines) and vertices $\mathrm{Tr}_{4;1}(\phi^4)$ which are non symmetric with respects to their strands. Vertices represent triangulation of spheres.

where each index contracted between a $\bar\phi$ and a ϕ is drawn as a segment between these two fields. A vertex represents in this framework a d-simplex (in the particular case of (3) a sphere) obtained by identifying the boundary of its fields, which we recall are dual to $(d-1)$ simplexes. Thus, a general TFT Feynman graph is a collection of stranded vertices joined by stranded propagator lines and represent the gluing of d-simplexes along their boundary $(d-1)$-simplexes. Therefore, they define simplicial complexes. An example of a TFT Feynman graph is given in Figure 2A.

Correlation functions or correlators of any interacting theory are generally difficult to evaluate. Hence one focuses on an expansion of correlators at small coupling constants evaluated with a Gaussian measure similar to $d\nu_C$ (2). In most cases, the result of that expansion diverges, and this calls for the regularization procedure called renormalization. Each correlator at small couplings expands in Feynman graphs via Wick's theorem. The understanding of the correlators of any QFT reduces to the study of Feynman graph amplitudes. As in usual QFTs, divergences occur in TFTs because of sums over infinite degrees of freedom. At the graphical level, these divergences can be localized by the presence of close loops also called internal faces (1 dimensional manifolds homeomorphic to circles, see Figure 2B).

Renormalizable models

We investigate some conditions on TFTs which yield a regularization of their Feynman amplitudes and then further lead to their perturbative renormalizability. We will restrict to $\phi : (U(1)^D)^{\times d} \to \mathbb{C}$, producing a $D \times d$ field theory. However the roles played by

(A) (B)

Figure 2: (A) A rank $d = 3$ TFT graph $\mathcal{G}$ made with two $\mathrm{Tr}_4(\phi^4)$ vertices and 2 external legs. (B) An internal face (in bold) of $\mathcal{G}$.

the two parameters, namely D and d, are different. The kernel K extends simply over $(U(1)^D)^{\times d}$.

As an approach for perturbative renormalization, we will use that of multiscale analysis [12]. The analysis starts by a slice decomposition of the propagator as $C = \sum_{i=0}^{\infty} C_i$ where C_i, the propagator in the slice i, satisfies the bound $C_i \leq kM^{-2i}e^{-\delta M^{-i}(\sum_{s=1}^{d}|p_s|^a+\mu)}$ for some constants k, $M > 1$ and $i > 0$, and $C_0 \leq k$. For all $a \in (0, 1]$, high i probes high momenta p_s of order $M^{\frac{i}{a}}$ that we call ultraviolet (UV) regime (corresponding to short distances on $U(1)$). Therefore, the slice 0 refers to the infrared (IR). The regularization scheme requires to introduce UV cut-off Λ on the sum over i. The cut-off propagator reads as $C^\Lambda = \sum_{i=0}^{\Lambda} C_i$.

As usual, we write any amplitude associated with a graph $\mathcal{G}(\mathcal{V}, \mathcal{L})$ as a product of propagators constrained by vertex kernels: $A_{\mathcal{G}} = \sum_{p_{s;v}} \prod_{\ell\in\mathcal{L}} C[\{\mathbf{P}_{v(\ell)}\}, \{\mathbf{P}'_{v'(\ell)}\}] \prod_{v\in\mathcal{V};s} \delta_{p_{s;v};p'_{s;v}}$. Slice all propagators using the slice decomposition, and collect the resulting momentum scales $i_\ell \in [0, \Lambda]$ in a multi-index $(i_\ell)_{\ell\in\mathcal{L}}$. The question is: what is the dependence of $A^\Lambda_{\mathcal{G}}$ in the cut-off Λ? The answer to that question can be given by an optimal integration of internal momenta. That analysis is rather involved. We shall only give the upshot of that analysis.

The following statement holds (power counting theorem [22]): Let $\mathcal{G}$ be a connected graph of the model (1), with set $\mathcal{L}(\mathcal{G})$ of lines with size $L(\mathcal{G})$, and set $\mathcal{F}_{\mathrm{int}}(\mathcal{G})$ of internal faces with size $F_{\mathrm{int}}(\mathcal{G})$, there exists a large constant $K_{\mathcal{G}}$ such that

$$|A_{\mathcal{G}}| \leq K_{\mathcal{G}}\, \Lambda^{\omega_{\mathrm{d}}(\mathcal{G})}, \qquad \omega_{\mathrm{d}}(\mathcal{G}) = -2aL(\mathcal{G}) + DF_{\mathrm{int}}(\mathcal{G}). \tag{4}$$

The quantity $\omega_{\mathrm{d}}(\mathcal{G})$ is called the superficial divergence degree of the graph $\mathcal{G}$ and indicates if the amplitude related to $\mathcal{G}$ is divergent (when $\omega_{\mathrm{d}}(\mathcal{G}) \geq 0$) or not.

The number of internal faces of $\mathcal{G}$ can be calculated in terms of the Gurau degree of the underlying colored graph [16] and of the degree of the boundary graph (encoding the boundary of the dual simplicial complex). Gurau's degree is a sum of genera of surfaces defined by a colored and canonical decomposition of the TFT graph. A graph is proved to be maximally divergent if its degree vanishes. In renormalization analysis, we exclusively deal with graphs with half-edges representing external fields. In TFT, the class of graphs which are diverging includes those which have a vanishing degree, with a vanishing degree of their boundary graph and a restricted number of external fields. These provide quite stringent conditions (a, D, d) and the maximal power $k_{\max}$ of the interactions $\mathrm{Tr}_{k_{\max}}(\phi^{k_{\max}})$ to identify a type of renormalizable model. The list of these conditions is called the locality principle of the model. Using this list, we can perform subtraction of the divergences by modifying (renormalizing) the coupling constants in order to make finite any amplitude when Λ is finally sent to infinity. The equations of the renormalized couplings in terms of the initial couplings define the so-called β-function equations which encode the renormalization group flow of the model.

The fine analysis of the previous degree of divergence of a TFT graph allows one to obtain the following table of renormalizable models as well as their UV asymptotic behavior after calculation of their β-functions:

Type	G_D	$\Phi^{k_{\max}}$	d	a	Renormalizability	UV behavior
TFT	$U(1)$	Φ^6	4	1	Just-	?
TFT	$U(1)$	Φ^4	3	$\frac{1}{2}$	Just-	AF
TFT	$U(1)$	Φ^6	3	$\frac{2}{3}$	Just-	?
TFT	$U(1)$	Φ^4	4	$\frac{3}{4}$	Just-	AF
TFT	$U(1)$	Φ^4	5	1	Just-	AF
TFT	$U(1)^2$	Φ^4	3	1	Just-	AF
TFT	$U(1)$	Φ^{2k}	3	1	Super-	AF
gi-TFT	$U(1)$	Φ^4	6	1	Just-	AF
gi-TFT	$U(1)$	Φ^6	5	1	Just-	?
gi-TFT	$SU(2)^3$	Φ^6	3	1	Just-	?
gi-TFT	$U(1)$	Φ^{2k}	4	1	Super-	AF
gi-TFT	$U(1)$	Φ^4	5	1	Super-	AF

Table 1: Updated list of renormalizable TFT models and their features; gi-TFTs are TFTs supplemented by the so-called gauge constraints [11][21]. AF $\equiv$ asymptotically free.

3 Conclusion and perspectives

We introduced TFTs in their simplest form and discuss their renormalizability. Renormalizability is a very desirable feature for any QFT, because it reveals a sort of economy which powers physical predictability: one does not need more than a finite number of coupling constants and their equations to determine the full physics of the model. To list renormalizable models and to inventory their features matter for understanding the tensor theory space, for instance, this will allow us to focus only on their universality class. Under some hypothesis, our work consisted in providing that list.

The statement of asymptotic freedom concerns the evolution and the limit of the model if one follows its evolution at higher and higher energy, towards the microscopic theory. As a result, we proved that several TFTs evolve towards the free theory. This implies that, at microscopic level, several TFT models are always well defined. Another interesting fact to discuss is the evolution of the model in the opposite regime, when energy decreases. Then, if there is asymptotic freedom, it generally means that, at some lower energy, the evolution of the model is subject to a drastic change, and perhaps a phase transition occurs. A well-known theory having this property is QCD: the coupling of elementary particles, called quarks, becomes larger and larger, which induces a binding of these particles to form new composite particles (hadrons), the most stable of which are involved in the formation of the atomic nucleus (these are protons and neutrons). For tensorial field theory, asymptotic freedom becomes interesting indeed because we do not want to stay in a phase where the geometrical spacetime is apparently discontinuous because spanned by building blocks. The hope here is that, as in QCD, asymptotic freedom will induce a new phase for tensorial field models, with new degrees of freedom ("binding" of tensors to draw a parallel with the binding of quarks) able to generate a nice geometrical universe.

The next big challenge in TFT is to go beyond perturbation theory. Already, interesting features of TFT arise in their most simple truncation: the existence of an IR fixed point [26]. If this IR fixed point is stable and generic, we could certainly provide greater details about the phase transitions and diagrams in the IR, towards new geometrical phases of discrete models.

The domain of applicability of our results remains again theoretical. As an attempt to quantize spacetime, our work might be meaningful for this set of approaches, shedding more light on how to tame divergences in frameworks studying discrete spacetimes (like causal dynamical triangulations, noncommutative/nonassociative geometry, spin-foam models etc.). Beyond the realm of quantum gravity, the results and techniques developed in tensorial models might be useful for field theories using nonlocal interactions, like effective field theories. In a different area, TFTs are statistical models, and our results might be important to extend probability theory to random matrix and tensor variables. We have also developed new combinatorial techniques generalizing results in graph theory and simplicial geometry. Our hope is that our results will be useful to combinatoricians interested in finding new invariants on graphs or invariants on simplicial manifolds.

Acknowledgements

J.B.G would like thank the International Union of Pure and Applied physics (IUPAP), and its Mathematical physics Commission for their interest on his work and prize. He is also deeply grateful and wishes to thank all people and institutions which have contributed and supported this approach and his own research on tensorial field theory which has led to this award.

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Parafermionic observables and their applications

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The author received the IAMP Early Career Award in 2015 for his fundamental contributions in the rigorous analysis of critical phenomena.

Lattice models were first introduced in discrete settings for real life experiments. They have been used to model a large variety of phenomena, ranging from ferroelectrics to lattice gases. They also provide discretizations of Euclidean and quantum field theories and are as such important from the point of view of theoretical physics. While the original motivation came from physics, they later appeared to be extremely complex and rich mathematical objects, whose study provided an area of cross-fertilization between different fields of mathematics (algebra, combinatorics, probability, complex analysis, spectral theory to cite a few) and physics (quantum field theory, condensed matter physics, conformal field theory).

The zoo of lattice models is very diverse: They arise in spin-glasses, quantum chains, random surfaces, spin systems, interacting percolation systems, percolation, polymers, etc. The special class of models interesting us here applies to interfaces defined on planar lattices. These models undergo a phase transition, at which an extraordinary rich behavior occurs. Through two fundamental examples, we try to illustrate an approach combining probabilistic techniques and ideas coming from analysis on graphs to describe this behavior.

A first example: the Self-Avoiding-Walk (SAW)

SAW was first introduced by Orr in 1947 as a combinatorial puzzle. In 1953, Nobel prize winner Paul Flory popularized (and rediscovered) SAWs by proposing them as a mathematical model for the spatial position of polymer chains. While very simple to define, the SAW has turned out to be a very interesting concept, leading to a rich mathematical theory helping develop techniques that found applications in many other domains of statistical physics. To name but a few examples of tools that emerged from the study of SAWs, the lace expansion technique was developed to understand the SAW in dimension $d > 5$, and the Schramm-Loewner Evolution was introduced to describe the scaling limit of the 2D loop erased random-walk, a model directly motivated by the SAW.

Let us describe the SAW more formally (see [MS93] for more details and references). Consider the hexagonal lattice $\mathbb{H}$ (one may also work with the square lattice, but some results presented below use some integrability properties of the model that are specific to the hexagonal lattice). A path is a sequence of neighboring vertices $\gamma_1, \ldots, \gamma_n$. It

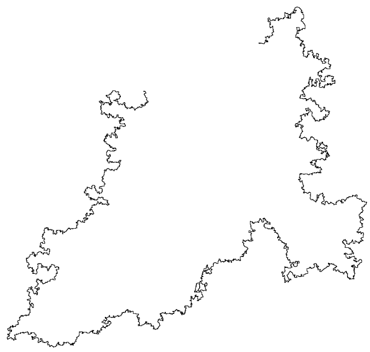

Figure 3: A typical 1000 steps self-avoiding walk.

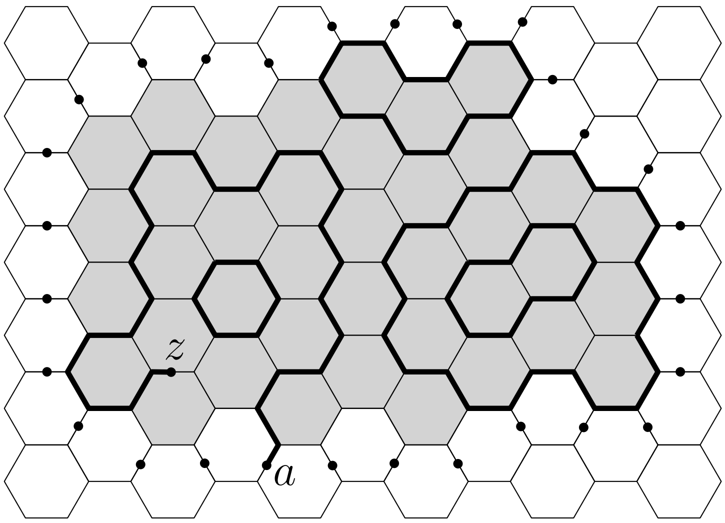

Figure 4: A loop configuration in $\widehat{\mathcal{E}}(\Omega, a, z)$ with an interface from a to z. The dots correspond to the boundary of Ω.

is self-avoiding if the map $k \mapsto \gamma_k$ is one-to-one. For each n, the model is defined by assigning equal probability to all self-avoiding paths with n vertices starting from 0.

Originally, Flory was interested in the geometric properties of the random path. In particular, he focused on the average distance $\|\gamma_n\|$ between γ_n and 0. Via a clever argument, he predicted that this average distance grows with n roughly like $n^{3/4}$. This prediction was important since it conjectures a behavior which is very different from the random walk. Interestingly, Flory's prediction was based on two assumptions which are not satisfied by the SAW. Nevertheless, destiny can be sweet and the actual behavior is indeed $n^{3/4}$: the two mistakes made by Flory (one for each assumption) miraculously cancel each other.

Before discussing the question of the mean displacement further, let us step back and focus on Orr's original contribution to the problem at hand. In his article, Orr computed the number of SAW (on the square lattice) of length less or equal to 6. With today's computers and clever algorithms, one may be able to enumerate SAWs on the hexagonal lattice up to length 105, but no exact formula giving the number of SAWs of length n in terms of n seems to emerge from such computations. Nevertheless, some nontrivial things can still be said about the number of SAWs. For instance, a simple sub-multiplicativity argument (the number of SAWs of length $n+m$ is smaller than the number of SAWs of length n times the number of SAWs of length m) implies that the number of SAWs grows exponentially fast, with a specific rate of growth μ_c depending on the lattice, and called the *connective constant of the lattice*. Much more elaborated physics arguments provided by the Coulomb gas formalism or conformal field theory refine this prediction, and suggest that this number is roughly $n^{11/32} \cdot \mu_c^n$. Interestingly, μ_c will not be the same for the square lattice as for the hexagonal lattice. Nonetheless, the polynomial correction $n^{11/32}$ is present in both cases: the exponent $11/32$ is universal.

Despite the precision of the previous predictions, the best results are very far from tight. Hammersley and Welsh proved that the number of SAWs of length n is between μ_c^n and $e^{O(\sqrt{n})}\mu_c^n$ without computing the constant μ_c (their argument dealt originally with the square lattice but it can easily be generalized to the hexagonal lattice). Concerning the mean-displacement, it is not rigorously known whether the average distance to the origin grows faster than $n^{1/2}$. Worse, while the radius of a SAW of length n is obviously larger than $n^{1/2}$, it does not imply much on the endpoint, and it is in fact unknown whether the average of $\|\gamma_n\|$ is larger than a constant times $n^{1/2}$, a statement most of us would consider tautological. Concerning upper bounds, it was proved only recently [DH13] that the SAW is sub-ballistic, in the sense that the average of $\|\gamma_n\|$ behaves like $o(n)$ as n tends to infinity. We encourage the reader to try to improve these results on his own (for instance to provide any type of quantitative upper bound). This should illustrate the intrinsic difficulty of the model.

The previous contributions on SAWs rely on techniques that were developed roughly fifty years ago. Since then, very few new tools have been discovered in two dimensions, with a notable exception that we want to mention now. This idea combines combinatorial techniques that are reminiscent of the original approach with intuition from the theory of discrete holomorphic functions. The main object of interested is a certain *observable* of the model, i.e. the average of a certain random variable. Let us spend some time to define it properly.

From now on, a *discrete domain* will be a collection of half-edges intersecting a family of faces of the hexagonal lattice forming the closure of a simply connected domain of the plane; see Fig. 4 (disregard the definition of $\widehat{\mathcal{E}}(\Omega, a, z)$, which will become relevant only later). Half-edges have two endpoints: one vertex of $\mathbb{H}$ and one *mid-edge*. From now on, a SAW will systematically run between two mid-edges (it boils down to extending the SAW by two half-edges).

Let Ω be a discrete domain and a be a mid-edge on the boundary, i.e. at the end of only one half-edge in Ω (see Fig. 4). Fix $x, \sigma \geq 0$ to be determined later. For a mid-edge

$z \in \Omega$, define the *parafermionic observable* via the formula

$$F(z) \;=\; F_{\Omega,a,x,\sigma}(z) \;:=\; \sum_{\omega} \;\exp(-\mathrm{i}\sigma \mathrm{W}_\omega(a,z))\; x^{\#\,\text{vertices in}\,\omega},$$

where the summation runs over SAWs from a to z staying in Ω. In the definition above, $\mathrm{W}_\omega(a,z)$ is the *winding* or total rotation of the direction in radians when the SAW ω is oriented from a to z. In other words, it is equal to $\pi/3$ times the difference between the number of left and right turns of ω.

The term involving $\mathrm{W}_\omega(a,z)$ may appear as an unnecessary complication. Indeed, for $\sigma = 0$, we obtain the generating function of the SAWs in Ω from a to z, which seems like a very natural object to consider. The advantage of this term is that, when σ and x are tuned properly, F satisfies nice local relations as a function of z. Namely, let v be a vertex in the interior of Ω and p, q and r be the three mid-edges next to v. We identify v,p,q,r with their complex affixes. If

$$x = \frac{1}{\sqrt{2+\sqrt{2}}} \quad \text{and} \quad \sigma = \frac{5}{8},$$

then F satisfies

$$(p-v)F(p) + (q-v)F(q) + (r-v)F(r) \;=\; 0. \tag{5}$$

The set of equations (5) indexed by vertices v in Ω has a beautiful interpretation in terms of discrete contour integrals. Indeed, fix a sequence $\Gamma := (f_0, \ldots, f_k = f_0)$ of adjacent faces of Ω and define the discrete contour integral of F along Γ by the formula

$$\oint_\Gamma F(z)dz = \sum_{i=0}^{k-1} (f_{i+1} - f_i)F\big(z_i\big) = 0,$$

where f_i denotes the affix of the center of the corresponding face, and z_i the center of the edge between the faces f_i and f_{i+1}.

Equation (5) corresponds to the fact that the integral of F along the "triangular" contour composed of the three faces around the vertex v is equal to 0. Since any contour integral can be written as the sum of the triangular contours inside it, the relations (5) imply that the integral of F along *any* discrete contour vanishes. This property is reminiscent of a classical property of holomorphic functions. For this reason, one may think of F as a discrete version of a holomorphic function.

A word of caution: imagine for a moment that we wish to determine F using only its boundary values and the relations (5). We have one unknown variable $F(z)$ by mid-edge, and one relation per vertex. For generic domains, this is vastly insufficient, and we are therefore apparently facing a dead end: the fact that the discrete contour integrals vanish is providing little information on the observable F. In conclusion, a function satisfying the relations (5) can be seen as some kind of weakly discrete holomorphic function, but the relations do not allow us to do as much as the standard notion of holomorphicity does.

Fortunately, the property above is not meaningless. A careful analysis of contour integrals going along the boundary of well chosen domains Ω implies that the value $\sqrt{2+\sqrt{2}}$ mentioned above has to be the connective constant of $\mathbb{H}$. We refer to [DS12b] for the proof of this result. Let us mention that the value of the connective constant was predicted by Nienhuis in [Nie82, Nie84] using completely different techniques. The fact that μ_c has such a simple form can almost be considered as an anomaly. Except for trees and one-dimensional lattices, the connective constant is not predicted to have any special form (except for the 3.12^2 lattice, which is obtained from the hexagonal lattice by a simple transformation). As an example, the connective constant of the square lattice can be approximated but no prediction currently exists concerning its exact value. In fact, it is even unknown whether it should be rational or algebraic for instance.

Computing the connective constant should be considered as a stepping stone towards a bigger goal since physicists and mathematicians are ultimately interested in the critical behavior of the model. Let us depart from our combinatorial question (counting SAWs) to enter the realm of phase transitions in statistical physics.

Consider a simply connected domain Ω together with two points a and b on its boundary. Also consider the graph $\Omega_\delta = \Omega \cap \delta\mathbb{H}$ for $\delta > 0$. Let a_δ and b_δ be two mid-edges on the boundary of Ω_δ close to a and b. We think of the family of triplets $(\Omega_\delta, a_\delta, b_\delta)$ as more and more refine (as $\delta \searrow 0$) discrete approximations of (Ω, a, b). Let us assume that the graphs Ω_δ are discrete domains[1]. We define a model of random interface $\gamma_{(\Omega_\delta,a_\delta,b_\delta)}$ as follows: SAWs from a_δ to b_δ in Ω_δ have probability proportional to $x^{\#\text{ vertices}}$ while other paths have probability zero.

If x is too small, the SAW is too penalized by its length, and $\gamma_{(\Omega_\delta,a_\delta,b_\delta)}$ converges in law to the geodesic between a and b in Ω. On the other hand if x is too large, then the SAW is not penalized enough and $\gamma_{(\Omega_\delta,a_\delta,b_\delta)}$ converges to a space-filling curve. The phase transition between these two possible behaviors occurs exactly at the value $x_c = 1/\mu_c$. While the previous statements about $x \neq x_c$ are now mathematical theorems, the behavior at the "critical value" x_c is still conjectural. Let us describe briefly what is expected to happen at this special value.

At $x = x_c$, conformal field theory predicts that $\gamma_{(\Omega_\delta,a_\delta,b_\delta)}$ converges *in the scaling limit* (i.e. as δ tends to 0) to a random, continuous, fractal, simple curve $\gamma_{(\Omega,a,b)}$ from a to b staying in Ω. Furthermore, the family of random curves $\gamma_{(\Omega,a,b)}$ indexed by the triplets (Ω, a, b) is expected to be conformally invariant in the following sense: for any (Ω, a, b) and any conformal (i.e. holomorphic and one-to-one) map $\psi : \Omega \to \mathbb{C}$,

$$\psi(\gamma_{(\Omega,a,b)}) \text{ has the same law as } \gamma_{(\psi(\Omega),\psi(a),\psi(b))}.$$

This prediction can be rephrased as follows: the random curve obtained by taking the scaling limit in $(\psi(\Omega), \psi(a), \psi(b))$ has the same law as the image by ψ of the random curve obtained by taking the scaling limit in (Ω, a, b). This is clear for a transformation corresponding to a symmetry of the lattice (for instance the rotation by $k\frac{2\pi}{3}$ for some

[1] Even though they obviously have no reason to be, one may easily alter the definition of Ω_δ so that the next discussion is still valid. We therefore prefer to ignore this difficulty.

$k \in \mathbb{Z}$), but this claim implies that the result is true for any conformal transformation (therefore in particular for a rotation by any angle).

The emergence of these additional symmetries in the scaling limit has tremendous implications. In particular, Schramm [Sch00] managed to identify a natural candidate for the possible conformally invariant family of continuous non self-crossing curves. Together with Lawler and Werner [LSW04], he was thus able to predict that $\gamma_{(\Omega,a,b)}$ should be the Schramm-Loewner Evolution (SLE) of parameter $8/3$. This object, which is directly related to many other lattice models in dimension 2 (in particular simple random walks), is very well understood. Proving the convergence of $\gamma_{(\Omega_\delta,a_\delta,b_\delta)}$ to SLE(8/3) would therefore provide deep insight into the behavior of the model at x_c, and as a byproduct into the behavior of the uniformly sampled SAW for large n (the two models are closely related). In particular, it would probably enable one to determine the critical exponents $11/32$ and $3/4$.

The previous discussion on conformal invariance seems to have carried us away from our original discussion concerning parafermionic observables, but in fact the two discussions are deeply related. Indeed, the parafermionic observable is expected to have a conformally covariant scaling limit. Namely, set F_δ for the observable in the domain Ω_δ with $a = a_\delta$, and $f_\delta = F_\delta(\cdot)/F_\delta(b_\delta)$ (which depends on Ω_δ, a_δ and b_δ). Smirnov conjectured that if $\sigma = 5/8$ and $x = x_c$, then

$$\lim_{\delta\to 0} f_\delta \;=\; (\psi')^{5/8}\,, \tag{6}$$

where ψ is the conformal map from Ω to the upper half-plane sending a to infinity, b to 0, and with $\psi'(b) = 1$ (conformal covariance follows readily). Above, the convergence is uniform on any compact of the domain Ω. To come back to the discussion about the fact that the observable shared the property of vanishing contour integrals with holomorphic maps, we see that it is in fact expected to converge (when properly renormalized) in the scaling limit to such a holomorphic map.

In fact the previous conjecture represents the main step in a program dedicated to the proof of convergence of $\gamma_{(\Omega_\delta,a_\delta,b_\delta)}$ to SLE(8/3). From this point of view, [DS12b] is indeed a first step towards a bigger goal. Unfortunately, proving convergence of the observable seems out of reach at the moment. Nevertheless, a similar program has been carried out for a different model, and we propose to switch now to this model to discuss parafermionic observables further. While the connection to the story above will not be immediately apparent, it will become clearer as the discussion progresses.

A second example: the Ising model

The Ising model was introduced by Lenz in 1920 to model the Curie temperature. It has been used to model a wide variety of phenomena in physics, ranging from ferromagnetism to spin glasses. In fact, the Ising model finds new applications in other fields of science (such as biology, neuroscience, etc) every single day. We will focus on the nearest-neighbor ferromagnetic Ising model on the hexagonal lattice. Let $G = (V, E)$ be a finite subgraph of $\mathbb{H}$. Define the Hamiltonian $H_G(\sigma)$ of a configuration $\sigma = (\sigma_u : u \in V)$ of spins $\sigma_u \in \{\pm 1\}$ by the formula

$$H_G(\sigma) := -\sum_{\{u,v\}\in E} \sigma_u\sigma_v.$$

For $\beta > 0$ and $f : \{\pm 1\}^V \longrightarrow \mathbb{R}$, let

$$\langle f\rangle_{G,\beta} := \frac{\displaystyle\sum_{\sigma\in\{\pm 1\}^V} f(\sigma)e^{-\beta H_G(\sigma)}}{\displaystyle\sum_{\sigma\in\{\pm 1\}^V} e^{-\beta H_G(\sigma)}}.$$

The measure $\langle\cdot\rangle_{G,\beta}$ is called the Ising measure on the graph G at inverse-temperature $\beta > 0$.

When working with the Ising model, one is usually interested in quantities of the form $\langle\prod_{u\in A}\sigma_u\rangle$, where $A \subset V$. The operator σ_u associated to a vertex u characterizes the phase transition and is as such an *order* operator. From the point of view of field theory, it is convenient to consider a different type of operators associated to faces, which is corresponding to *disorder* operators. Let f, g be two faces and introduce a cut $\mathcal{C}$ from f to g, i.e. a sequence of adjacent faces starting from f and ending at g. Consider $\mu_f\mu_g$ to be the operator reversing the value of the coupling constants of the edges between successive faces of the cut (we identify the cut with this set of edges). In other words,

$$\mu_f(\sigma)\mu_g(\sigma) := \exp\Big(-2\beta\sum_{\{u,v\}\in\mathcal{C}}\sigma_u\sigma_v\Big).$$

Observe that the operator depends on the cut $\mathcal{C}$ and on β. The use of such disorder operators goes back as far as the original exact solutions to the 2D Ising model and is fundamental in the study of the critical behavior of the model (since it pops up everywhere, we do not give a specific reference).

We would like to manipulate order and disorder operators. To do this, we consider the high-temperature expansion of the Ising model, which we present briefly now. As observed by van der Waerden, the identity

$$\exp(\beta\sigma_u\sigma_v) = \cosh(\beta)(1 + \tanh(\beta)\sigma_u\sigma_v)$$

allows the partition function of the Ising model to be expressed as follows:

$$\begin{aligned}\sum_{\sigma\in\{\pm 1\}^V} e^{-\beta H_G(\sigma)} &= \cosh(\beta)^{|E|}\sum_{\sigma\in\{\pm 1\}^V}\prod_{e=\{u,v\}\in E}(1 + \tanh(\beta)\sigma_u\sigma_v)\\ &= \cosh(\beta)^{|E|}\sum_{\omega\subset G}\Big(\prod_{e\in\omega}\tanh(\beta)\Big)\Big(\sum_{u\in\omega}\sigma_u^{|\{v:\{u,v\}\in\omega\}|}\Big).\end{aligned}$$

For any $u \in V$, associating the configuration σ with the same configuration except that the spin at u is flipped implies that the last sum is equal to

$$\begin{cases} 2^{|V(G)|} & \text{if } \omega \in \mathcal{E}(G), \\ 0 & \text{otherwise,} \end{cases}$$

where $\mathcal{E}(G)$ denotes the set of even subgraphs of G, that is, the set of subgraphs ω of G such that every vertex of G is incident to an even number of edges of ω. Note that on a subgraph of the hexagonal lattice, $\omega \in \mathcal{E}(G)$ is the disjoint union of self-avoiding loops. We deduce that

$$\sum_{\sigma \in \{\pm 1\}^V} e^{-\beta H_G(\sigma)} = \cosh(\beta)^{|E|}\, 2^{|V|} \sum_{\omega \in \mathcal{E}(G)} x^{|\omega|}, \tag{7}$$

where $x := \tanh(\beta)$. A similar computation shows that for $A \subset V$,

$$\sum_{\sigma \in \{\pm 1\}^V} \Big(\prod_{u \in A} \sigma_u\Big) e^{-\beta H_G(\sigma)} = \cosh(\beta)^{|E|}\, 2^{|V|} \sum_{\omega \in \mathcal{E}(G,A)} x^{|\omega|},$$

where $\mathcal{E}(G, A)$ denotes the set of subgraphs ω of G such that every vertex not in A (resp. in A) is incident to an even (resp. odd) number of edges in ω. Altogether, we get

$$\langle \prod_{u \in A} \sigma_u \rangle_{G,\beta} = \frac{\sum_{\omega \in \mathcal{E}(G,A)} x^{|\omega|}}{\sum_{\omega \in \mathcal{E}(G)} x^{|\omega|}}.$$

In other words, correlations between order operators can be expressed in terms of ratios of weighted sums over subgraphs of G. But what happens when one mixes order and disorder operators? Let us take a specific example. Consider a discrete domain Ω and a vertex u on its boundary. Also consider a vertex $v \in \Omega$ and a cut $\mathcal{C}$ between a face f outside Ω and bordered by u and a face g bordered by v. When doing the same expansion as above, one obtains that

$$\langle \sigma_u \sigma_v\, \mu_f \mu_g \rangle_{G,\beta} = \frac{\sum_{\omega \in \mathcal{E}(G,\{u,v\})} (-1)^{|\omega \cap \mathcal{C}|} x^{|\omega|}}{\sum_{\omega \in \mathcal{E}(G)} x^{|\omega|}}. \tag{8}$$

Since $\omega \in \mathcal{E}(G, \{u, v\})$ is the disjoint union of self-avoiding loops and a self-avoiding path from u to v, the loops do not surround v and therefore contribute an even number to $|\omega \cap \mathcal{C}|$. As a consequence, only the self-avoiding path from u to v can contribute an odd number, which corresponds modulo 2 to the number of turns that the path does around the face f.

Now, Smirnov introduced an observable at mid-edges by considering the following quantity: let Ω be a discrete domain, a a mid-edge on its boundary and z a mid-edge inside. Consider the set $\widehat{\mathcal{E}}(\Omega, a, z)$ of "subgraphs of Ω" obtained as the union of disjoint self-avoiding loops plus a SAW from a to z avoiding the loops. Let $|\omega|$ be the number of vertices in ω (note that it is also the number of vertices, if the two half-edges arriving at a and z contribute 1/2). Also set $\mathrm{W}_\omega(a,z)$ for the winding of the SAW from a to z. Then define

$$F(z) \;=\; F_{\Omega,a,x}(z) \;:=\; \sum_{\omega\in\widehat{\mathcal{E}}(\Omega,a,z)} \exp(-\tfrac{\mathrm{i}}{2}\mathrm{W}_\omega(a,z))\; x^{|\omega|}.$$

The observable has a structure similar to the one of the SAW, with $\sigma = 1/2$ instead of $\sigma = 5/8$ and the sum on SAWs replaced by a sum on subgraphs $\omega \in \widehat{\mathcal{E}}(\Omega,a,z)$. Consider the specific case of configurations for which the SAW arrives from one endpoint (say v) of the edge corresponding to z. In such case, the term corresponding to the winding contributes $-\lambda$ or λ depending on the parity of the number of turns around the mid-edge z. A small leap of faith (or a small computation using the previous observation, which we leave to the reader) shows that F is in fact a complex linear combination of quantities of the form (8), where v is one of the two endpoints of the edge of z, and g one of the two faces bordered by z. To summarize, an observable similar to the parafermionic observable for SAWs can be defined in the Ising model as a linear combination of order-disorder operators.

The similarity between the observables for SAW and Ising is uncanny. It does not come as a surprise that for a certain value x_c of x, the Ising observable also satisfies the relations (5). This value is in fact equal to $1/\sqrt{3} = \tanh(\beta_c)$, where β_c is the critical inverse-temperature of the Ising model on $\mathbb{H}$. Exactly as in the case of the SAW, one may ask whether, when considering a sequence $(\Omega_\delta, a_\delta, b_\delta)$ approximating (Ω, a, b), $f_\delta = F_\delta(\cdot)/F_\delta(b_\delta)$ converges.

The Ising model has a tactical advantage compared to SAWs. The value of σ is 1/2 instead of 5/8. This apparently small difference was harvested by Chelkak and Smirnov to prove that the observable f_δ satisfies additional relations, and that it is now discrete holomorphic in the standard sense, not only weakly. In particular, f_δ is determined uniquely by its boundary conditions and these relations. Let us mention that discrete holomorphicity goes far back. Discrete holomorphic functions have also found several applications in geometry, analysis, combinatorics, and probability. We refer the interested reader to [DS12a] for more references on this beautiful theory.

Anyway, Chelkak and Smirnov [CS12] were able to describe f_δ as the solution of a discrete "Riemann-Hilbert" boundary value problem. With some additional work, they also showed that such a solution must converge to the holomorphic solution of the corresponding continuous Riemann-Hilbert boundary value problem. As a consequence, they were able to rigorously prove that

$$\lim_{\delta\to 0} f_\delta \;=\; \sqrt{\psi'},$$

where ψ was defined in (6).

Using a program similar to the one that could potentially be used for SAW, interfaces of the Ising model with Dobrushin boundary conditions were proved to converge to SLE(3) in [CDH+14]. In other words, conformal invariance of interfaces can be proved rigorously in the case of the Ising model.

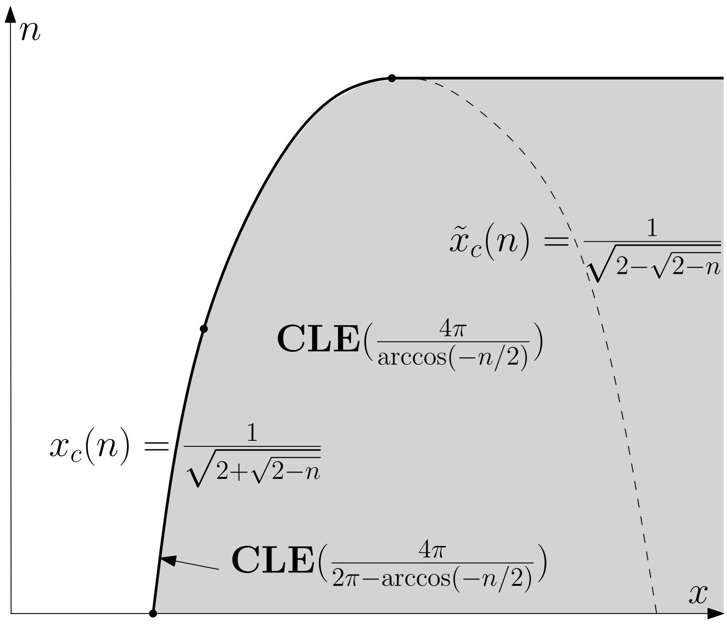

Figure 5: Phase diagram of the loop $O(n)$ model on the hexagonal lattice.

Let us conclude this part by mentioning that since the breakthrough of [CS12], conformal invariance of many observables of the Ising model has been derived: crossing probabilities [BDH14], energy and spin fields [HS13, Hon10, CI13, CHI15]), etc.

The parallel between the stories of the SAW and the high-temperature expansion of the Ising model leaves little doubt about a connection between the two models. A model indeed interpolates between the two examples above, and we propose to discuss it briefly below.

The loop $O(n)$-model

The high-temperature expansion of the Ising model and the SAW are both part of a wider family of statistical models, called the loop $O(n)$-model. In this model, a configuration ω is an element of $\mathcal{E}(G)$ and the probability of ω is proportional to $x^{\#\text{ edges}}n^{\#\text{ loops}}$. For $n = 0$ and $n = 1$, we recover the SAW and the high-temperature expansion of the Ising model respectively. The phase diagram (Fig. 5) of the loop $O(n)$ model on the hexagonal lattice was predicted by Nienhuis in [Nie82, Nie84]:

1. For $n \leq 2$ and $x = x_c(n) := 1/\sqrt{2+\sqrt{2-n}}$, the probability of having a loop of length ℓ passing through the origin decays as an inverse power of ℓ. Furthermore, the scaling limit of the loops is described by a conformally invariant family of *simple* loops called CLE(κ) (where κ depends on n and ranges from 8/3 to 4).

2. For $n \leq 2$ and $x > x_c(n)$, the probability of having a loop of length ℓ passing through the origin decays as an inverse power of ℓ. Furthermore, the scaling limit

of the loops is described by a conformally invariant family of *self-touching* loops called CLE(κ) (where κ depends on n but not on $x > x_c(n)$ and ranges from 4 to 8). Except for $n = 2$, the exponent in the inverse power is not the same as the one at $x_c(n)$.

3. Otherwise, the probability decays exponentially fast. In particular, for $n > 2$ the probability of having large loops is always decaying exponentially fast.

Most of the previous diagram is still conjectural. Nevertheless, a generalization of the previous observables provides some understanding on what is going on. Exactly like in the examples of the SAW and Ising, one may introduce an observable

$$F(z) \;=\; F_{\Omega,a,x,n,\sigma}(z) \;:=\; \sum_{\omega\in\widehat{\mathcal{E}}(\Omega,a,z)} \exp(-\mathrm{i}\sigma \mathrm{W}_\omega(a,z))\; x^{\#\,\mathrm{edges}}\; n^{\#\,\mathrm{loops}}.$$

For $n \le 2$, two values of (x, σ) play a special role in the sense that the corresponding observable has vanishing contour integrals. The first one is for $x = x_c(n)$ and $\sigma = \sigma(n)$ (the value is irrelevant here). The other value is at $x = \tilde{x}_c(n) = 1/\sqrt{2-\sqrt{2-n}}$ and $\tilde{\sigma} = \tilde{\sigma}(n)$. One expects that the observable f_δ defined as above would converge to $(\psi')^\sigma$ for $(x_c(n), \sigma)$ and $(\psi')^{\tilde{\sigma}}$ for $(\tilde{x}_c(n), \tilde{\sigma})$. The values of σ and $\tilde{\sigma}$ allow to predict the dependency of the value κ of the CLE(κ) on n (see Fig. 5 for the precise values). Furthermore, proving convergence of the observable represents the main step towards a proof of conformal invariance for the whole family of loops.

Interestingly, no good observable seems to be available for $n > 2$. It is therefore unclear how to prove that there is exponential decay at every x for $n > 2$. Nevertheless, we should mention a recent result proving this for $n \gg 1$ [DPSS14]. This result should be compared to a conjecture of Polyakov concerning the spin $O(n)$ models, that yields that spin-spin correlations decay exponentially fast at every inverse temperature in the 2D spin $O(n)$ model as soon as $n > 2$. While the previous result does not answer this conjecture, it is worth noting that the loop $O(n)$ model can be seen as an approximative high-temperature expansion of the spin $O(n)$ model for integer values of n.

Conclusion

The take-home message is the following: the order-disorder operators of the Ising model give rise, when written in terms of the high-temperature expansion, to discrete holomorphic observables. As a consequence, one may prove that they converge in the scaling limit to conformally invariant objects, a fact which leads to conformal invariance of interfaces. Certain generalizations of these quantities to loop models are still discretizations of conformal maps. Proving their convergence in the scaling limit would imply conformal invariance of loops in the corresponding model, but unfortunately, in basically any case except the Ising model, the properties of the observables are insufficient to derive rigorously the convergence. Still, weaker properties of the observables can be used to derive interesting features such as critical points and bounds for critical exponents.

Let us conclude by mentioning that the name parafermionic observable was coined in [FK80], where these observables were introduced initially.

Let us mention that parafermionic observables are not restricted to the loop $O(n)$ model and can be used in many other models. Maybe the most notable example is provided by the Fortuin-Kasteleyn percolation and Potts models, where they were used to determine the order of the phase transition, see [DST15].

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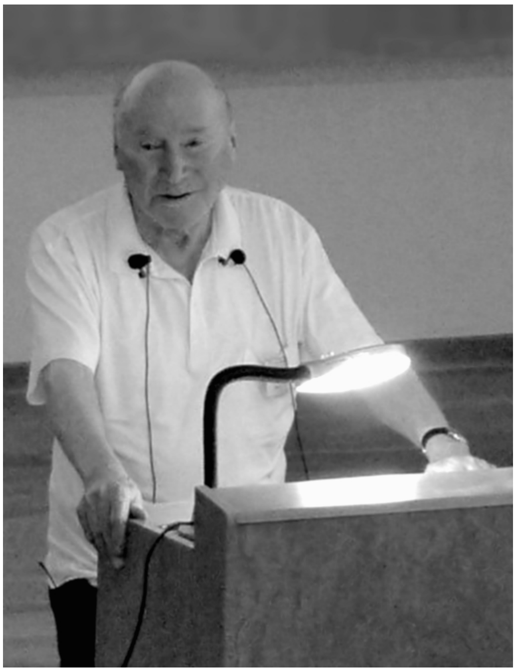
Rudolf Haag

(1922 – 2016)



Rudolf Haag was born on August 17, 1922 in the university town of Tübingen. His father was a mathematics teacher, his mother was actively engaged in social and gender politics. After finishing school he was visiting his sister in London just when World War II started. As a consequence, he had to spend the time of the war in a camp of civil prisoners in Canada.

After the war he returned to Germany. He studied physics in Stuttgart and Munich, where he got his PhD in 1951 under Fritz Bopp and became his assistant. Two visits to Copenhagen allowed him to have extensive discussions with Niels Bohr and his group, and after his habilitation he went to Werner Heisenberg in Göttingen. Subsequently, he was visiting professor in Marseille and Princeton, and in 1960 he became professor in Urbana, Illinois. In 1966, he accepted a call from the University of Hamburg, where he stayed until his retirement in 1987. After his retirement he moved to Fischhausen-Neuhaus, a little village on lake Schliersee in the Bavarian Alps. There he stayed till the end of his life. He passed away on January 5, 2016.

Rudolf Haag received the Max-Planck medal of the Deutsche Physikalische Gesellschaft in 1970 and the Henri Poincaré Prize in 1997. He was member of several academies (Leopoldina, Göttingen, Bavaria and Austria). Together with Res Jost, in 1965 he founded the journal *Communications in Mathematical Physics*, and he acted as its editor in chief up to 1973.

Rudolf's early work was concerned with the way particles and fields are related to each other. The question how to construct scattering states in quantum field theory (QFT) was his concern since the times with Heisenberg. With him he had an exchange on that problem of the following sort, as he told us.

Haag: "How do you construct the scattering states say of two particles?"
Heisenberg: "Just take the product of the state vectors!"
Haag: "But in a Hilbert space we have linear combinations, not products!"
Heisenberg: "Never mind, take the product!"

Rudolf thought that Heisenberg was right, except that one had to construct a product. That he achieved in the fifties along with the development of the fundamental idea of *locality* of fields and observables, which he identified as the most characteristic feature of relativistic QFT.

His paper "On Quantum Field Theories" [1955] marks a crucial breakthrough for the understanding of this fundamental topic. In particular, his result on the impossibility of using the Fock representation of the canonical time zero fields to describe interaction soon became known as Haag's Theorem.

Shortly afterwards, Rudolf realized that the idea of a one-to-one correspondence between particles and fields was misleading; particle states could well be reached by polynomials of field operators acting on the vacuum. This led him to recognize the central role of the *local algebras* generated by field operators, which are smeared over bounded spacetime regions and commute or anticommute at spacelike separations. This idea, presented in 1957 at the Lille Conference [1957], was essential for the actual construction of scattering states which Rudolf achieved in those years. The methods and ideas presented by Rudolf later became a key ingredient in theorems of David Ruelle, Klaus Hepp and Huzihiro Araki, and are now called Haag-Ruelle scattering theory. The origin of the particle structure of QFT as a consequence of locality was fully clarified in this work, and a proof was given of the LSZ asymptotic condition, whose basic form was anticipated by Rudolf already in [1955].

In those years, Rudolf's keen insight that the local algebras are the central objects of the theory, was also made mathematically precise. Taken up by Araki, who had characterized the local algebras of fields and observables in his 1962/63 Zürich lecture notes as von Neumann algebras, it took eventually the perfect shape in a paper by Rudolf with Daniel Kastler [1964]. The new point of view was the emphasis on observables whose characteristic feature is their commutativity at spacelike separations. Thus the central concept was the algebras generated by local observables. The understanding of the approximate nature of our knowledge of physical states, translated into precise mathematical terms, led the authors to describe those algebras as C*-Algebras, *i.e.* as norm-closed *-algebras of bounded operators on some Hilbert space.

That was the beginning of a new science, the algebraic approach to quantum field theory, better named "local quantum physics" (LQP) by Rudolf. This new paradigm was reached by Rudolf following a path which also went along collaborations with Hans-

Jürgen Borchers and Bert Schroer. It proved to be essential for the understanding of the central role of the locality postulate and the unraveling of its unreasonable effectiveness in determining a surprisingly large part of the structures appearing in QFT. Moreover, a new and fruitful contact with mathematics was initiated by it. After the previous exchange of QFT with the theories of distributions and holomorphic functions in the Wightman framework, it now was the *theory of operator algebras.*

This contact first culminated in the work on the characterization of equilibrium states in the statistical mechanics of infinite quantum systems by the KMS condition [1967] (together with Nicolaas Hugenholtz and Marinus Winnink). It strongly influenced modular theory, developed by Minoru Tomita and Masamichi Takesaki. In collaboration with Daniel Kastler and Ewa Trych-Pohlmeyer, Rudolf later characterized the KMS states in terms of stability properties under local perturbations of the dynamics. Furthermore, in a collaboration with Araki, Kastler, and Takesaki, the chemical potential was explained in terms of extensions of KMS states from the observables to charged fields.

Then, on the side of QFT, the contact with mathematics became manifest in his work on superselection sectors (with Sergio Doplicher and John Roberts [1971]), which has proved to have common roots with the theory of subfactors, later invented by Vaughan Jones. It also provided the basic frame for the operator-algebraic approach to conformal field theory by Jürg Fröhlich, Yasuyuki Kawahigashi, Roberto Longo and Karl-Henning Rehren. In that work the amazing effectiveness of LQP manifested itself in the explanation of the appearance of particle statistics and of its restriction to the cases of (para) Bose or (para) Fermi. (The singular case of infinite statistics was later excluded by Detlev Buchholz and Klaus Fredenhagen in theories of massive particles.) These results hold in all theories describing local observables in four-dimensional Minkowski space which are devoid of massless particles and long-range forces.

That fruitful ground continued to sprout. In the course of time the structure of superselection sectors of the local observables proved to reveal the existence of a unique compact group of global internal symmetries and to determine uniquely a field algebra obeying ordinary commutation or anticommutation relations at spacelike separations (Doplicher and Roberts). More recently, in the special case of simple sectors (ordinary statistics), the analysis of superselection sectors could be extended to theories with massless particles (Buchholz and Roberts). All of these results are implied by the locality of observables and do not require any *ad hoc* assumptions about unobservable fields.

On a different side, while constructive field theory had been successfully developed within the Wightman framework by James Glimm, Arthur Jaffe, and many others, the algebraic approach seemed ideal for deducing general structures from first principles, but unfit for a constructive approach. Yet, on the contrary: thanks to works by Hans-Jürgen Borchers, Gandalf Lechner, and Bert Schroer, it was possible to establish by algebraic means large families of interacting (integrable) models in two spacetime dimensions which were not covered by the previous constructive approach. This work continues and is a remarkable example how the quest for the understanding of first principles and their structural analysis might pay in the end.

Rudolf gave a lot of inspiration to mathematics, but his main interest was physics.

He was deeply concerned about a lack of understanding, in terms of local observables, of gauge theories (a missed opportunity, as, quoting Dyson, he termed it in his address at the Göttingen conference "Algebraic Quantum Field Theory – the first 50 Years," where also his picture shown above was taken), and he was enthusiastic when the idea of supersymmetry appeared. Actually, his paper on the possible supersymmetries of the S-Matrix (with Jan Lopuszanski and Martin Sohnius) [1975] is his most cited paper.

In later years, Rudolf became interested in the relation between QFT and gravity. First results on the connection between stability and thermal properties of states with their small-scale structure were obtained in collaborations with Heide Narnhofer and Ulrich Stein [1984], and with Klaus Fredenhagen [1990]. These papers provided an important foundation for the modern approach to QFT on curved spacetimes which developed during the last twenty years.

More recently, Rudolf's interest moved to the foundations of quantum physics, where he tried to base the theory on the concept of events and their causal relations. In this context, he studied in detail the detection process in elementary particle physics and also in quantum optics. His insights and ideas also found expression in the second edition of his book, which he wrote after his retirement [1992]. In this book he presented his view on quantum physics in a coherent way; it is now a standard reference for the subject, and it is an indispensable reading for everybody interested in the conceptual foundations and the accomplishments of LQP.

Rudolf, being first and foremost interested in physics, always searched for the underlying deeper structures and identified concepts which were amenable to a precise mathematical formulation. He was well aware of the intricacies of that ground, requiring subtle thoughts. During discussions with his collaborators he liked to rub between his fingers a thin leaf of grass, recalling a conductor's baton, which in the most delicate passages of the discussion he would use sometimes to titillate his ear, as an aid to hear the music of ideas. He would begin discussions sitting down and invite others to sit, saying, with a smart smile: "Let's think".

His invaluable example, and school for his students, was indeed a lesson of thinking: never being satisfied with the achievements reached, but aware of the ever more relevant problems yet lying beyond the borders; loving mathematical precision and elegance, but knowing that mathematics cannot think for us nor provide by itself solutions to the questions of physics.

In the last few years Rudolf had serious problems with his sight; almost fully deficient in one eye, largely impaired the other. Yet he tried to read papers on his computer using huge magnifications and to attend talks watching the screen through special lenses, and he succeeded in writing scientific papers. In 2013 he had a stroke, which did not affect at all his mental clarity and scientific curiosity, but limited his left-side mobility. He tried to keep updated also by conversations with and reports by his former students. And he kept pursuing his idea of a central role of "events" till the end.

Rudolf's subtle thinking allowed him to show often a very subtle humor, with a world view that was probably influenced also by his deep love for music; on which he often would make non trivial comments. Listening with one of us to a Beethoven sonata, played by a famous artist, he would say: "Why does he try to make it sweeter? It has to be sharp, even unpleasant!". He was right!

At younger ages Rudolf had played the violin, but later he favored the piano, which he played on almost every day. He did not refrain from it even after his stroke, playing then with his right hand only, and improvising on music which was close to his heart. He had wished to do that on his last night, too, when he passed away peacefully, surrounded by the love of his family.

Rudolf was twice a widower, of Käthe first, then of Barbara; with his first wife he had four children, Albert, Friedrich, Elisabeth and Ulrich.

His death is a tremendous loss for our scientific community. It is the end of an era for science, for theoretical physics, for quantum field theory, not just for local quantum physics alone; and the end of a paramount important part of the life for those who had the privilege of being his collaborators and friends.

Detlev Buchholz, Sergio Doplicher and Klaus Fredenhagen

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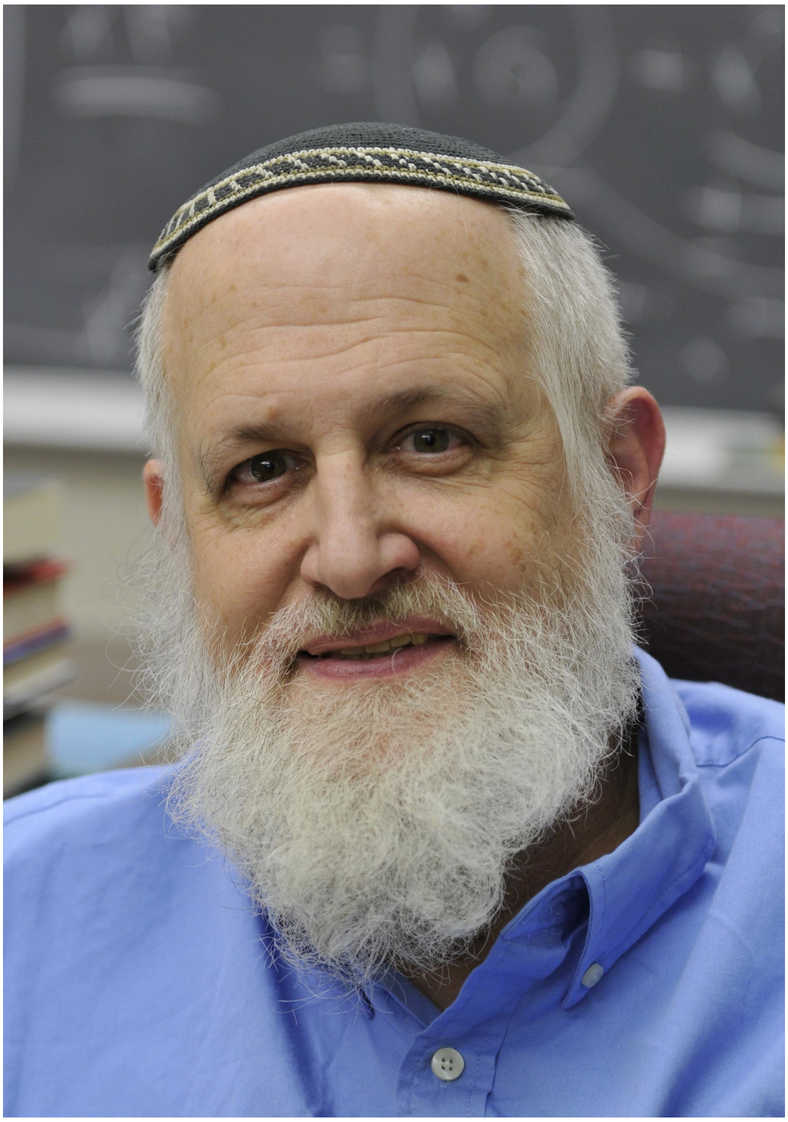
Barry Simon to Receive 2016 AMS Steele Prize for Lifetime Achievement



Barry Simon of the California Institute of Technology will receive the 2016 AMS Leroy P. Steele Prize for Lifetime Achievement for “his impact on the education and research of a generation of mathematical scientists through his significant research achievements, his highly influential books, and his mentoring of graduate students and postdocs.”

Simon's mathematical talent showed early in life. In 1962, at the age of 16, he was the subject of a short article in *The New York Times*, which recounted the story of Simon's participation in an exam contest sponsored by the Mathematical Association of America and the Society of Actuaries. After missing one question, he argued that the wording of the question had been ambiguous. The contest sponsors agreed, and Simon was awarded a perfect score.

An alumnus of Harvard University, Simon received his PhD from Princeton University in 1970 and was immediately appointed as an assistant professor. In the decade that followed, as Simon rose to the rank of full professor in 1981, Princeton became a thriving center for mathematical physics, particularly in statistical mechanics, quantum field theory, and non-relativistic quantum mechanics. One of Simon's PhD students from that time, Percy Deift, described the atmosphere this way: "Barry was a dynamo, challenging us with open problems, understanding every lecture instantaneously, writing paper after paper, often at the seminars themselves, and all the while supervising 7 or 8 PhD students." Deift made these remarks in the laudatio for the Poincaré Prize, awarded to Simon in 2012.

Simon's prodigious productivity continued after he moved to Caltech in 1981 to take his present position as the IBM Professor of Mathematics and Theoretical Physics. Today his list of research publications includes over 400 items. His secret? He needs "only five percent of the time ordinary mortals need" to write a research paper, quipped his collaborator Jürg Fröhlich, in a reminiscence prepared for a conference celebrating Simon's 60th birthday. Simon has had 31 graduate students, many of whom have gone to become leaders in mathematical physics and other areas, and he has mentored about 50 postdoctoral researchers.

Simon's own research contributions range over several areas of pure mathematics and mathematical physics. One of his most important contributions still stands as a landmark today: After nearly 40 years, work done by Simon and 4 co-authors (Fröhlich, Thomas Spencer, Freeman Dyson, and Elliott Lieb) still stands as the only rigorous proof of symmetry breaking in certain regimes fundamental to physics.

Simon was the first to give a mathematically precise definition of resonance that allowed linking of time-independent and time-dependent perturbation theory and the first to use differential-geometric invariants to understand Berry's phase and some other quantum phenomena. In work with Lieb, Simon produced the first rigorous proofs and interpretations of theories central to quantum mechanics. A leading contributor to the construction of quantum fields in two space-time dimensions, Simon (together with Francesco Guerra and Lon Rosen) established an analogy with classical statistical mechanics that led to deep new insights. Simon also proved several definitive results in the general theory of Schrödinger operators.

In addition to his outstanding contributions at the forefront of research, Simon is known for several books that have had a major influence on generations of students entering the field of mathematical physics. His 4-volume work *Methods of Modern Mathematical Physics*, written with Michael Reed during the 1970s, is where many of today's top researchers first learned this subject. Simon's uncanny ability to extract the key elements in a proof "is expressed in his books as a signature combination of economy and clarity, which accounts, I believe, for their usefulness and great popularity," remarked Deift in the Poincaré laudatio. Simon's two-volume set *Orthogonal Polynomials on the Unit Circle*, published by the AMS in 2005, became instant classics, connecting the theory of orthogonal polynomials with the spectral theory of Schrödinger operators and other topics in mathematical physics.

On top of all of his other contributions, Simon is also the co-author of two highly popular manuals for Windows computers: The Mother of All Windows Books and The Mother of All PC Books, which appeared in the 1990s. Written with Woody Leonhard, the books provided clear and practical advice in a witty and irreverent style, making them highly popular with computer users struggling to make sense of their costly machines.

In addition to the aforementioned Poincaré Prize (2012), Simon's previous awards include several honorary degrees and the Bolyai Prize of the Hungarian Academy of Sciences (2015). He was named a Fellow of the AMS in 2013.

Presented annually, the AMS Steele Prize is one of the highest distinctions in mathematics. The prize will be awarded on Thursday, January 7, 2016, at the Joint Mathematics Meetings in Seattle.

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The editors recall that two conferences in Canada will celebrate Barry Simon's 70th birthday next summer; see http://www.fields.utoronto.ca/programs/scientific/16-17/modern-physics/ and http://www.crm.umontreal.ca/2016/Simon16/index_e.php for details.

News from the IAMP Executive Committee

New individual members

IAMP welcomes the following new members

1. Prof. Virginie Bonnaillie-Noël, ENS Paris - CNRS, France
2. Dr. Muhammad Roshanzamir-Nikou, Urmia University, Iran
3. Dr. Alessia Nota, University of Bonn, Germany

Recent conference announcements

Geometrical aspects of spectral theory

April 4-6, 2016. BCAM workshop, Bilbao, Spain.

This conference is partially supported by the IAMP.

Organized by D. Krejčiřík, T. Ourmières-Bonafos and L. Vega.

http://gast2016-bcam.blogspot.com.es

Conference - DimaScat : Scattering Theory and Spectral Asymptotics of Differential Operators - in Honour of Dimitri Yafaev

April 20-22, 2016. University of Rennes, France.

Organized by N. Raymond, K. Pravda-Starov, S. V. Ngọc.

http://www.lebesgue.fr/content/semYafaev-Yafaev

Fifty Years of Hearing Drums: Spectral Geometry and the Legacy of Mark Kac

May 16-20, 2016. Universidad Católica de Chile, Santiago, Chile.

Organized by Rafael Benguria and the Nucleo Milenio de Física Matematica

http://www.fis.puc.cl/ icmsmag/SpectralGeometryConference

Mathematical Physics Days in Hagen

May 17-19, 2016. FernUniversität in Hagen, Germany.

Organized by P. Hislop, W. Spitzer and S. Warzel

This conference is partially supported by the IAMP.

https://www.fernuni-hagen.de/mathphysdays2016/

Analysis and beyond

A conference celebrating Jean Bourgain's work and impact.

May 21-24, 2016. IAS Princeton.

This conference is partially supported by the IAMP.

Organized by A. Gamburd, S. Jitomirskaya, A. Naor, P. Sarnak, T. Tao, G. Staffilani, P. Varju.

https://www.math.ias.edu/bourgain16

Mathematical Many-Body Theory and its Applications

June 13-19, 2016. BCAM, Bilbao, Spain.

This conference is partially supported by the IAMP.

Organized by S. Adams, M. Benitez, S. Breteaux, J.-B. Bru, W. de Siqueira Pedra, M. Merkli.

http://www.bcamath.org/es/workshops/mmbta

Great Lakes Mathematical Physics Meeting 2016

June 17-19, 2016. Michigan State University.

This conference is partially supported by the IAMP.

Organized by J. Schenker and P. Hislop.

http://instmathphys.msu.edu/glamp2016

Quantum Roundabout 2016

Student conference on the mathematical foundations of quantum physics.

This conference is partially supported by the IAMP.

July 6-8, 2016. The University of Nottingham.

Organized by P. Liuzzo-Scorpo, R. Nichols and B. Regula.

The 9th MSJ-SI "Operator Algebras and Mathematical Physics"

August 1-12, 2016. Tohoku University, Sendai, Japan.

Organized by M. Izumi, Y. Kawahigashi, M. Kotani, H. Matui, N. Ozawa.

http://www.ms.u-tokyo.ac.jp/ yasuyuki/msj-si2016.htm

Sirince Summer School in Mathematical Physics

August 22 - September 4, 2016. Sirince, Turkey.

This conference is partially supported by the IAMP.

Organized by A. Mardin, T. Turgut, A. Yilmaz.

https://matematikkoyu.org/eng/events/2016-fizik/index.php

QMath13: Mathematical Results in Quantum Physics

October 8-11, 2016. GeorgiaTech, Atlanta, USA.

This conference is partially supported by the IAMP.

Organized by F. Bonetto, E. Harrell, M. Loss.

http://qmath13.gatech.edu/

Open positions

Postdoctoral Position in Random Matrix Theory, Bielefeld

Applications are invited for a postdoctoral position in the area of random matrix theory and its applications in particle physics, statistical mechanics and mathematics. The successful applicant will work in the Mathematical Physics group of Professor Gernot Akemann at the Department of Physics, Bielefeld University, Germany. The deadline for applications is Feb. 15, 2016. More information on how to apply can be found at

`https://www2.physik.uni-bielefeld.de/1387.html`

More job announcements are on the job announcement page of the IAMP

`http://www.iamp.org/page.php?page=page_positions`

which gets updated whenever new announcements come in.

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