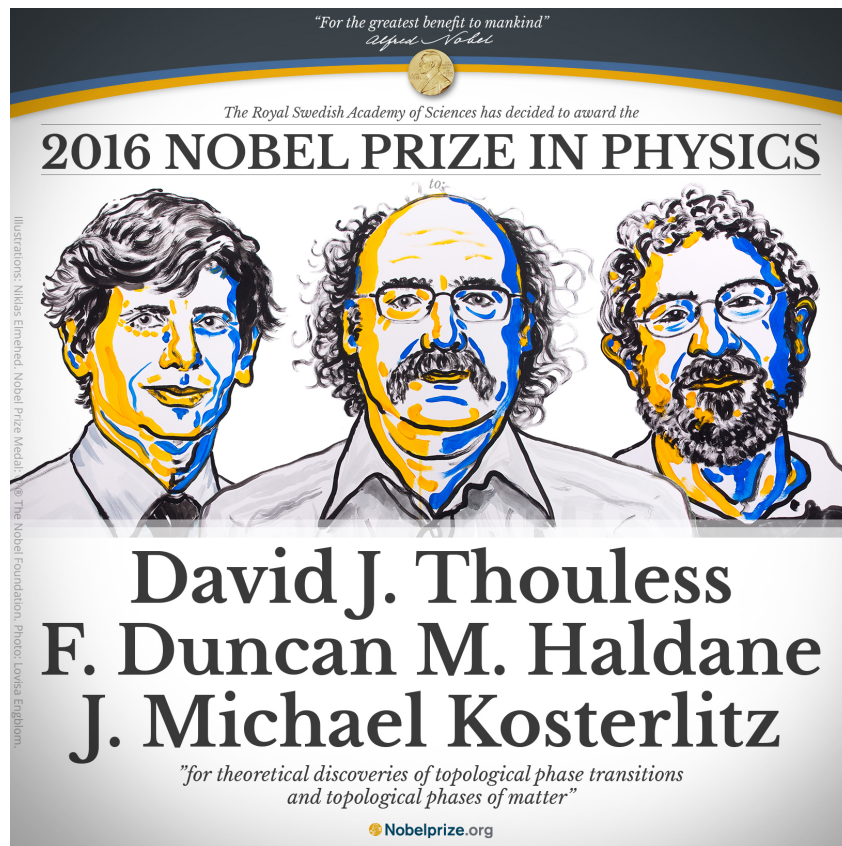


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Cover picture: Winners of the 2016 Nobel Prize:
D. J. Thouless, F. D. M. Haldane, J. M. Kosterlitz.
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News Bulletin (International Association of Mathematical Physics)

Topological quantum states, the quantum Hall effect, and the 2016 Nobel prize

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1 The 2016 Physics Nobel Prize

The 2016 Nobel prize was awarded to David J. Thouless, who got half the prize, and to John M. Kosterlitz and F. Duncan Haldane, who shared the other half. Another way of counting the fractions, a passtime in the business of the quantum Hall effect, is to say that Kosterlitz and Thouless shared half the prize for their joint discovery of the eponymous phase transition, while Thouless and Haldane shared the other half for their works on the quantum Hall effect and topological quantum phases. The prize committee put the two discoveries under the same umbrella with the citation: “For theoretical discoveries of topological phase transitions and topological phases of matter”.

Kosterlitz and Thouless made their joint discovery while in Birmingham, UK. Their paper [33] appeared in 1972 in the *Journal of Physics C*, impact factor 2.2. This shows that in the last count it is the work that matters, not where it appeared.

Thouless and Haldane did not work together and did not coauthor any paper. However, both pioneered a new era in condensed matter physics: The era of topological phases of quantum matter.

The seminal work of Thouless on the quantum Hall effect is a 1982 *Physical Review Letters* article [44], coauthored with (then) three post docs at Seattle : Mahito Kohmoto, Peter Nightingale and Marcel den Nijs. The work is known by the acronym TKN^2 . (Ruedi Seiler, Barry Simon and I share the blame for spreading the acronym.) TKN^2 showed that the Hall conductance in a model of the Hall effect, the Hofstadter model, takes integer values.

Let me now come to Haldane. In 1981, in a manuscript that had been rejected by two journals, and eventually evolved to a 1983 *Physical Review Letters* article [24],

Haldane discovered topological quantum phases in quantum spin chains that distinguish integer spin chains from half-integer chains. In a 1988 *Physical Review Letters* article [25], Haldane freed the Chern numbers from the bonds of the magnetic field and fathered “Chern insulators”.

D. Thouless is the oldest of the three, born in 1938. Haldane, is the youngest, born in 1951 and Kosterlitz was born in 1942. This may not be the impression one gets from the Fig. 1 where Haldane looks like the senior member of a triumvirate.

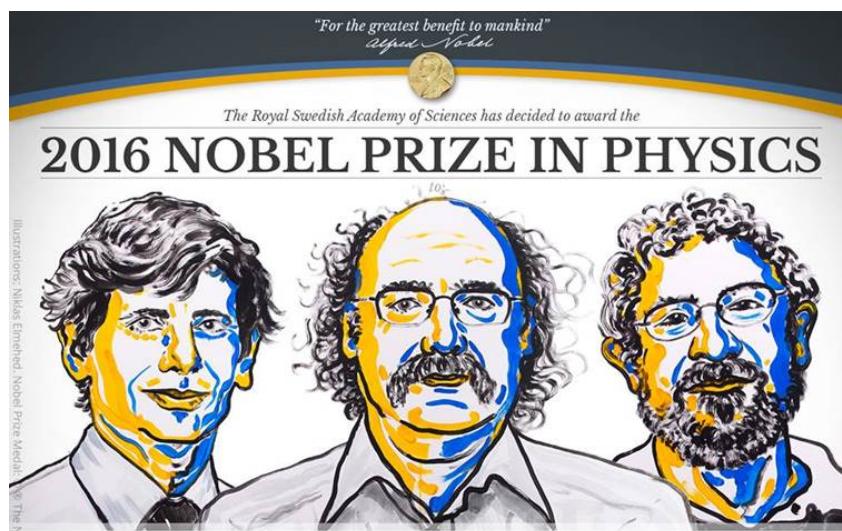


Figure 1: Left: D. Thouless, center D. Haldane, right M. Kosterlitz

The official web site at nobelprize.org gives additional information as well as videos of Haldane’s and Kosterlitz’s Nobel lectures. The first part of Haldane’s talk covered Thouless’s work. Thouless attended the ceremony but gave no lecture due to poor health.

The three laureates have in common that they were all subjects of her Majesty the Queen of England (Kosterlitz was born in Scotland) who emigrated to the U.S. around 1980, the dawn of the Thatcher era in Great Britain. The three apparently followed the advice of James Callaghan, UK foreign minister, who said in 1974: “If I were a young man, I would emigrate.”

While I shall restrict myself to the story of the quantum Hall effect and the TKN^2 integers, it is appropriate to mention that in 1991 Jürg Fröhlich and Thomas Spencer received the Dannie Heineman Prize for proving rigorously the existence of the Kosterlitz-Thouless phase transition in [23], and that in 1987 Ian Affleck, Tom Kennedy, Elliott Lieb, and Hal Tasaki, introduced their eponymous spin 1 chains, where topological order of the kind pioneered by Haldane could be studied on a rigorous level [1].

1.1 Trivia and stories

Two years before the TKN^2 paper, in a 1980 paper entitled “Ground states of a two-dimensional electron in a periodic magnetic field” [22], B. A. Dubrovin and S. P. Novikov,

discovered that the bundle of ground states over the Brillouin zone is topologically non-trivial. The paper does not explicitly mention “Chern numbers,” and none were computed, but Novikov, a 1970 Fields medalist, must have known all there is to know about Chern numbers. Novikov told Ruedi Seiler that he went around the Landau Institute asking what might be the physical significance of the topological invariants they found. Nobody had a good idea. It was left to TKN^2 to discover the interpretation as Hall conductance two years later. This agrees with Berry’s law of discovery¹: “Nothing is ever discovered for the first time.”

In 1980 Barry Simon moved from Princeton to Caltech as a Fairchild scholar and I tagged along as his *Fairbaby scholar*. In 1982-83 Ruedi Seiler spent a part of his sabbatical at Caltech and the three of us happily studied TKN^2 . In the summer of 1983 Barry went to Australia, where he met Michael Berry and had the good fortune to learn about Berry’s ground breaking paper [14] “Quantal phase factors accompanying adiabatic changes,” which was to only appear a year later, in 1984. Barry Simon’s *Physical Review Letters* article [42] unearthed the differential-geometric underpinning of Berry’s work, fathered the subject of adiabatic holonomy, and coined the term “Berry’s phase,” which stuck. A whim of history and the helping hand of a lethargic referee for *Proc. Roy. Soc.*, lead to what might appear as a reversal of the arrow of time, where the effect, Barry Simon’s 1983 paper, [42], preceded its cause, Michael Berry’s 1984 paper [14].

Barry Simon’s 1983 paper [42] made two important contributions to TKN^2 . First, it identified the TKN^2 integers as Chern numbers: Topological invariants named after the Chinese-American mathematician Shiing-Shen Chern, 1911-2004. This makes Barry the vector responsible for spreading the Chern epidemic, to the condensed matter community. (Barry called the disease *Bundle Fibrosis*.) Secondly, Barry’s paper identified the bundle (and its associated curvature and holonomy) underlying Berry’s paper with TKN^2 . (Barry credits Bernard Souillard with the suggestion that the two might be related.) The works of Berry and Barry lie at the heart of the geometrization of the theory of the quantum Hall effect [9, 7, 8].

In 1988, Lorenzo Sadun and Jan Segert were Barry’s post docs at Caltech, and I on sabbatical from the Technion. The four of us coauthored a paper in CMP [6] whose abstract, initially an internal joke of Jan, was: “**Abstract:** *Yes, but some parts are reasonably concrete.*” The paper was accepted with the condition that we write a real abstract. Barry threw his weight and the editor gave in.

The paper made good on a promise in [7] to give a topological classification of non-degenerate matrices. Among the results it contains is

$$\pi_5(M_n(\mathbb{H})) = \mathbb{Z}_2^{n-1}, \quad (1)$$

where $M_n(\mathbb{H})$ are the non-degenerate, Hermitian, $n \times n$ matrices with quaternionic entries. This means that energy bands of time-reversal invariant spin-1/2 systems are classified by \mathbb{Z}_2 . Like Dubrovin and Novikov before us, we missed the physical interpretation of the \mathbb{Z}_2 invariant for spin-systems. It was Kane and Mele who discovered its significance for topological insulators in 2005 [30].

¹<https://michaelberryphysics.wordpress.com/quotations/>

2 The Hall effect

2.1 Maxwell, Hall, and Rowland

Maxwell's *A Treatise on Electricity and Magnetism* appeared in 1873 [37]. The article 501 in volume II part IV, chapter 1 has the title “Electromagnetic force is a mechanical force acting on the conductor, not the electric current itself.”

Edwin Hall, a graduate student of Henry Rowland at Johns Hopkins, found Maxwell's article 501 puzzling and brought up the matter with Rowland. Rowland told Hall that he doubted Maxwell was right and, in fact, had made preliminary experiments to test it.

Since Rowland was too busy with other things Hall picked up the challenge. He initially tried to measure the change in the resistance of the conductor due to a magnetic field. This change is now known as magneto-resistance. It is a very small effect, being second order in the magnetic field. Hall's measurements turned out to be inconclusive. Hall then went back to the original design of Rowland to measure the voltage perpendicular to the current (parallel to the force), which is first order in the magnetic field, Fig 2. If Maxwell was right the voltage would vanish. This voltage is now known as the Hall voltage.

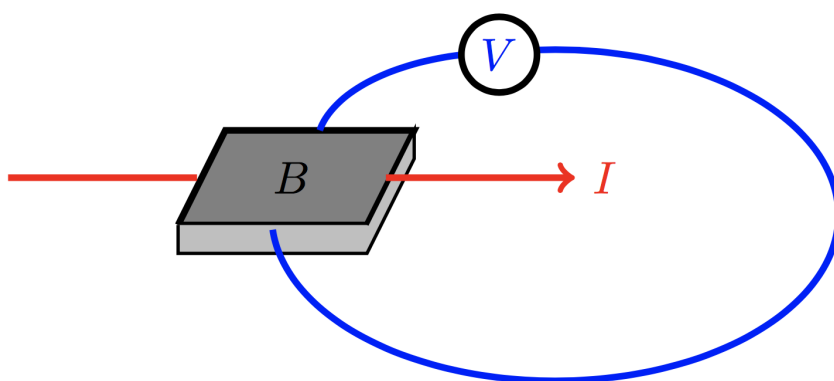


Figure 2: The Hall effect

It turned out to be a difficult experiment: An appreciable Hall voltage needs the conductor to be essentially two-dimensional. Rowland advised Hall to measure the voltage of thin gold plates. Eventually Hall succeeded in measuring appreciable Hall voltage and prove Maxwell wrong. His paper “On a new action of the magnet on electric currents” appeared in 1879 in the *American Journal of Mathematics* [26], the year Maxwell died, aged 48.

It is perhaps worthwhile to remember that all of this took place well before the electron was discovered in 1897, by J.J. Thomson.

Epilogue: The third edition of Maxwell's treatise appeared posthumously in 1903, and on page 157 the editor put the polite footnote: "Mr. Hall has discovered that a steady magnetic field does slightly alter the distribution of currents in most conductors, so that the statement must be regarded as only approximately true".

2.2 Peierls: Understanding the sign is a sign of understanding

In his book "Brilliant blunders" [36], Mario Livio portrays five scientific giants who made great discoveries and brilliant blunders. Einstein and Darwin are among the five Livio picked but he left Maxwell out. Maxwell was a giant all right, and article 501 was, as we shall see, not a silly mistake, but a source of a sequence of surprises of increasing intellectual depths: A brilliant blunder. Hall's finding was not the end of the story. It was the start.

The next surprise came at the beginning of the 20th Century: Electrons had already been discovered and semi-conductors were recognized as an intermediate class between insulators and metals. (The term semi-conductor was coined by Josef Weiss in 1910.) Semi-conductors have low density of charge carriers and hence a large Hall effect. It turned out that the Hall voltage for different semiconductors took different signs. This meant that the charge carriers in semi-conductors were of either sign. What were the anti-electrons in semi-conductors?

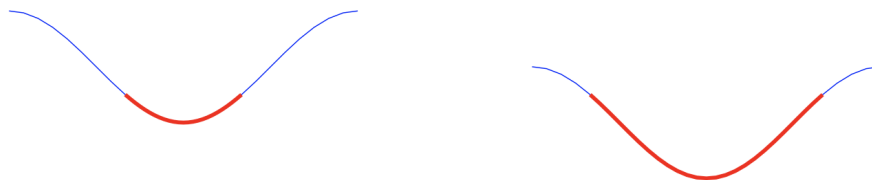


Figure 3: Electrons, left, are the filled states of the band, shown red. Holes, right, are the empty states, shown in blue. The Fermi energy at $T = 0$ is the top filled energy level.

The resolution of the puzzle required quantum mechanics. In 1928 Felix Bloch, a graduate student of Werner Heisenberg in Leipzig, established the quantum theory of solids, where electrons were confined to energy bands. (Bloch got the Nobel prize in 1952.) Bloch's work prepared the ground for Rudolph Peierls, a graduate student at the ETH of Wolfgang Pauli and Werner Heisenberg, who in 1929 resolved the sign problem [19]: The anti-electrons were holes, namely, missing electrons, in the Bloch bands, Fig. 3.

Peierls is probably best known for his memorandum with Otto Frisch from 1940, showing the feasibility of airborne atomic weapons. But in the mathematical physics community he is best known for his proof of the existence of phase transitions in Ising like systems.

We shall meet Peierls again as the grandfather of the Hofstadter model and the post-doctoral advisor of D. Thouless.

2.3 The quantum Hall effect

The next surprise hidden in the Hall effect had to wait for two scientific and technological revolutions: The first goes back to the 1913 Nobel prize of Heike Kammerlingh-Onnes for liquefying helium and discovering superconductivity. This gave birth to the era of low-temperature physics. The second is associated with the 1956 Nobel prize to John Bardeen, Walter Brattain, and William Shockley for the discovery of the transistor. This was the death knell for vacuum tubes, and the birth of the immensely successful semiconductor industry. It brought in its wake the development of increasingly sophisticated and exquisite fabrication techniques. In the 1970s it became possible to fabricate very thin 2-dimensional conducting sheets of electrons sandwiched between insulators. Rowland's ideal system has been realized.

There are two energy scales in the Hall effect: The energy scale given by the magnetic field B and the energy scale given by the temperature T . The studies of Hall and Peierls were in the regime $T \gg B$. The regime $T \ll B$, often referred to as the quantum Hall regime, is where the quantum ground state holds its sway.

In the late seventies von Klitzing measured the Hall resistance of several Hall transistors, made by different manufacturers, in different countries, over a period of time. He made these measurements at low temperatures and large magnetic fields. Fig. 4 shows a caricature of the experimental graphs.

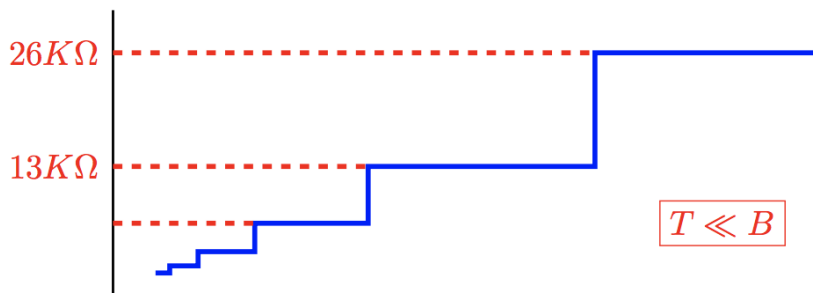


Figure 4: The quantized plateaus in the quantum Hall effect

Von Klitzing noted that measurements of different samples seem to have certain universal feature: A plateau at about $26 K\Omega$ and another at $13 K\Omega$ and yet another at about $8 K\Omega$, etc., with remarkable accuracy of about one part in 10 million. Since the samples were quite different the precise agreement was a mystery. A clue that this precision might have fundamental significance came from the observation that $26 K\Omega$ is the quantum unit of resistance

$$h/e^2 = 25812.807557 \Omega \approx 26 K\Omega$$

The current accuracy in the Hall conductance at plateaus is of order of 1 part in 10^{10} .

One immediate application of this finding is the redefinition of the standard of resistance, the Ohm. This makes the Ohm universal in the sense that one does not need

to pick a mother standard to calibrate its daughters. Quantum Hall transistors made anywhere, by any manufacturer, are all equally good standards.

Von Klitzing had his share of difficulties getting his paper [32] past the referees of *Physical Review Letters*. A story I heard is that the referees did not appreciate the significance to fundamental physics of a new standard for the Ohm. However, since c is an absolute quantity this could also be viewed as a new precision measurement of the fine-structure constant $\alpha = e^2/\hbar c$ in the (defunct) Gaussian units². Changing the title to “New Method for High-Accuracy Determination of the Fine-Structure Constant Based on Quantized Hall Resistance” did the job.

In 1985 Klaus von Klitzing was awarded the Nobel prize for this discovery.

3 The Hofstadter model

The low-energy properties of metals and insulators are governed by a single energy band; the “valence band” for insulators (and p-type semiconductors) and the “conduction band” for metals (and n-type semiconductors). In either case, (adiabatic) transport properties are governed by a single energy band. In periodic systems the Bloch momentum \mathbf{k} plays the role of the momentum \mathbf{p} for free particles, and the band function $\epsilon(\mathbf{k})$ plays the role of the kinetic energy $\mathbf{p}^2/2m$ for free particles. The mother of all band functions (in two dimensions) is the “tight-binding” dispersion relation,

$$\epsilon(\mathbf{k}) = 2(\cos k_1 + \cos k_2).$$

The Hilbert space associated to a single band is the space of square-integrable functions on the Brillouin zone, which is topologically a torus. To unravel interesting quantum states, we need to incorporate gauge fields $\mathbf{A}(\mathbf{x})$ in the single band framework. This is done by “Peierls substitution”: the band function $\epsilon(\mathbf{k})$ is replaced by the “minimal substitution”,

$$H = \epsilon(\mathbf{k} - \mathbf{A}(i\nabla_k)). \quad (2)$$

This makes the one-band Hamiltonian a pseudo-differential operator.

The 2-dimensional tight-binding model in a constant magnetic field, with gauge field $\mathbf{A} = (0, Bx)$ (the Landau gauge) is associated with a Hamiltonian which is a pseudo-differential operator on $L^2(\mathbb{T}^2)$,

$$H = 2 \cos k_1 + 2 \cos(k_2 - iB\partial_{k_1}). \quad (3)$$

Peierls’s single-band theory is related to lattice gauge theories: $L^2(\mathbb{T}^d) \simeq \ell^2(\mathbb{Z}^d)$, by Fourier; The tight-binding dispersion corresponds to nearest neighbors hopping on the lattice; and Peierls substitution translates to an external (abelian) gauge field $U_b \in U(1)$ that lives on the (directed) bonds of the lattice.

Fourier transforming Eq. (3) with respect to k_1 gives a family of operators on $\ell^2(\mathbb{Z})$ parametrized by B and k_2 :

$$(H\psi)(n) = \psi(n+1) + \psi(n-1) + 2 \cos(Bn + k_2)\psi(n). \quad (4)$$

² In SI units, the impedance of the vacuum, $Z_0 \approx 377 \, \Omega$, is exact, and the fine-structure constant is $\alpha = Z_0 e^2/(2\hbar)$. I thank Duncan Haldane for this note.

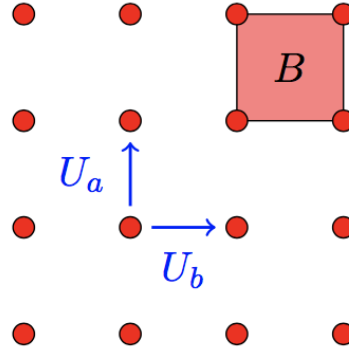


Figure 5: Hopping on a lattice with flux B per plaquette and gauge fields U on the bonds.

Several names are attached to this equation: This first is P. G. Harper, a student of Peierls, who had the equation in 1955. It is reasonable to expect that Peierls had some part in this.

In the math-physics literature Eq. (4) is known as the almost Mathieu operator, after Emil Leonard Mathieu (1835-1890), who studied the corresponding ODE. Barry Simon is responsible for the pun-intended appellation.

The common name in the physics literature, and the one I shall use, is the Hofstadter model. Douglas Hofstadter, the son of the Nobel laureate Robert Hofstadter, was a graduate student of Gregory Wannier. He made one of the early uses of computers and computer graphics to study the spectral properties of H . He discovered swarms of fractal butterflies in the spectrum of H [28]. The fractal butterfly is reproduced in his Pulitzer-prize winning book “Gödel, Escher Bach” [29].

When $B/2\pi$ is rational, H is a periodic operator, and its spectrum is a collection of bands. When B is irrational, the spectrum is a Cantor-type set. This was conjectured by Mark Azbel in 1964, and advertised by Mark Kac and Barry Simon as the “Ten Martini Problem”. This conjecture was settled, in its full glory, by Artur Avila and Svetlana Jitomirskaya [3] in 2005 (following up on a breakthrough by Puig [41]).

One may, of course, wonder if the delicate spectral features of the Hofstadter model are artifacts of Peierls’s single-band approximation. The single-band approximation is expected to be reasonable for infrared properties, but not in the ultraviolet. An ultraviolet artifact is the periodicity of Eq. (4) in B . The period 2π corresponds to magnetic fields that are 9 orders of magnitude stronger than the earth magnetic field. Peierls substitution is not expected to be a good approximation for such large fields.

The sensitivity of the spectral properties of Hofstadter model is an infrared property and so is not believed to be an artifact of the single-band approximation.

From a physical point of view the sensitive dependence of spectral properties on the number-theoretic characterization of B reflects the formulation of the Hofstadter model on an infinite system. No physical system is ever infinite. Finite systems have a built-in infrared cutoff. However, as it is physically meaningful to consider finite systems of

increasing size, the small-scale structure of the spectrum is physically meaningful.

3.1 TKN^2 and the elephant in the room

Marcel den Nijs gave a colloquium at Washington University celebrating Thouless winning the Nobel Prize, where he told a few inside stories about the TKN^2 discovery. The genius of Thouless, he said, was to focus on the Hofstadter Hamiltonian as a model for the quantum Hall effect.

The Hofstadter model is a problematic model of the quantum Hall effect in the following sense: The model does not reproduce the plateaus in Fig. 4 as a function of the density of particles. (It reproduces the plateaus as function of the chemical potential³. I shall come back to this issue in §5.) A more faithful model of the quantum Hall effect would replace the periodic lattice by a random background. Had TKN^2 done so, the TKN^2 integers would probably have had to wait for someone else to discover them.

The starting point of TKN^2 was a paradox. In 1981 Robert Laughlin (Nobel prize 1986) argued that by gauge invariance the Hall conductance should be an integer whenever the Fermi energy lies in a spectral gap [35]. I shall not explain Laughlin's argument which, on even days, I find brilliant and on odd days circular. TKN^2 however, trusted Laughlin, and this led them to the following paradox: Consider a free particle in a constant magnetic field in two dimensions. The Hamiltonian is quadratic and solvable. The spectrum is the same as the Harmonic oscillator, except that each eigenvalue has infinite multiplicity. By explicit calculation one finds that the Hall conductance of a fully occupied lowest energy (the Landau level) is unity. This is the good part, in agreement with Laughlin. Now imagine that translation invariance is broken by a weak periodic potential so that the unit cell carries rational flux. This leads to a Hofstadter model where the mother Landau level splits into daughters mini-bands separated by gaps. If you put the Fermi energy in one of those gaps, the Hall conductance should be an integer, by Laughlin. But, a daughter mini-band surely carries a fraction of the current carried by the mother Landau band. The Hall conductance must then be both an integer and a fraction.

To resolve the paradox, TKN^2 resorted to a numerical computation of the Hall conductance for the Hofstadter model. This can be done whenever $B = 2\pi p/q$. The periodicity $n \mapsto n + q$ reduces H to a $q \times q$ matrix valued function, $H(\mathbf{k})$, on the 2-torus, $\mathbf{k} \in \mathbb{T}^2$:

$$H(\mathbf{k}) = \begin{pmatrix} \cos(k_1 + B) & 1 & 0 & \dots & \dots & e^{ik_2} \\ 1 & \cos(k_1 + 2B) & 1 & \dots & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 1 & \cos(k_1 + qB) & 1 \\ e^{-ik_2} & 0 & \dots & \dots & 1 & \cos(k_1 + qB) \end{pmatrix}, \quad (5)$$

whose spectrum is a collection of q bands of $\mathbf{k} = (k_1, k_2)$. Given the spectral projections $P_n(\mathbf{k})$ on the bottom n -bands, the Hall conductance can be computed by a formula of

³This is because the spectral gaps imply that there is no 1-1 correspondence between the chemical potential and the density at $T = 0$.

Kubo. The textbook version of Kubo used by TKN^2 is ugly and opaque⁴. The version I like better, for objective reasons, of course, is a formula of Seiler, Simon and myself [7],

$$\sigma_n = \frac{i}{2\pi} \int_{\mathbb{T}^2} \text{Tr} \left(P_n \left[\frac{\partial P_n}{\partial k_1}, \frac{\partial P_n}{\partial k_2} \right] \right) d^2k, \quad (6)$$

where the argument is identified with the adiabatic curvature [42, 13]. It gives Kubo the right geometric flavor. However, TKN^2 did not know this. Adiabatic curvature had not been invented yet.

By general principles, $P_n(\mathbf{k})$ is a smooth function of \mathbf{k} as long as the n -th gap does not close. This implies that σ_n is well defined provided that the n -th gap is open. (This is known to be the case for the Hofstadter model for all gaps other than the middle gap at 0 for even q [18, 45].)

Since Kubo's formula depends on spectral projections it is oblivious to the choice of the phase of the wave functions. This is good since the phase is a gauge freedom that must not affect physical quantities. It is important both conceptually and also for reproducible numerical computations, because different algorithms for diagonalizing matrices can make different choices for the phases of the eigenvectors.

When q is small the numerical computation of σ_n is, in principle, straightforward. But when q is large it can be tricky because the Hofstadter model can have exponentially small gaps.

TKN^2 found that Laughlin was always right: When a mother spectral band splits into daughter bands, some daughter bands may carry more current than the mother, and some may carry opposite currents, but all are quantized. TKN^2 invested great efforts to find out how a mother integer splits into daughter integers and searched for a simple equation to describe this. I shall say more about this in §3.2.

Quantized Hall conductance was a surprise for von Klitzing, but given Laughlin's argument it was what TKN^2 expected to find. As a consequence, TKN^2 initially paid little attention to the remarkable fact that Kubo's formula actually yielded an integer. Marcel den Nijs called this “the elephant in the room.” Their eventual realization that the Kubo formula counts rotations came during the write-up of the manuscript. Marcel den Nijs described this as “an afterthought.”

The rotation turns out to be the rotation of the phase of the wave function when going around the Brillouin zone. The fermionic wave function associated with Range $P_n(\mathbf{k})$ is

$$|\Psi(\mathbf{k})\rangle = \frac{1}{\sqrt{n!}} \mathcal{A} |\psi_1(\mathbf{k})\rangle \otimes \cdots \otimes |\psi_n(\mathbf{k})\rangle. \quad (7)$$

\mathcal{A} denotes the anti-symmetrization that takes care of Pauli exclusion principle, and the wave function lives in the fermionic Fock space $\Lambda^n \mathbb{C}^q \sim \mathbb{C}^{\otimes \binom{n}{q}}$.

To count the rotation of the phase of the wave function, we need to pay attention to the continuity properties in \mathbf{k} . It turns out that one can not, in general, choose the phase to be a smooth (periodic) function on the Brillouin zone. The obstruction is fundamental

⁴In units of e^2/h .

(it is related, of course, to the fact that ground state bundle is non-trivial). It reflects a conflict between the requirements that $|\Psi(\mathbf{k})\rangle$ be normalized, for each \mathbf{k} , and smooth (and periodic) in \mathbf{k} . To count the rotations we insist that the wave function be normalized and smooth and give up the periodicity. This means that we embed the Brillouin zone in \mathbb{R}^2 as a domain \blacksquare , with boundary \square .

By known identities for the adiabatic curvature, Eq. (6) can be rewritten in terms of the eigenfunctions:

$$\sigma_n = \frac{1}{\pi} \int_{\blacksquare} d^2k \operatorname{Im} \langle \partial_{k_1} \Psi | \Psi \partial_{k_2} \Psi \rangle. \quad (8)$$

The integrand is a curl of a vector field:

$$2 \operatorname{Im} \langle \partial_1 \Psi \partial_2 \Psi \rangle = -i \nabla_k \times \langle \Psi \nabla_k \Psi \rangle.$$

Using Stokes, the integration over the Brillouin zone can be reduced to integration over its boundary:

$$\sigma_n = -\frac{i}{2\pi} \oint_{\square} d\mathbf{k} \cdot \langle \Psi | \nabla_k \Psi \rangle. \quad (9)$$

The integrand is called the “canonical connection” in mathematics and “Berry’s gauge connection” in physics. The integral is, by definition, Berry’s phase. I shall say more on Berry’s phases in §4.

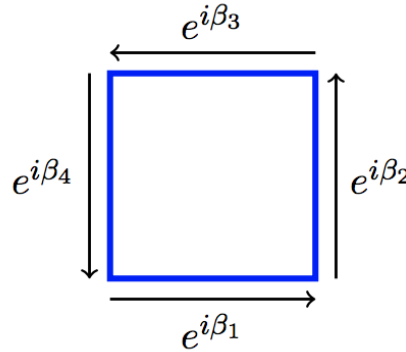


Figure 6: Berry’s phases around the edges of the Brillouin zone.

In general, Berry’s phase is not forced to make an integral number of rotations for any old closed path. However, the path going around the Brillouin zone is special: The spectral projections $P_n(\mathbf{k})$ are periodic and therefore coincide on opposite pieces of the boundaries of the Brillouin zone $1 \leftrightarrow 3$ and $2 \leftrightarrow 4$ (Fig. 6). This implies that Berry’s phases associated with opposite ends are related ⁵

$$e^{i\beta_1} = e^{-i\beta_3}, \quad e^{i\beta_2} = e^{-i\beta_4} \implies e^{i(\beta_1+\beta_2+\beta_3+\beta_4)} = 1. \quad (10)$$

This proves that $\sigma_n \in \mathbb{Z}$. This is special case of a much more general result of Chern [17].

⁵This follows from Eqs. (17),(20) of §4.

3.2 A Diophantine equation and the Colored Hofstadter Butterfly

TKN^2 invested great efforts to find a formula that would encapsulate their numerical findings for σ_n . To find such a formula they turned the Hofstadter model into a problem in perturbation theory by replacing 2 in Eq. (4) by a parameter λ , which they made very small. This allowed them to find a Diophantine equation for the Chern numbers for small λ . It turned out that the formula agreed with their numerical finding for $\lambda = 2$. Since the TKN^2 integers can only change when gaps close this suggests that no gaps close as λ is tuned. This was later shown to be the case by Choi and Elliott [18] and van Mouche [45].

There is an entertaining puzzle associated with TKN^2 's serendipity. Wigner and von Neumann [46] told us that one needs to tune 3 parameters to cause a gap closure in an Hermitian matrix. The Hofstadter model with λ tunable appears to be a three-parameter family involving (k_1, k_2, λ) . Why do the gaps stay open? The protection comes from Chambers formula [16], which implies that the band edges are at $\mathbf{k} \in (0, 0), (0, \pi), (\pi, 0), (\pi, \pi)$. \mathbf{k} is not a free parameter and Wigner von Neumann is consistent with no gap closure.

TKN^2 's Diophantine equation can be more simply derived from symmetry considerations [20]. Let the density of electrons ρ , and the magnetic field B be such that the pair correspond to a gap in the Hofstadter spectrum. Then σ satisfies the Diophantine equation

$$\rho - \frac{B}{(2\pi)}\sigma \in \mathbb{Z}. \quad (11)$$

For example, if $B = 2\pi/q$ and the density is such that n bands are full, $\rho = n/q$, the equation reads

$$\sigma_n = n \bmod q. \quad (12)$$

The equation determines σ_n in the n -th gap up to a multiple of q , which is not enough to determine σ_n uniquely.

TKN^2 complemented the Diophantine equation with the window condition, $|\sigma_n| < q/2$, which determines σ_n uniquely⁶. Picking the right model was a stroke of genius. The window condition for the hexagonal or triangular lattice is not known [5].

Using the Diophantine equation and the window condition Daniel Osadchy made a colored Hofstadter butterfly for his MSc, which appeared on the cover of *Physics Today* with our joint paper with Ruedi Seiler [8]. Since only the gaps are colored, the spectrum is the complement of the colored area. Evidently the spectrum is a small set: It is a set of measure zero⁷.

4 The geometry of adiabatic transport

TKN^2 used Kubo's formula as the definition of the Hall conductance and the finding that it counts rotations was therefore a surprise. One can simply accept this surprise, and one

⁶Except for the middle gap, $n = q/2$ when q is even. Fortunately, this gap is closed.

⁷The Hausdorff dimension of the spectrum of the Hofstadter model has been studied in [34, 21].

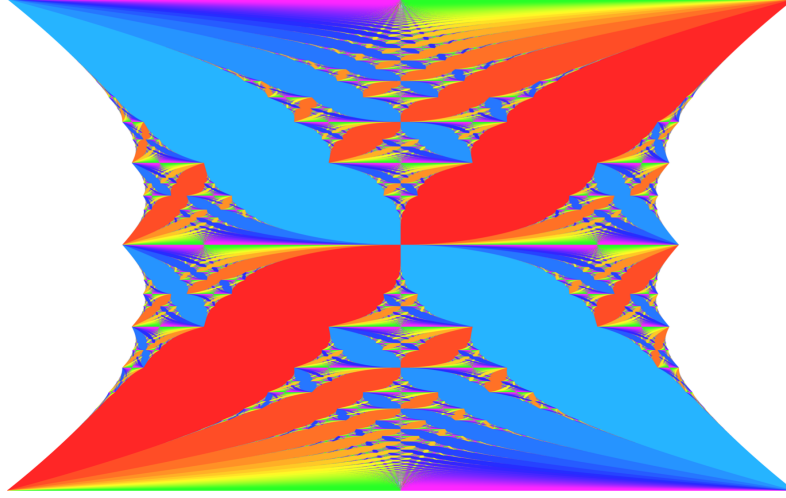


Figure 7: A colored Hofstadter butterfly. The colors represent Chern numbers. The vertical axis is the magnetic field B and the horizontal axis is the chemical potential. The picture was made by Daniel Osadchy.

can wonder if, at least in hindsight, there is a good reason for this. Using Kubo as a black box masks its geometric origin in adiabatic evolutions. For reasons of space and time I shall not try and describe the theory of adiabatic quantum transport [15, 9, 38]. Instead, I shall describe parallel transport in adiabatic evolutions and the Berry connection.

Adiabatic evolutions lead to motions that adhere to spectral subspace. Let $P(\mathbf{k})$ be a smooth family of orthogonal projections, and $|\psi(\mathbf{k})\rangle \in \text{Range } P(\mathbf{k})$. We want to define parallel transport $|\psi(\mathbf{k})\rangle$ along a curve $\mathbf{k}(t)$ with $t \in [0, 1]$.

The constraint $P|\psi\rangle = |\psi\rangle$ determines the motion of the perpendicular component of the vector:

$$P_{\perp} d|\psi\rangle = (dP)|\psi\rangle. \quad (13)$$

The motion parallel to P is free. Parallel transport fixes it. A natural choice is to insist that there is no motion, i.e.

$$0 = Pd|\psi\rangle. \quad (14)$$

The evolution generated by an adiabatic H agrees with this notion of parallel transport provided $HP = 0$.

The two equations, Eqs. (13,14), combine to

$$d|\psi\rangle = (dP)|\psi\rangle = (dP)P|\psi\rangle = [dP, P]|\psi\rangle, \quad (15)$$

where the last step used the useful identity

$$P(dP)P = 0. \quad (16)$$

Since $[dP, P]$ is anti-Hermitian it generates a unitary evolution along the path $\mathbf{k}(t)$

$$dU = [dP, P]U, \quad U(0) = \mathbb{1}. \quad (17)$$

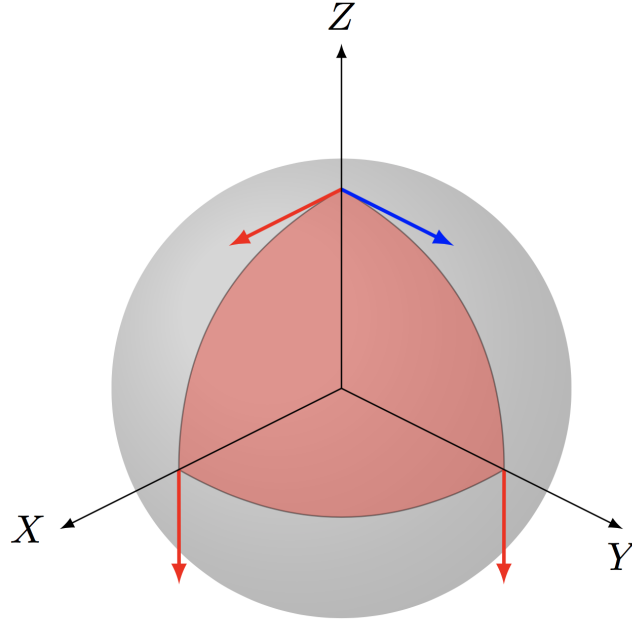


Figure 8: Parallel transport of the sphere: A point moving along a meridian at constant speed has a velocity vector which is parallel transported. The acceleration is radial in accordance with Eq. (14). The spherical triangle illustrates the holonomy, a.k.a. Berry's phase, associated with parallel transport.

U preserves the length of the vector and respects the splitting into P and P_\perp , i.e.,

$$U_t P_0 = P_t U_t. \quad (18)$$

In particular, if the path is closed,

$$U_1 P_0 = P_1 U_1 = P_0 U_1. \quad (19)$$

so U_1 reduces on P_0 and defines a unitary map on $\text{Range } P_0$. This is the adiabatic holonomy [13, 42]. In the case that P is one-dimensional, it reduces to Berry's phase,

$$U_1 P_0 = e^{i\beta(1)} P_0. \quad (20)$$

Given a continuous frame $|\Psi(\mathbf{k}(t))\rangle \in \text{Range } P(\mathbf{k}(t))$ along the closed path, Berry's phase $\beta(1)$ can be computed by integrating the canonical, a.k.a. Berry's, gauge connection⁸

$$d\beta(t) = i \langle \Psi(\mathbf{k}(t)) | d\Psi(\mathbf{k}(t)) \rangle \quad (21)$$

independently of the choice of frame. This follows from the equation of parallel transport (14), for the vector $|\psi(t)\rangle = e^{i\beta(t)} |\Psi(\mathbf{k}(t))\rangle$ along the path $\mathbf{k}(t)$.

⁸In the case $|\Psi(\mathbf{k}(0))\rangle = e^{i\varphi} |\Psi(\mathbf{k}(1))\rangle$ the integral of the connection gives $\beta(1) - \varphi$. This is the case for β_j in Eq. (9). Fortunately, the phases φ_j drop upon summation over the four edges.

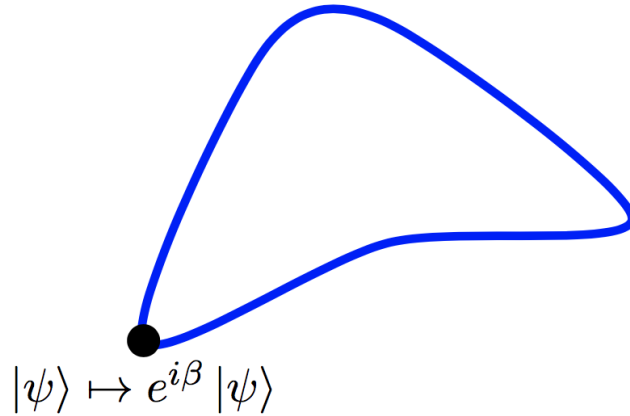


Figure 9: Berry's phase as failure of parallel transport

Epilog: In 1950, Tosio Kato gave a proof of the adiabatic theorem of quantum mechanics where he introduced the unitary evolution Eq. (17). Kato realized the significance of the parallel transport in adiabatic evolution but he missed the holonomy for closed paths found by Berry. Berry's rule of scientific discoveries is partly self-referential.

4.1 Geometric response

How come the Kubo formula (6) can be expressed in terms of the adiabatic curvature? An honest answer [15, 9, 38] needs more space and patience than the reader and writer can afford. Instead, let me explain an elementary general identity that shows how the response of driven quantum systems is intimately related to the rate of change of Berry's gauge field.

Consider the response of driven quantum systems described by the Hamiltonian $H(k, t)$. The explicit time-dependence reflects the external driving of the system. The role of the parameter k is to define the observable associated with the response. The time-dependent Schrödinger equation determines the state:

$$i \frac{d}{dt} |\psi\rangle = H(k, t) |\psi\rangle. \quad (22)$$

We are interested in the response associated with the observable of virtual work $\frac{\partial H}{\partial k}$. For example, if the parameter k has the interpretation of momentum, the virtual work is the velocity.

By Leibniz and Eq. (22),

$$\left\langle \psi_t \left| \frac{\partial H}{\partial k} \right| \psi_t \right\rangle = i \frac{d}{dt} \left\langle \psi_t \left| \frac{\partial \psi_t}{\partial k} \right\rangle \right. \quad (23)$$

The equation relates the expectation of virtual work to the rate of change of Berry's gauge field. This relation captures the flavor of the geometric view of transport. I believe that it is due to Ruedi Seiler and myself. It is a time-dependent generalization of the Feynman-Hellmann theorem.

To see how this connects to the quantum Hall effect recall that the Hall conductance relates the current in the y direction to the electric field in the x direction. The electric and magnetic fields are determined by the gauge field $\mathbf{A} = (-Et, Bx)$. Peierls's substitution gives the time-dependent Hofstadter model

$$(H\psi)(n) = e^{-iEt}\psi(n+1) + e^{iEt}\psi(n-1) + 2\cos(Bn+k_2)\psi(n), \quad (24)$$

which has the structure of $H(k, t)$ in (22). Since k_2 is (Bloch) momentum, the observable $\partial_{k_2}H$ is, by definition, the velocity in the y direction. Eq. (23) then relates the Hall velocity to Berry's gauge potential .

5 The take-home moral

In the old days every quantum system was characterized by a wave function. There is, of course, no interesting topology associated with wave functions, since any vector in the Hilbert space can be continuously deformed to any other vector. TKN^2 ushered in a new era where certain condensed-matter systems are characterized instead by a *bundle* of eigenfunctions, (often with the Brillouin zone as its base). The bundle allows one to describe the otherwise ill-defined notion of the wave function of infinitely many fermions that fill a (Bloch) band. This opens the door to a topological characterization of quantum states, since bundles of eigenfunctions are not connected by smooth deformations, for the same reason that a Möbius strip cannot be continuously deformed to a cylinder.

A different angle on the TKN^2 integers was developed by Jean Bellissard [13]. In this theory the quantum state of infinitely many non-interacting fermions is described by an infinite-dimensional projection in the one-particle Hilbert space. (By Pauli's principle, fermions populate orthogonal states in the one particle Hilbert space: An N -dimensional projection accommodates N fermions.) Here, the TKN^2 integers are interpreted as Fredholm indices. Bellissard was influenced by Alain Connes's Non-commutative geometry, where Fredholm indices play the role of Chern numbers.

In Bellissard's theory the Brillouin zone and spectral gap play a minimal role: The gap condition is replaced by decay properties of the kernel of the spectral projection, and the theory was originally devised to deal with the Hofstadter model for irrational magnetic field, which has no Brillouin zone. Its application that I like the best is to the quantum Hall effect with a random background potential. This gives a better model than Hofstadter's. (It reproduces the plateaus in Fig. 4, which Hofstadter does not.) The application is a confluence of the two great mathematical-physics problems of the time: The quantum Hall effect and Anderson localization [2, 40].

Seiler, Simon and myself [10] developed a version of Bellissard's theory that dispensed with much of the C^* algebra in Connes's theory and gave birth to the *Relative index of projections* [4].

Associating quantum states with projections leads naturally to topological obstructions: Projections must have the same dimension to be continuously related. Infinite-dimensional projections have similar, but more subtle, obstructions [4]. Quantum states described by infinite-dimensional projections can have interesting topological properties.

5.1 Where are we now?

B. Simon’s list of “Fifteen problems in mathematical physics” [43] includes the problem to prove, or find an alternative to, Kubo’s formula. The issue has to do with the putative existence of a steady state. The formal derivations of Kubo assume that the thermodynamic, the long time limit, and the weak driving exist and commute. Sven Bachmann, Wojciech de Roeck, and Martin Fraas, [11] recently solved this problem, proving Kubo’s formula for interacting electrons, for arbitrarily large number of particles. This closes a gap in the theory of the quantum Hall effect.

Topological quantum phases were born in the context of two-dimensional semiconductors with broken time reversal. It is now understood that there are interesting topological quantum phases all the way from insulators to superconductors, with and without time reversal, parity, and charge conjugation. The field is so large that it is impossible for me to say anything meaningful in a paragraph, see e.g. [27] and Alexei Kitaev, “Periodic table of topological insulators and superconductors” [31]. (Kitaev makes use of K-theory pioneered, in the context of condensed matter physics, by Jean Bellissard [12].)

Entanglement in topological phases [39] and the interface between different topological phases are vibrating topics in condensed matter physics today. The era of topological quantum states of matter initiated by Thouless and Haldane is blooming.

Acknowledgment

I owe a debt of gratitude to M. Berry, J. Bellissard, A. Elgart, M. Fraas, J. Fröhlich, G. M. Graf, D. Osadchy, L. Sadun, J. Segert, R. Seiler, B. Simon, and P. Zograf for all they taught me. I thank J. Bellissard, M. Fraas, D. Haldane, O. Kenneth, M. den Nijs, M. Reznikov, L. Sadun, R. Seiler, B. Simon, and A. Stern for comments on a draft of this paper. The work is supported by ISF.

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International Prize “Tullio Levi-Civita” for the Mathematical and Mechanical Sciences

In honor of the famous Italian mathematical physicist Tullio Levi-Civita,* in 2010, the International Research Center for Mathematics & Mechanics of Complex Systems (M&MoCS) established the international prize “Tullio Levi-Civita.” The prize recognizes the high quality and undisputed originality of the scientific research of up to two distinguished Italian or foreign scientists in the field of mechanics and/or mathematics. Every winner must have contributed to the development of at least one young researcher and is expected to hold a series of lectures and join the research activities of the Center during a short stay. In 2016 the prize was awarded to Tudor Ratiu (École Polytechnique Fédérale de Lausanne EPFL, Switzerland) and Mauro Carfora (Università degli Studi di Pavia, Italy). Past winners of the prize include Lucio Russo (2010, Università degli Studi di Roma Tor Vergata), Pierre Seppecher (2010, Université de Toulon et du Var), Jean-Jacques Marigo (2011, École Polytechnique), Eric Carlen (2011, Rutgers University), Félix Darve (2012, Institut National Polytechnique de Grenoble), Alexander Seyranian (2012, Institute of Mechanics, Moscow State Lomonosov University), Kazuo Aoki (2013, Kyoto University), David Steigmann (2013, University of California, Berkeley), Marcelo Epstein (2014, University of Calgary), Errico Presutti (2014, GSSI Gran Sasso Science Institute), Graeme W. Milton (2015, University of Utah), Andrea Braides (2015, Università degli Studi di Roma Tor Vergata).

Moreover, every year, during the Levi-Civita Lectures, the recipients of the International Levi-Civita Prize and up to two young promising researchers are invited to give a talk. Finally, there is no need for applications, since every scientist working in mechanics and mathematics will be considered for the prize by the scientific committee. Any further enquiries can be sent to memocs.cisterna@gmail.com, and more information can be found on the website <http://memocs.univaq.it>.

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* Tullio Levi-Civita (1873-1941) was born into an Italian Jewish family and enrolled in 1890 at the University of Padua. He graduated in 1892 and his final dissertation, which was supervised by Ricci Curbastro, dealt with absolute invariants and tensor calculus. Levi-Civita was appointed to the Chair of Rational Mechanics at Padua in 1898. After the end of World War I, the University of Rome made strenuous efforts to attract many leading scientists and hence become an internationally recognized first-tier institution. Levi-Civita was always very international in his outlook and the ability of Rome to attract top quality students from abroad must have been a reason for him choosing to

move there. In 1918 he was appointed to the Chair of Higher Analysis at Rome and, two years later, he was appointed to the Chair of Mechanics. Levi-Civita had a great command of pure mathematics, with a particularly strong geometric intuition, which he exploited in addressing a variety of problems in applied mathematics. He is best known for his work on absolute differential calculus, with its applications to Einstein’s theory of relativity, and on the calculus of tensors including covariant differentiation, continuing the work of Christoffel. Levi-Civita was also interested in the theory of stability and qualitative analysis of differential equations (because of his interest in geometry and geometric models) and classical and celestial mechanics. Indeed, he published many papers dealing with analytic dynamics. He examined special cases of the three-body problem and, near the end of his career, he became interested in the n -body problem. In the field of systems of partial differential equations, he extended the theory of Cauchy and Kovalevskaya. In addition, Levi-Civita made a major contribution to hydrodynamics, proving the existence of periodic finite-amplitude irrotational surface waves in a monodimensional fluid flow.

Note: This announcement is reprinted with minor corrections from the June, 2017, issue of the *Newsletter of the European Mathematical Society*.

Call for Nominations for the 2018 Henri Poincaré Prize

The Henri Poincaré Prize, sponsored by the Daniel Iagolnitzer Foundation was created in 1997 to recognize outstanding contributions in mathematical physics, and contributions which lay the groundwork for novel developments in this broad field. The prize was also created to recognize and support young people of exceptional promise who have already made outstanding contributions to the field of mathematical physics. The prize is awarded every three years at the International Congress of Mathematical Physics (ICMP), and in each case is awarded usually to three individuals.

The prize winners are chosen by the Executive Committee of the IAMP upon recommendations given by a special Prize Committee. The Executive Committee has made every effort to appoint to the Prize Committee prominent members of our community that are representative of the various fields it contains. However, to be able to do its job properly the Prize Committee needs input from the members of IAMP. For this purpose the Executive Committee calls IAMP members to provide nominations for the Henri Poincaré Prize to be awarded at ICMP 2018 in Montreal, Canada.

A proper nomination should include the following:

- A description of the scientific work of the nominee emphasizing the key contributions
- A recent C.V. of the nominee
- A proposed citation should the nominee be selected for an award

Please keep the length of your nomination within a page and submit it to the President (president@iamp.org) or the Secretary (secretary@iamp.org).

A list of previous winners can be found at: <http://www.iamp.org>.

To ensure full consideration please submit your nominations by **September 30, 2017**.

Call for Nominations for the 2018 IAMP Early Career Award

The IAMP Executive Committee calls for nominations for the 2018 IAMP Early Career Award. The prize was instituted in 2008 and will be awarded for the fourth time at the ICMP in Montreal, Canada in July, 2018.

The Early Career Award is given in recognition of a single achievement in mathematical physics. The total prize value amounts to 3000 Euro and is reserved for scientists whose age in years since birth on July 31 of the year of the Congress is less than 35.

The nomination should include the name of the candidate accompanied by a brief characterization of the work singled out for the prize. Members of the IAMP should send their nomination or nominations to the President (president@iamp.org) and to the Secretary (secretary@iamp.org). A list of previous winners and the details of the award selection process can be found at <http://www.iamp.org>.

Nominations should be made not later than on **January 31, 2018**.

Call for Nominations for the IUPAP Young Scientist Award in Mathematical Physics 2018

The Young Scientist Award in Mathematical Physics of [IUPAP](#) recognizes exceptional achievements in mathematical physics by scientists at relatively early stages of their careers. It is awarded triennially to at most three young scientists satisfying the following criteria:

- The recipients of the awards in a given year should have a maximum of 8 years of research experience (excluding career interruptions) following their PhD on January 1 of that year, in the present case 2018.
- The recipients should have performed original work of outstanding scientific quality in mathematical physics.
- Preference may be given to young mathematical physicists from developing countries.

Further information about the prize can also be found [here](#).

The awards will be presented at the ICMP in July 2018 in Montreal.

Please submit your nomination by e-mail to the officers of the [IUPAP C18 Commission for Mathematical Physics](#), Manfred Salmhofer (salmhofer@uni-heidelberg.de), Rainer Dick (rainer.dick@usask.ca), and Patrick Dorey (p.e.dorey@durham.ac.uk).

The deadline for nominations is **August 31, 2017**.

News from the IAMP Executive Committee

New individual members

IAMP welcomes the following new members

1. DR. MICHAŁ WROCHNA, University of Grenoble Les Alpes, France
2. DR. JAN MAAS, IST Austria
3. DR. JEAN-FRANÇOIS GILLE, CPT Marseille-Luminy, France

Recent conference announcements

Classical and quantum motion in disordered environment. A random event in honour of Ilya Goldsheid's 70th birthday

Dec. 18-22, 2017. Queen Mary, University of London, UK.

This conference is partially supported by IAMP.

Organized by L. Parnowski, M. Shamis, S. Sodin, A. Soshnikov.

<http://www.maths.qmul.ac.uk/~s.sodin/Ilya70.html>

Advances in Mathematics and Theoretical Physics

Sept. 19-22, 2017. Accademia Nazionale dei Lincei, Rome, Italy.

Organized by F. Ciolli, A. De Sole, A. Giuliani, D. Guido, R. Longo, G. Morsella, N. Pinamonti, A. Pizzo, G. Ruzzi.

<http://www.mat.uniroma2.it/tlc/17SIMP/index.php?p=home>

Open positions

PhD positions at Ostrava University, Czech Republic

The Department of Mathematics of the University of Ostrava is offering PhD position. These are 4-year, fully funded positions, covering tuition fee and featuring a competitive salary. The language of instruction is English. The starting date is the beginning of the 2017/2018 academic year or shortly thereafter.

Topics: Spectral Theory, Eigenvalue Problems in Mathematical Physics, Differential Operators. The deadline is 31 August 2017. For further information:

<http://katedry.osu.cz/kma/agmp/phd/>

Multiple theory PhD and postdoc positions at Nanyang Technological University in Singapore

Multiple theory PhD and PostDoc positions will be available from November 2017 in the group of Marco Battiato at the Nanyang Technological University in Singapore.

The research will focus on

- the development of a massively parallel solver for the Boltzmann equation for strongly out-of-equilibrium states in real band structures and extended heterostructures;
 - the study of ultrafast spin transport in metals and semiconductors;
 - absorption and production of THz radiation in heterostructures;
 - thermalisation dynamics and transport in topological insulators and perovskites.
- The remuneration and travel budgets are competitive.

For more information:

<https://sites.google.com/site/battiatomarco/open-positions>

Postdoctoral positions in mathematical physics/probability at Helsinki University

The Mathematical Physics group at the University of Helsinki

(<http://mathstat.helsinki.fi/mathphys/>) is looking for postdocs in the following fields:

- Rigorous Statistical Mechanics and Renormalization Group
- Singular Stochastic PDE's
- Conformal Field Theory and Liouville Quantum Gravity

The positions are funded through the European Research Council (ERC) Advanced Grant “Quantum Fields and Probability” and they are available from September 2017 on.

An updated list of academic job announcements in mathematical physics and related fields is available at

http://www.iamp.org/page.php?page=page_positions

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