IAMP News Bulletin
July 2018

International Association of Mathematical Physics

International Association of Mathematical Physics News Bulletin, July 2018

Contents

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Cover picture: Welcome to the ICMP 2018

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News Bulletin (International Association of Mathematical Physics)

Welcome to the ICMP!

On behalf of the International Association of Mathematical Physics, the International Scientific Committee, and the Local Organizing Committee we welcome participants to the XIXth International Congress on Mathematical Physics in Montréal!

In addition to the technical scientific program, we invite you to attend the public lectures by Rainer Weiss and Elliott Lieb, and we hope that you will acquaint yourself with the rich cultural and touristic offerings you will find here during your visit!

VOJKAN JAKSIC
Chair of the Local Organizing Committee

ROBERT SEIRINGER
President of the International Association of Mathematical Physics, and
Chair of the International Scientific Committee

EVANS HARRELL
Chief Editor of the IAMP News Bulletin

Public Lectures

IAMP is pleased to present two free Public Lectures which are intended for a wider audience from the Montréal area.

Monday, July 23, 2018, 16:45, symposia auditorium

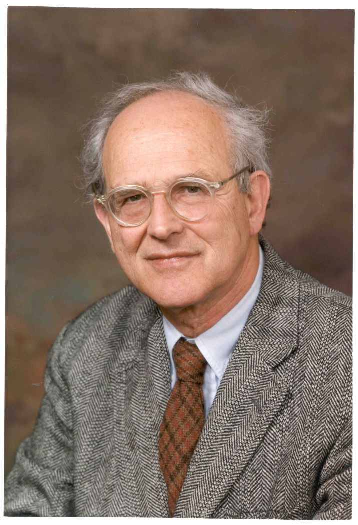
RAINER WEISS (Massachusetts Institute of Technology)

Exploration of the Universe with Gravitational Waves

Thursday, July 26, 2018, 16:45, symposia auditorium

ELLIOTT LIEB (Princeton University)

Facets of Entropy



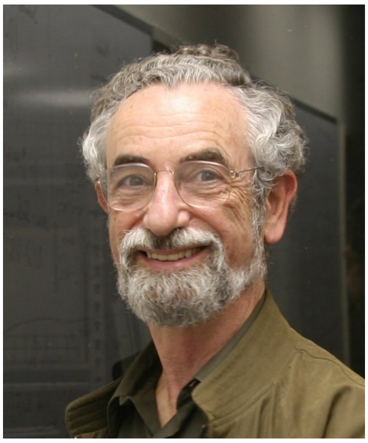
Rainer Weiss

Massachusetts Institute of Technology
2017 Nobel Laureate

RAINER WEISS (NAS) is a Professor Emeritus at Massachusetts Institute of Technology (MIT). Previously Dr. Weiss served as an assistant physics professor at Tufts University and has been an adjunct professor at Louisiana State University since 2001. Dr. Weiss is known for his pioneering measurements of the spectrum of the cosmic microwave background radiation, his inventions of the monolithic silicon bolometer and the laser interferometer gravitational wave detector, and his roles as a co-founder and an intellectual leader of both the COBE (microwave background) Project and the LIGO (gravitational-wave detection) Project. He has received numerous scientific and group achievement awards from NASA, an MIT excellence in teaching award, the John Simon Guggenheim Memorial Foundation Fellowship, the National Space Club Science Award, the Médaille de l'ADION Observatoire de Nice, the Gruber Cosmology Prize, and the Einstein Prize of the American Physical Society. Dr. Weiss is a fellow of the American Association for the Advancement of Science, the American Physical Society, The American Academy of Arts and Sciences; and he is a member of the American Astronomical Society, the New York Academy of Sciences, and Sigma Xi. He received his B.S. and Ph.D. in physics from MIT. Dr. Weiss is a member of the NAS and has served on nine NRC committees from 1986 to 2007 including the Committee on NASA Astrophysics Performance Assessment; the Panel on Particle, Nuclear, and Gravitational-wave Astrophysics; and the Task Group on Space Astronomy and Astrophysics.

Exploration of the Universe with Gravitational Waves

The observations of gravitational waves from the merger of binary black holes and from a binary neutron star coalescence followed by a set of astronomical measurements is an example of investigating the universe by 'multi-messenger' astronomy. Gravitational waves will allow us to observe phenomena we already know in new ways as well as to test general relativity in the limit of strong gravitational interactions – the dynamics of massive bodies traveling at relativistic speeds in a highly curved space-time. Since the gravitational waves are due to accelerating masses while electromagnetic waves are caused by accelerating charges, it is reasonable to expect new classes of sources to be detected by gravitational waves as well. The lecture will start with some basic concepts of gravitational waves, then briefly describe the instruments and the methods for data analysis that enable the measurement of gravitational wave strains of the order of 10^{-21}, and will then present the results of recent runs. The lecture will end with a vision for the future of gravitational wave astrophysics and astronomy.



Elliott Lieb

Princeton University

ELLIOTT LIEB is an eminent American mathematical physicist and professor of mathematics and physics at Princeton University, who specializes in statistical mechanics, condensed matter theory, and functional analysis. In particular, his scientific works pertain to: the quantum and classical many-body problem, the stability of matter, atomic structure, the theory of magnetism, and the Hubbard model. He is a prolific author in mathematics and physics with over 300 publications. He received his B.S. in physics from MIT (1953) and his Ph.D. in mathematical physics from the University of Birmingham in England (1956). Lieb was a (1956–1957) Fulbright Fellow at Kyoto University, Japan, and for some time worked as the Staff Theoretical Physicist for IBM. He has been a professor at Princeton since 1975, following a leave from his professorship at MIT. Lieb has been awarded several prizes in mathematics and physics, including the 1978 Heineman Prize for Mathematical Physics of the American Physical Society and the American Institute of Physics (1978), the Max Planck Medal of the German Physical Society (1992), the Boltzmann medal of the International Union of Pure and Applied Physics (1998), the Schock Prize (2001), and the Henri Poincaré Prize of the International Association of Mathematical Physics (2003). He is a member of the U.S. National Academy of Sciences and has twice served (1982–1984 and 1997–1999) as the President of the International Association of Mathematical Physics. His Erdös number is 2. He is married to fellow Princeton professor Christiane Fellbaum.

Facets of Entropy

I will discuss and compare several ways in which the word entropy is used and the confusion that is sometimes generated. For example, to what extent is entropy well-defined outside of equilibrium and what does it mean for systems that have no thermodyamic limit? Does statistical mechanics define the concept and does it have an independent meaning outside of the self-referential concepts temperature and heat? A simple definition of entropy for macroscopic systems by Jakob Yngvason and myself that makes no mention of heat, temperature, and Carnot cycles will be described briefly.

About Montréal

Montréal is one of Canada's most dynamic cities. Culturally diverse and a designated UNESCO City of Design, Montréal has something for everyone. Places such as the Quartier International and Downtown Art and Architecture showcase Montréal's avant-garde character. Hundreds more examples of the city's innovation and free-spiritedness flourish in fashion boutiques and interior and industrial design spaces.

There are several neighbourhoods in Montréal, each with its own distinct feel. Old Montréal is beautifully preserved and offers a peak into the history of the city. Encircling the "Mountain" lie cozy, charming neighbourhoods, each with its own commercial centers and three-storey row housing. The Metro easily services these areas and beyond.

Renowned for its sensational and original restaurants and as a center for French cuisine par excellence, Montréal is part of a select group of 15 cities to join the prestigious World Good Food Cities Network in 2007. Over 80 different kinds of food are served in 6,000 restaurants, from 5-star to soul-comforting smoked-meat counters.

Montréal is the most bilingual metropolis in North America and the second largest French-speaking city in the world after Paris. Located on an island 499 km^2 in the middle of the St. Lawrence River, it was colonized by the French and later the British. The population of Greater Montréal is approximately 3.6 million, with 53% of its inhabitants bilingual (French and English) and 20% using a third language in addition to French and English.

For more information, please visit

http://www.tourisme-Montreal.org/.

You may also find some of the following web resources useful for visiting Montréal:

https://www.mtl.org/en → https://www.mtl.org/en/contact-us

http://ville.Montreal.qc.ca/ → Activities and recreation → Tourist information

https://www.quebecoriginal.com/en-ca/listing/tourist-organizations-and-transportation/tourist-organizations/regional-tourism-associations/tourism-Montreal-greater-Montreal-convention-and-tourism-bureau-gmctb-420985

https://www.tripadvisor.ca/Attractions-g155032-Activities-Montreal_Quebec.html

50 Years of Mathematical Research at the CRM

by VÉRONIQUE HUSSIN (Centre de Recherches Mathématiques, Montréal)

To celebrate its 50th anniversary, Montréal's Centre de Recherches Mathématiques (CRM) has decided to offer a diversified and intensive scientific program that should have a great impact on research in mathematical sciences. It will be spread over 18 months starting from March 2018. This program includes a thematic semester on "Mathematical challenges in many-body physics and quantum information," and 13 one-month periods on topics of great interest as well as a series of activities of shorter duration.

At the CRM's 50th anniversary launch day held on March 9, 2018, the CRM had the pleasure to welcome a sole speaker in Jean-Pierre Bourguignon. Professor Bourguignon is a French mathematician specializing in differential geometry and physical mathematics. He is also an honorary professor at the IHES (Institut des Hautes Études Scientifiques, Université Paris-Saclay) and Director Emeritus of the CNRS in France. He currently serves as President of the European Research Council. The CRM, proud to promote its scientific activities in French, had chosen to give him the floor for a talk entitled "Mathematics, science and technology, the new data – examples inspired by activities of the CRM."

The CRM was created thanks to the enthusiasm of the mathematician Maurice Labbé, who in 1968 convinced Vice-Rector Roger Gaudry to support the initiative of forming a research pool in mathematical sciences, which would have an exposure on an international and national level. The renowned statistician Jacques St-Pierre was the first director of the Centre de Recherches Mathématiques from 1969 to 1971.

In 1990, Professor Francis Clarke, the then director, introduced the first thematic programs that enabled the gathering at the CRM of hundreds of researchers from around the world to work on specific themes at the cutting edge of mathematics. They participate in workshops, conferences, mini-courses, and schools, the duration of which is from one to four months, up to one year. Extended stays of visiting researchers at the CRM are also welcomed, and conferences given by Aisenstadt Chairs are also prevalent.

In 1991 Professor Clarke oversaw the creation of the ISM (Institut des Sciences Mathématiques) project, a doctoral school based in Montréal. This institute united mathematical forces in Québec in a single body in order to optimize with efficiency the training of students in mathematics, statistics, and actuarial sciences in the province. He was using, in particular, faculty resources of six universities in Québec.

In 1997 the CRM instigated the Network for Computing and Mathematical Modeling, ncm2, a network of centers in the Montréal area. The ncm2, supported by Canada's NSERC, helped to meet the needs of industry in a great variety of fields related to computation and mathematical modeling.

In 1999 the three Canadian institutes (CRM, Fields, and PIMS) created MITACS (Mathematics of Information Technology and Complex Systems) with the help of a grant from the Federal government. The goal of MITACS, the only network of centers of excellence in mathematical sciences in Canada, was to channel Canadian efforts to develop, apply and market new tools and mathematical methodologies within a world-class research program. The MITACS network was a huge success: it brought together up to 300 researchers and 600 students from almost 50 Canadian universities. It expanded its activities to sciences other than mathematics, and, consequently, the MPrime network took over the mathematical operations in 2011. But since MPrime no longer exists, industrial collaborations of the three Canadian mathematical institutes are being held within the framework of the Institutes Innovation Platform (IIP), an initiative supported by NSERC.

In 2001, under the new strategic plan of the FRQNT, the CRM consolidated groups of researchers from partner universities by creating laboratories. There are curently 13 of them: Mathematical Analysis, CAMBAM (Centre for Applied Mathematics in Bioscience and Medicine), CICMA (Centre Interuniversitaire en Calcul Mathématique Algébrique), CIRGET (Centre Interuniversitaire de Recherches en Géométrie et Topologie), GIREF (Groupe Interdisciplinaire de Recherche en Éléments Finis), LaCIM (Laboratoire de Combinatoire et d'Informatique Mathématique), MILA (Montréal Institute for Learning Algorithms), Mathematical Physics, PhysNum, Probability, Quantact (Laboratoire de Mathématiques Actuarielles et Financières), and Statistics.

In 2009 the CRM joined forces with French partners to create the Unité Mixte Internationale (UMI) of the CNRS. This UMI, whose official name is "Centre de Recherches Mathématiques UMI 3457", was founded in October, 2011, and has been scoring a huge success. It gives financial support to long-term and short-term French research visitors at the CRM as well as Québec research visitors in France (in the form of 'postes rouges' or visits lasting a few weeks). Furthermore, the UMI subsidizes meetings and workshops, either by providing funds or by taking charge of the speaker's visit (for example). In this way, the UMI supports thematic activities and other CRM activities.

In 2013, on Christiane Rousseau's initiative, the CRM contributed to the event "Mathematics of Planet Earth," which became highly successful.

In 2015 the Simons Foundation started to offer a 5-year subsidy to help the CRM welcome senior and junior researchers within its programs.

The CRM has a long tradition of supporting the mathematical physics community of researchers. This field represents one of the traditional forces of the CRM, and the vitality of the mathematical-physics laboratory has remained undimmed to this day. It comprises around 20 regular members, tens of associate local members (all full-time faculty in one or the other partner universities of the CRM), and external associate members working permanently in universities or research laboratories in Europe, the United States, or Mexico. The laboratory performs research in the most active fields of mathematical physics, namely, coherent nonlinear systems in fluid mechanics, optic and physics of plasmas, classical and quantum integrable systems, spectral theory of random matrices, percolation, conformal field theories, quantum statistical mechanics, spectral theory and diffusion of random Schrödinger operators, quasicrystal, relativity, spectral transformation methods, asymptotic properties of eigenstates, fundamental questions in quantification, coherent states, wavelets, supersymmetry, equation symmetry analysis of nonlinear stochastic PDE and difference equations, representation theory of Lie group and quantum groups, and the mathematical structure of classical and quantum field theory.

In particular, this year's program comprises the thematic semester "Mathematical Challenges in Many-Body Physics and Quantum Information" (from September to December, 2018). Organized by Jacques Hurtubise (McGill), Dmitry Jakobson (McGill), Vojkan Jaksik (McGill), Dmitry Korotkin (Concordia), and Luc Vinet (Montréal), it consists of five workshops and one school held at the CRM as well as a joint CRM-Princeton workshop. This thematic program will be inaugurated at the XIX International Congress on Mathematical Physics and the related satellite meetings. Long-term researchers at the CRM, notably those in the Simons CRM Scholar-in-Residence program, will give daily seminars and mini-courses between workshops.

The five workshops will focus on the following themes: many-body quantum mechanics, entanglement, integrability and topology in many-body systems, quantum information and quantum statistical mechanics, and spectral theory of quasi-periodic and random operators.

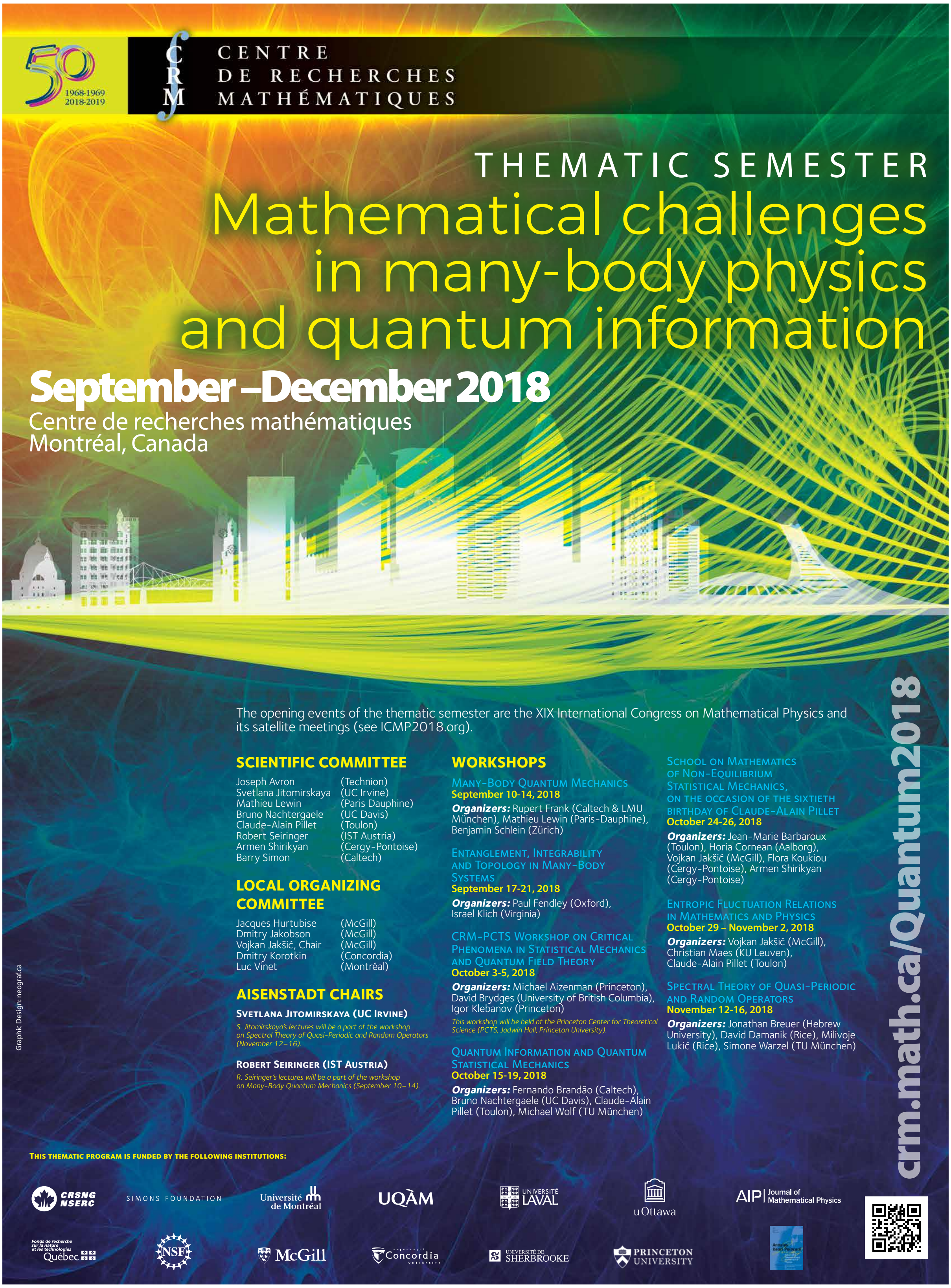
The school on mathematics of non-equilibrium statistical mechanics, on the occasion of the sixtieth birthday of Claude-Alain Pillet, will serve as preparation for the workshop on "Entropic Fluctuation Relations in Mathematics and Physics."

The joint CRM-Princeton workshop will be held at Princeton. It will focus on critical phenomena in statistical mechanics and quantum field theory.

This thematic semester will also welcome three Aisenstadt Chairs. Professor Michael Aizenman (Princeton) is the principal organizer of the joint CRM-Princeton workshop. He will give his lectures at the CRM in September. Professor Svetlana Jitomirskaya (UC Irvine) will give a series of three talks as part of the workshop on "Spectral Theory of Quasi-Periodic and Random Operators." Professor Robert Seiringer (IST Austria) will give a series of three conferences within the workshop on "Many-Body Quantum Mechanics."

In short, since its creation 50 years ago, the Centre de Recherches Mathématiques set itself apart with its great enthusiasm and progressiveness in the face of mathematical advances. As Professor Jean-Pierre Bourguignon said during the inauguration of the 50th anniversary, thanks to the CRM's visionary leadership, it has anticipated changes that occurred in mathematics during recent years and set an example by developing partnership programs between fundamental and applied research. By the same token, the CRM opened up new avenues for talented researchers and has allowed the City of Montréal to be immortalized in the mathematical community.

Acknowledgment
The author is grateful for the assistance of Suzette Paradis in preparing this article.


1968-1969
2018-2019
CENTRE
DE RECHERCHES
MATHÉMATIQUES
THEMATIC SEMESTER
Mathematical challenges in many-body physics and quantum information
September – December 2018
Centre de recherches mathématiques
Montréal, Canada
The opening events of the thematic semester are the XIX International Congress on Mathematical Physics and its satellite meetings (see ICMP2018.org).
SCIENTIFIC COMMITTEE
Joseph Avron (Technion)
Svetlana Jitomirskaya (UC Irvine)
Mathieu Lewin (Paris Dauphine)
Bruno Nachtergaele (UC Davis)
Claude-Alain Pillet (Toulon)
Robert Seiringer (IST Austria)
Armen Shirikyan (Cergy-Pontoise)
Barry Simon (Caltech)
LOCAL ORGANIZING COMMITTEE
Jacques Hurtubise (McGill)
Dmitry Jakobson (McGill)
Vojkan Jakšić, Chair (McGill)
Dmitry Korotkin (Concordia)
Luc Vinet (Montréal)
AISENSTADT CHAIRS
Svetlana Jitomirskaya (UC Irvine)
S. Jitomirskaya's lectures will be a part of the workshop on Spectral Theory of Quasi-Periodic and Random Operators (November 12–16).
Robert Seiringer (IST Austria)
R. Seiringer's lectures will be a part of the workshop on Many-Body Quantum Mechanics (September 10–14).
WORKSHOPS
Many-Body Quantum Mechanics
September 10-14, 2018
Organizers: Rupert Frank (Caltech & LMU München), Mathieu Lewin (Paris-Dauphine), Benjamin Schlein (Zürich)
Entanglement, Integrability and Topology in Many-Body Systems
September 17-21, 2018
Organizers: Paul Fendley (Oxford), Israel Klich (Virginia)
CRM-PCTS Workshop on Critical Phenomena in Statistical Mechanics and Quantum Field Theory
October 3-5, 2018
Organizers: Michael Aizenman (Princeton), David Brydges (University of British Columbia), Igor Klebanov (Princeton)
This workshop will be held at the Princeton Center for Theoretical Science (PCTS, Jadwin Hall, Princeton University).
Quantum Information and Quantum Statistical Mechanics
October 15-19, 2018
Organizers: Fernando Brandão (Caltech), Bruno Nachtergaele (UC Davis), Claude-Alain Pillet (Toulon), Michael Wolf (TU München)
School on Mathematics of Non-Equilibrium Statistical Mechanics, on the occasion of the sixtieth birthday of Claude-Alain Pillet
October 24-26, 2018
Organizers: Jean-Marie Barbaroux (Toulon), Horia Cornean (Aalborg), Vojkan Jakšić (McGill), Flora Koukiou (Cergy-Pontoise), Armen Shirikyan (Cergy-Pontoise)
Entropic Fluctuation Relations in Mathematics and Physics
October 29 – November 2, 2018
Organizers: Vojkan Jakšić (McGill), Christian Maes (KU Leuven), Claude-Alain Pillet (Toulon)
Spectral Theory of Quasi-Periodic and Random Operators
November 12-16, 2018
Organizers: Jonathan Breuer (Hebrew University), David Damanik (Rice), Milivoje Lukić (Rice), Simone Warzel (TU München)
crm.math.ca/Quantum2018
Graphic Design: neograf.ca
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UQÀM
Université Laval
uOttawa
AIP Journal of Mathematical Physics
Québec
NSF
McGill
Concordia
Université de Sherbrooke
Princeton University

Universality, Condensed Matter, and Quantum Field Theory

by VIERI MASTROPIETRO (Milano)

1 Universality

Matter is composed of atoms or molecules mutually interacting and obeying the laws of quantum mechanics; the delicate interplay of quantum-mechanical effects and collective phenomena related to the enormous number of particles is the origin of the remarkable properties exhibited at low temperatures by several materials, superconductivity being a classical example.

The structure of matter is extremely complex and depends on an enormous number of parameters, and one could expect that this is reflected by a very complicated dependence of macroscopic observables on such parameters. It is an experimental fact, on the contrary, that certain quantities exhibit independence from the underlying structure, having the same value in large classes of systems with different microscopic composition. This *universal* behavior suggests that certain macroscopic properties are sensitive more to abstract mathematical structures than to the microscopic details, and can therefore be the same as in the idealized models accessible to theoretical analysis. This opens the possibility of exact quantitative predictions for real systems starting from ideal or very approximate models. The situation recalls the emergence of Gaussian law in central limits in probability, where a similar independence from details is found in a much simpler setting; remarkably, universality in condensed matter regards even the precise value of certain physical observables and not simply distribution laws.

A classical example of universality appears in the second-order phase transitions occurring at a certain critical temperature, where several physical properties have a non-analytic behavior driven by a *critical index*. While the critical temperature depends on microscopic details, the exponents are typically universal; for instance [1] the value of the index β at the ferromagnetic transition is $0.119(8)$ in Rb_2CoF_4, $0.123(8)$ in K_2CoF_4, $0.135(3)$ in Ba_2FeF_6. Although such materials are composed of very different molecules, the exponents are essentially identical (up to small computational or experimental errors); in addition, even if the Ising model in 2 dimensions is rather idealized and has only a vague resemblance to the microscopic structure of such materials, it has essentially the same exponent $1/8$. This universal behavior is a collective effect due to the interaction of a large number of particles, and has found an explanation in the renormalization-group approach, developed starting from the deep work of Kadanoff [2] and Wilson [3].

While in the above example universality is rather independent from the classical or quantum nature of particles, in other cases it is a truly quantum phenomenon. This is the case in the quantization of the Hall conductivity (see Fig.1), which is expressed by a universal constant e^2/h times an integer or fractional number; here one has not only perfect independence from all the microscopic parameters but also a quantization phenomenon. The explanation has been found, at least neglecting the effect of the interaction, identifying the Hall conductance with a topological invariant [4],[5]: the first Chern number of a certain bundle associated with the ground state of the quantum Hamiltonian. Quantized and universal conductance is observed in Chern or topological insulators [6].

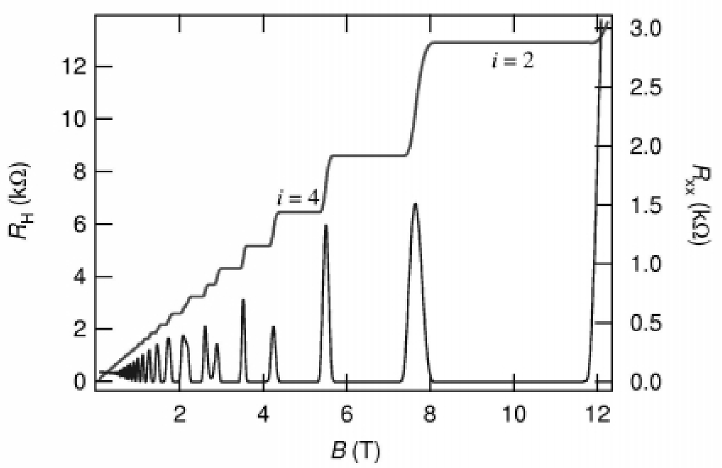

Figure 1: Quantization of the Hall conductivity

A more recent example of universal conductivity has been found in graphene, the newly discovered material [7] composed of a monoatomic crystal of carbon atoms. An experiment [8], see Fig. 2, shows that the optical longitudinal conductivity is equal to $\sigma_0 = \frac{e^2}{h}\frac{\pi}{2}$, an expression obtained by a very idealized model of noninteracting electrons on the honeycomb lattice. This agreement is again a striking example of universality as the charge carriers in graphene are surely strongly interacting via Coulomb forces, and many-body effects are experimentally seen in several physical properties, like the Fermi velocity, but not in the conductivity. A similar universality phenomenon is present also in the Hall conductivity: experiments do not show any correction due to the interaction, at least if weak enough.

There are also more subtle forms of universality; in certain cases the observables do depend on microscopic details but there exist suitable relations between them which are universal. This is what happens in a class of one-dimensional conductors named Luttinger liquids by Haldane [9]; their conductivity, exponents and susceptibility depend on microscopic details but satisfy universal relations, allowing for instance to predict the exponents once the conductivity and susceptibility are known. One-dimensional fermionic systems are not easily realized in nature, and among their clearer realization are the edge states of 2d topological insulators.

The understanding of universality phenomena like the ones briefly mentioned above (and many others) has stimulated an enormous theoretical activity and the explanations require a combination of physical intuition with surprisingly abstract mathematical developments. In addition, it also turns out that universality has quite close relations to other deep phenomena happening in apparently unrelated fields, like quantum field theory (QFT); even if QFT describes quantum relativistic particles at extreme energies and the microscopic components of matter obey nonrelativistic quantum mechanics, several universal phenomena have a common mathematical origin. Despite enormous progress, we are still at the beginning, and many basic questions and observations need an explanation.

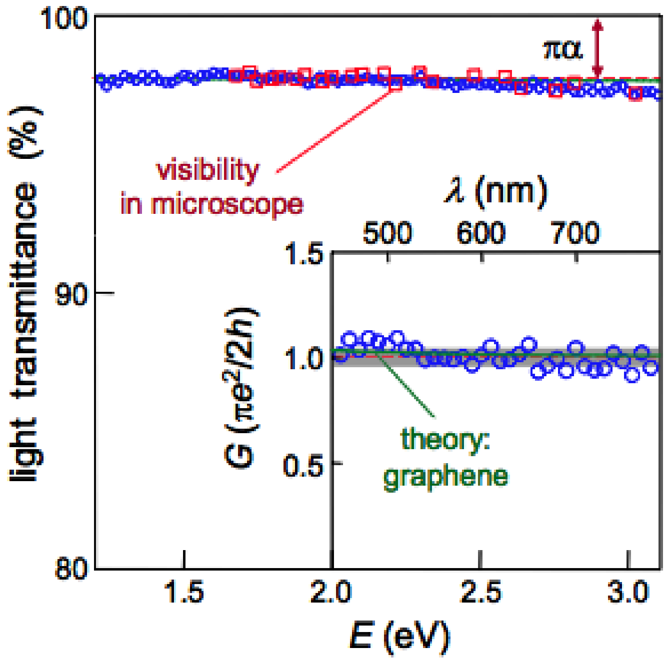

Figure 2: The optical conductivity in graphene.
(Reprinted from https://arxiv.org/ftp/arxiv/papers/0803/0803.3718.pdf)

2 Honeycombs

A major role in our understanding is played by simple but extremely deep models, accessible to exact or rigorous analysis, the classical example being the Ising model. More recently, a system that had a major role in many-body theory is given by a tight-binding model of fermions hopping in the honeycomb lattice; see Fig.3. The model was proposed in [10]; if $\Lambda = \Lambda_A \bigcup \Lambda_B$ is the honeycomb lattice, $x+\delta_i$ the nearest-neighbor sites to x, and $a^\pm$, $b^\pm$ are fermionic creation or annihilation operators, the Hamiltonian in second quantization is, at half filling,

$$H_0 = -t \sum_{\vec{x}\in\Lambda_A, i=1,2,3} \sum_{\sigma=\uparrow\downarrow} \left(a^+_{\vec{x},\sigma} b^-_{\vec{x}+\vec{\delta}_i,\sigma} + b^+_{\vec{x}+\vec{\delta}_i,\sigma} a^-_{\vec{x},\sigma} \right).$$

This system was studied at the beginning essentially for its theoretical interest, as at low energies it admits an effective description in terms of massless Dirac fermions in $2+1$ dimensions, and this suggests the possibility of observing in condensed matter the analogue of QFT phenomena. Far from a mere theoretical curiosity, this model later provided a quite accurate description of the charge carriers of graphene; the theoretical computation of the optical conductivity in Fig.2 is obtained using this simple model. Note that in the above model the only microscopic parameter is the hopping t, but the conductivity is independent from it. It is of course a simplification of a more realistic system describing quantum particles in the continuum subject to a periodic potential with honeycomb symmetry [11].

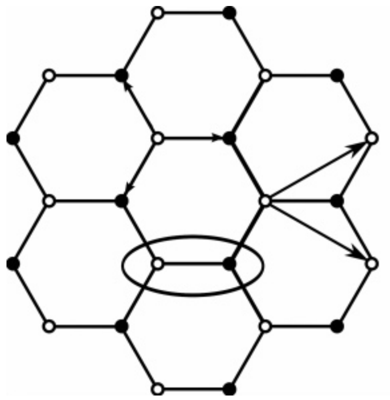

Figure 3: The honeycomb lattice Λ

Subsequently, Haldane [12] provided another key step adding an extra interaction with next-to-nearest-neighbor sites, that is $H_0 + H_1$, where

$$H_1 = -t_2 \sum_{\alpha=\pm} \left(e^{i\alpha\phi} a^+_{\vec{x},\sigma} a^-_{\vec{x}+\alpha\vec{\ell}_j,\sigma} + e^{-i\alpha\phi} b^+_{\vec{x}+\vec{\delta}_1,\sigma} b^-_{\vec{x}+\vec{\delta}_1+\alpha\vec{\ell}_j,\sigma} \right) + \frac{M}{3} \left(a^+_{\vec{x},\sigma} a^-_{\vec{x},\sigma} - b^+_{\vec{x}+\vec{\delta}_j,\sigma} b^-_{\vec{x}+\vec{\delta}_j,\sigma} \right) .$$

This model is a prototype of Chern insulators. The term H_1 induces a gap in the spectrum so that if the chemical potential is in the gap the system is insulating, that is, the longitudinal conductivity is vanishing. However, the system has a transversal Hall conductivity which is universal (that is, independent from t, M, ϕ) and perfectly quantized, which can be expressed in terms of a Chern number; that is, it exhibits a nontrivial Hall effect without a net external magnetic field. Fig.4 shows the region in parameter space where the transversal equals the value $\pm 2e^2/h$; at the origin one recovers instead the value of the optical conductivity of graphene, $\frac{e^2}{h}\frac{\pi}{2}$. Universality also persists adding a stochastic term describing disorder, as a consequence of the topological interpretation [13],[14]. The Haldane model has been experimentally realized in cold-atom experiments [15]. In the gapped region the longitudinal conductivity is vanishing,

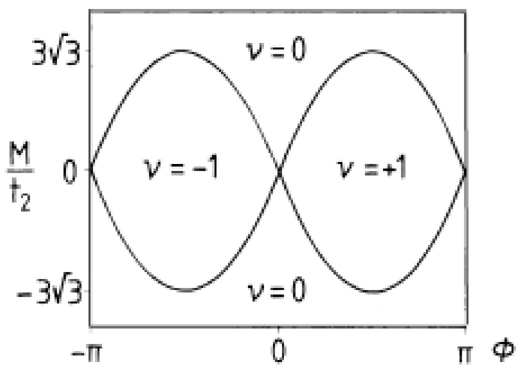

Figure 4: Phase diagram of the Haldane model

but the system can still support nonvanishing currents at the surface [9]; remarkably the edge conductance is equal to the transversal conductivity, and therefore quantized and universal.

This is a manifestation of a rather general fact known as bulk-edge correspondence, which in this model can be explicitly verified. Note also that the electrons circulating at the surface provide a physical realization of a one-dimensional electronic system.

Finally Kane and Mele [16] considered the sum of two Haldane models describing particles with opposite spin, with phase ϕ and $-\phi$ respectively; this model is the prototype of topological insulator and has a quantized universal spin Hall conductivity without breaking time reversal.

Therefore, the simple model of electrons on the honeycomb lattice is a prototype for several universality phenomena in condensed matter, ranging from graphene to topological or Chern insulators, and, via the bulk-edge correspondence, to one-dimensional electron systems. The absence of interaction makes the model exactly solvable, and its properties fully determined by the single-particle Schrödinger equation.

Experiments, however, refer to real materials in which interactions cannot be neglected and are often quite strong, and one needs to investigate the interplay of interactions and universality in the interacting versions of models of fermions on the honeycomb lattice. The role of interaction in the above universality phenomena is a major question. In the case of weak interactions, some progress has been recently achieved, which will be briefly reviewed in the rest of this note.

3 Condensed Matter and QFT

An interacting version of systems of electrons on the honeycomb lattice is obtained for instance by adding to the Hamiltonian H_0 or $H_0 + H_1$ a term of the form

$$V = U \sum_{\vec{x}} \left(n_{\vec{x},\uparrow} - \frac{1}{2} \right) \left(n_{\vec{x},\downarrow} - \frac{1}{2} \right) ,$$

where $n_{\vec{x},\sigma}$ is the fermionic density, obtaining respectively the graphene-Hubbard or Haldane-Hubbard model. This term describes a density-density interaction with coupling U, like a screened Coulomb potential or more generically some effective force mediated by the lattice. It is an on-site ultralocal interaction but one could consider also nearest-neighbor or short-range interactions, and no qualitative differences are expected in the weak coupling regime; in contrast long-range interactions, like Coulomb, are expected to produce radically different behavior.

A major difficulty is that, in the presence of interaction, the physical properties cannot be explicitly computed, not being derivable from the single-particle ones. A way in which progress can be made is by the methods of an apparently distant subject, namely constructive QFT. The reason is that thermodynamical averages at zero temperature, like the conductivity, can be expressed by Euclidean functional integrals of the form

$$\int P(d\psi) e^{V+B}$$

where $P(d\psi)$ is a Gaussian Grassmann integration, V is an interaction quartic in the fields, and B is a source containing external fields. Such objects (and their bosonic counterparts) are exactly the ones used to represent QFT models. In the case of fermions on the honeycomb

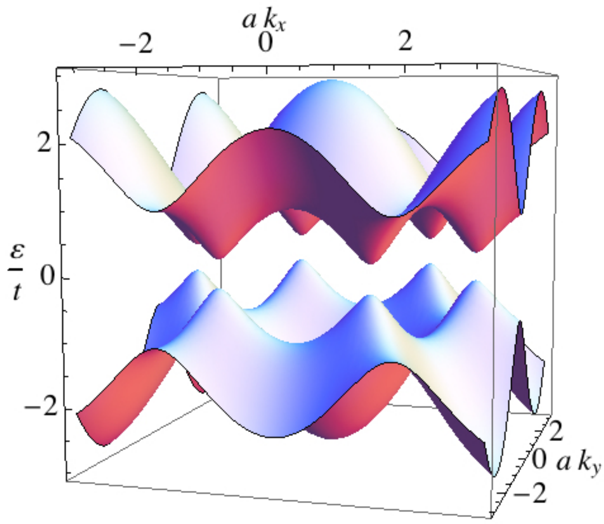

Figure 5: Dispersion relation of the Haldane model

lattice the similarity with QFT is even more striking. Fig. 5 represents the energy dispersion relation of the Haldane model, and one can notice that it resembles closely the relativistic energy of a massive relativistic particle, that is, $\pm\sqrt{v_F^2|\vec{k}|^2 + M^2}$, the Fermi velocity $v_F = \frac{3}{2}t$ playing the role of the light velocity. In the case of graphene the gap closes and the dispersion relation is similar to that of massless Dirac fermions $\pm v_F|\vec{k}|$. As a consequence of this fact, the functional integrals for the interacting graphene or Haldane model resemble regularizations of systems of Dirac particles with a current-current interaction. It is also somewhat natural to relate universality in condensed matter to the physics of quantum anomalies in QFT, which also display universality features [19], see, e.g., [20], [21].

In the case of models with exponentially decaying covariance, like the Haldane model, convergence of the series expansion for the corresponding functional integrals is valid uniformly in the thermodynamic limit; this can be achieved for instance using a convenient representation in terms of sum of determinants and Gram bounds, see e.g. [22], [23]. In the case of graphene, instead, the covariance has power-law decay, and one needs a multscale analysis based on renormalization-group ideas. One uses the basic property that the sum of Grassmann Gaussian variables is still Gaussian; this allows the integration to be performed in several steps, integrating fields of lower and lower energy scales. When the fields with higher energy scales are integrated out, one gets an expression similar to the initial one, with the difference that the covariance is restricted to lower energies and the interaction V is replaced by a different expression called the effective potential [28] (see also [29]). In this case, the coupling of the effective potential is no longer what we called U, but is smaller and smaller at each iteration. In other words, at low momenta the theory is closer and closer to a noninteracting one.

More complicated is the case of the edge states of Hall insulators, which is a model of interacting fermions in one spatial dimension. A typical system of this kind is the Gross-Neveu model with $N = 1$, or the massless Thirring model. The $N > 1$ Gross-Neveu is asymptotically free, that is, the strength of the interaction decreases at each scale [24]; in contrast, the $N = 1$

case, constructed in [25], [26], [27] is such that the effective coupling remains close to the initial value at each scale as consequence of subtle cancellations in the renormalized expansion, and the same happens in the edge states of topological insulators.

The above considerations show that it is quite natural to face the universality problems in statistical physics using methods inspired by constructive QFT. The first applications of such methods were the proof of the universality of next-to-nearest neighbor 2D Ising model [30] (see also [31]) and of the universal scaling relations in models like the 8 vertex or the Ashkin-Teller model, see [32], [33], [34].

4 Ward Identities and Graphene

Fig. 2 represents experimental data showing that the optical conductivity of graphene at zero temperature is universal and equal to its noninteracting value, that is, insensitive to the presence of many-body interactions. Why is it so? A renormalization-group analysis shows that the system is closer and closer to a noninteracting theory integrating the high energy modes. This fact is however not sufficient by itself to explain the phenomenon, as interactions can and actually do modify in general the value of physical observables, as is experimentally seen for the Fermi velocity.

The answer relies on a subtle interplay between regularity properties of the Fourier transform of the Euclidean current correlations and lattice symmetries, and is the content of a theorem proved in [35], [36], which will be briefly reviewed below.

Theorem *For U small enough the graphene zero temperature longitudinal conductivity computed by the Kubo formula with Hamiltonian $H_0 + V$ is*

$$\sigma_{lm} = \frac{e^2}{h}\frac{\pi}{2}\delta_{lm}.$$

The starting point is the Euclidean representation of Kubo formula. If $\hat{K}_{lm}(p_0, \vec{p})$ is the Fourier transforn of the Euclidean current-current correlation $< J_i(\mathbf{x})J_j(\mathbf{y}) >_\beta$, where $\mathbf{x} = x_0, \vec{x}$, $\langle O\rangle_\beta = \frac{\mathrm{Tr} e^{-\beta H} O}{\mathrm{Tr} e^{-\beta H}}$ and J_i is the current in the i direction, $i = 1, 2$, then

$$\sigma_{lm} = -\frac{2}{3\sqrt{3}} \lim_{p_0\to 0^+} \frac{1}{p_0}\Big[\hat{K}_{lm}(p_0, \vec{0}) - \hat{K}_{lm}(0, \vec{0})\Big]$$

In a system with an extended Fermi surface the above quantity is infinite and the same happens in one dimension. The fact that the longitudinal conductivity in graphene is finite follows from dimensional considerations relying on the fact that the Fermi surface is pointlike and the spatial dimensions is 2; this implies that $\hat{K}$ is continuous in the momenta.

The correlations in coordinate space can be written as power series in U; convergence follows by multiscale analysis combined with determinant bounds. By such an expansion one obtains that the correlations decay for large distances like a power-law with power 4. From this bound one could not even conclude the finiteness of the conductivity, connected to the derivative of the 3-dimensional Fourier transform. The fact that the integral expressing the

Fourier transform is not absolutely convergent is not accidental; indeed the density correlations are even in the coordinates, therefore if its derivative were continuous, then it would have to vanish; no finite conductivity would be present. It is however possible to perform a suitable resummation of the convergent expansion obtaining the following expression:

$$\hat{K}_{lm}(\mathbf{p}) = \frac{Z_l Z_m}{Z^2} \langle \hat{\jmath}_{\mathbf{p},l}; \hat{\jmath}_{-\mathbf{p},m} \rangle_{0,v_F} + \hat{R}_{lm}(\mathbf{p}),$$

where $\langle \cdot \rangle_{0,v_F}$ is the average associated to a noninteracting system with Fermi velocity $v_F(U)$ and Z_l, Z are nontrivial analytic functions of U; moreover $\hat{R}_{lm}(\mathbf{p})$ is continuously differentiable while the first term is not. In the noninteracting case $U = 0$, then $R = 0$, $Z = 1$, $Z_l = v_F(0)$; the conductivity appears to depend on $v_F(0)$, but an explicit computation shows that it is equal to $1/4$ (in units $\hbar = e = 1$).

The parameters Z_l, Z, v are expressed by nontrivial series in U ; for instance the Fermi velocity is given by $v_F = 3/2t + aU + O(U^2)$ with $a > 0$, that is, $v_F(U) > v_F(0)$, in agreement with experiments. Universality is present only if intricate cancellations occur between them, impossible to check directly by the series expansion. Note in passing that such a difficulty is rather peculiar; in a QFT model the velocity would not be affected by the interaction, while here it is, and despite this fact universality of conductivity must persist.

Universality in the end follows from the following two facts. First of all, the contribution from R to the conductivity vanishes exactly; it is differentiable and even. The second crucial point is the validity of the Ward identities, that is, nontrivial relations between correlations following from the conservation of the current; in particular,

$$\sum_{\mu=0}^{2} (i)^{\delta_{\mu,0}} p_\mu \hat{G}_\mu(\mathbf{k}, \mathbf{p}) = \hat{S}_2(\mathbf{k} + \mathbf{p}) - \hat{S}_2(\mathbf{k}),$$

where $\hat{S}_2$ is the Fourier transform of the average of two Fermi fields, and $\hat{G}_\mu$ the average of two Fermi fields and the density $\mu = 0$ or currents $\mu = 1, 2$. By inserting the explicit form for such averages one gets the following relations:

$$Z_0 = Z \ , \qquad Z_1 = Z_2 = v_F Z \ .$$

Therefore the first term reduces to $v_F^2 \langle \hat{\jmath}_{\mathbf{p},l}; \hat{\jmath}_{-\mathbf{p},m} \rangle_{0,v_F}$, that is, the same expression as in the noninteracting case but with a different velocity. But any such expression is independent of v_F, and as we said there is no contribution from R; hence universality follows.

Note that the proof makes transparent why in experiments only the conductivity is universal while other observables are not. The theorem above is one of the few rigorous results in the theory of graphene, and it helped to settle some debates in the physical literature [17].

5 Interacting Hall conductivity

Let us consider now the effect of the interaction on the Hall conductivity σ_{12} in the Haldane-Hubbard model. In the noninteracting case we have seen that there are three different phases in the parameter space, with Hall conductivity which is vanishing or perfectly quantized having value $\pm 1/\pi$. In the presence of an interaction, the spectrum of the many-body problem cannot be exactly computed. Despite this fact, a rather complete universality picture can be derived, as explained by the following theorem [37].

Theorem *In the Haldane-Hubbard model $H = H_0 + H_1 + V$, for small U there exist functions $m^R_\omega = W + \omega 3\sqrt{3}t_2 \sin\phi + \delta_\omega$, with $\delta_\omega \neq 0$, analytic in U and continuously differentiable in W, ϕ such that*

$$\sigma_{12}(U) = \frac{1}{2\pi}\left[\operatorname{sign}(m^R_+) - \operatorname{sign}(m^R_-)\right]$$

On the critical line, the longitudinal conductivity is $\sigma_{ii} = 1/8$.

A many-body interaction destroys the single-particle description, and it could produce new phases, mainly close to the critical lines where the gap vanishes. However, the above theorem excludes this scenario; the Hall conductivity remains perfectly quantized and universal, the only effect of the interaction being the modification of the critical lines. The result is in agreement with [18], where it has been shown, using a topological approach, that if the interaction does not close the spectral gap above the ground state, then the Hall coefficient remains the same as the one of the noninteracting case. As can be seen from Fig. 5, the region where the conductivity is nonvanishing is enlarged by a repulsive interaction.

The proof is again based on the combination on constructive QFT methods and Ward identities. A power-series expansion cannot prove a statement like the one above; as the critical lines move, necessarily the estimate of the convergence radius tends to vanish going close to the noninteracting critical lines. Therefore, a power series approach can lead at best to results far from the lines, well inside the topological regions. In order to get results valid in all the parameter space one writes in the Hamiltonian the masses $m_\pm = m^R_\pm + \delta_\pm$, where $\delta_\pm$ are chosen as functions of $m^R_\pm$ and U so that the results are uniform in m^R; inverting the above relation one gets the renormalized critical lines. This idea is borrowed from the theory of critical phenomena; in Ising models the critical temperature is shifted by the interaction, and one can study the critical behaviour by suitably choosing a counterterm.

The proof of universality starts from the identity

$$\widehat{K}^{\mathrm{R}}_{i,j}(\mathbf{p}) = \widehat{K}^{\mathrm{R},0}_{i,j}(\mathbf{p}) + \int_0^U dU' \frac{d}{dU'}\widehat{K}^{\mathrm{R},U'}_{i,j}(\mathbf{p}),$$

where the last term is proportional to the average $\widehat{K}^{\mathrm{R}}_{i,j,V}$ of two currents and an interaction. Some manipulation of the Ward identities, valid if correlations are at least three time differentiable, implies that

$$\frac{\partial}{\partial p_0}\widehat{K}^{\mathrm{R}}_{i,j,V}((p_0,\vec{0}),(-p_0,\vec{0})) = \frac{\partial}{\partial p_0}\left[p_0^2 \frac{\partial^2}{\partial p_i \partial q_j}\widehat{K}^{\mathrm{R}}_{0,0,V}((p_0,\vec{0}),(-p_0,\vec{0}))\right].$$

The right side vanishes as $p_0 \to 0$ and this implies that the contribution of such terms to the conductivity vanishes. This implies that all the interaction corrections to the Hall conductivity are vanishing. The regularity property of $\widehat{K}^{\mathrm{R}}_{i,j,V}$ in momentum space, on which the cancellation

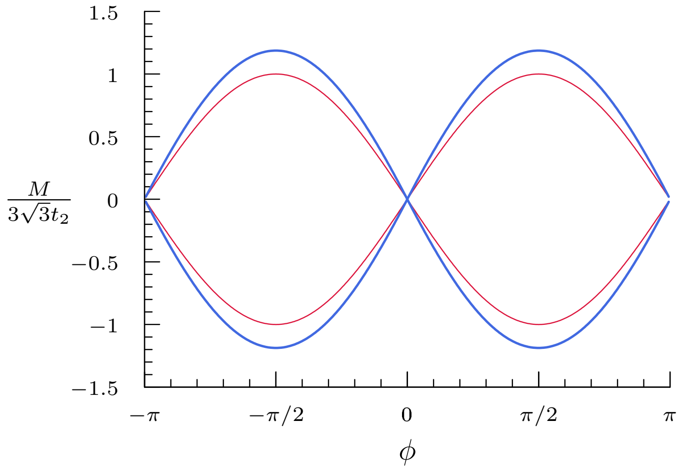

Figure 6: Red: $U = 0$. Blue: $U > 0$.

is based, is due to the exponential decay of the correlations in coordinate space, due to the presence of a gap. The above use of Ward identities resembles what is done in QED, see e.g. [20], but here a non-perturbative result is proved.

The proof relates universality of the Hall conductivity to good regularity properties of the correlations in momentum space, due to the presence of a gap in the (renormalized) noninteracting Hamiltonian. This property is of course not necessary for the universality of transport coefficients, and can be present also in gapless systems. We already have seen an example in the longitudinal conductivity in graphene, in which the Euclidean correlations decay with a power law.

In gapless one-dimensional systems the interaction has a much more dramatic effect than in Chern insulators or in graphene, qualitatively modifying several physical properties. The conductivity is infinite, the current correlations appearing in the Kubo formula being discontinuous as functions of the frequency. (In graphene they are continuous and nondifferentiable). On the other hand, the edge states of topological insulators are one dimensional, and if the bulk-edge correspondence is valid also in the presence of interactions, the edge conductance must be universal even in the presence of interactions. This actually happens to be true but the mechanism explaining it is much more subtle and is generated by a complicated interplay of lattice Ward identities and properties of the anomalies of the emerging QFT description, somewhat similar to what happens in Luttinger liquids [34] ,[38].

6 Topological insulators and chiral anomalies

The Kane-Mele model, a paradigmatic system for topological insulators, is obtained by summing two Haldane models describing particles with spin $\sigma = \pm$ and with parameters ϕ and $-\phi$. In the noninteracting case the Hall conductivity is vanishing while the spin Hall conductivity is quantized and equal to $\sigma^s_{12} = \pm\frac{1}{\pi}$. With periodic boundary conditions in the 1 direction and Dirichlet boundary conditions in the 2 direction there are eigenfunctions exponentially localized at the boundary. Such edge states can carry a current, whose conductance is the same as the Hall conductivity, a phenomenon known as bulk-edge correspondence.

The edge transport properties can be obtained by suitable limits of $G_{h_1,h_2}(\underline{p}) = \sum_{y_2=0}^{a}\sum_{x_2=0}^{\infty}\langle\hat{h}_{1,\underline{p},x_2};\hat{h}_{2,-\underline{p},y_2}\rangle$, with $\underline{p} = p_0, p_1$ and $\langle ...\rangle$ the limit $\beta \to \infty$ of the thermodynamical average; such an expression measures the response of an observable $\sum_{y_2=0}^{a}\hat{h}_{2,-\underline{p},y_2}$ localized in a strip of width a around the boundary $y_2 = 0$ to an external potential proportional to $\sum_{x_2}\hat{h}_{1,\underline{p},x_2}$. Introducing the charge or spin density ρ^i, $i = c, s$ and the charge or spin currents j_1^i, the edge spin conductance is given by $\sigma^s = \lim_{p_0\to 0^+}\lim_{p_1\to 0} G_{\rho^c,j_1^s}(\underline{p})$. Other interesting physical quantities are the susceptibility $\kappa^i = \lim_{p_1\to 0}\lim_{p_0\to 0^+} G_{\rho^i,\rho^i}$ and the Drude weight $D_i = \lim_{p_0\to 0^+}\lim_{p_1\to 0} G_{j_1^i,j_1^i}$.

The interaction modifies dramatically the correlation decay of the edge correlations, with the presence of anomalous exponents η which are nontrivial functions of the interaction strength. Despite this strong effect of the interaction, the spin edge conductance is still perfectly universal and quantized; the bulk-edge correspondence holds true also in the presence of interactions, as stated by following theorem, proven in [39]

Theorem *For U small, the edge spin conductance in the Kane-Mele model plus an interaction V is*

$$\sigma^s = \pm\frac{1}{\pi}.$$

Moreover, the Drude weights and the susceptibilities satisfy the relations:

$$\kappa^c = \frac{K}{\pi v}\ , \quad D^c = \frac{vK}{\pi}, \kappa^s = \frac{1}{K\pi v}\ , \quad D^s = \frac{v}{K\pi},$$

with $K = 1 + O(U) \neq 1$ and $v = v_F + O(U)$. Finally, the 2-point function decays with anomalous exponent η which is related to K defined above by $\eta = (K + K^{-1} - 2)/2$.

The spin edge conductance is still perfectly universal and quantized, but the other thermodynamical quantities are nontrivial functions of the coupling. They verify however universality relations known as Haldane relations, saying that $\kappa^i/D^i v^2 = 1$ and $\eta = (K + K^{-1} - 2)/2$. Therefore the interacting Kane-Male model exhibits two kinds of universality; the spin conductance or bulk conductivity are exactly independent from the interaction while the Drude weight, the susceptibility, and the exponents are interaction-dependent but verify nontrivial universal relations; similar relations are true in vertex models or spin chains [34] or in $d = 1$ Hubbard models with repulsive interactions [38].

The proof is based on a multiscale analysis leading to a representation of the correlations as sums of two terms, one of a QFT model and a more regular part, similar to the one already discussed for graphene but with two crucial differences; first the reference QFT of the dominant part lives in a space with $1+1$ instead $2+1$ dimensions (a consequence of the fact that we are studying the edge states), and second the QFT is an interacting theory and not a free one. The decomposition has the following form, if $i = c, s$, $\mu = 0$ is the density and $\mu = 1$ is the current

$$\langle h^i_{\mu,\underline{p},x_2} h^{i'}_{\nu,-\underline{p},y_2}\rangle = Z_\mu(x_2)Z_\nu(y_2)\langle J^i_{\mu,\underline{p}} J^{i'}_{\nu,-\underline{p}}\rangle_H + \hat{R}^{i,i'}_{\mu,\nu}(\underline{p}, x_2, y_2),$$

where $\langle\cdot\rangle_H$ denotes the expectations of a QFT model of interacting chiral fermions in $d = 1+1$, known as the helical model, with current-current interaction, velocity v and coupling λ_H, and

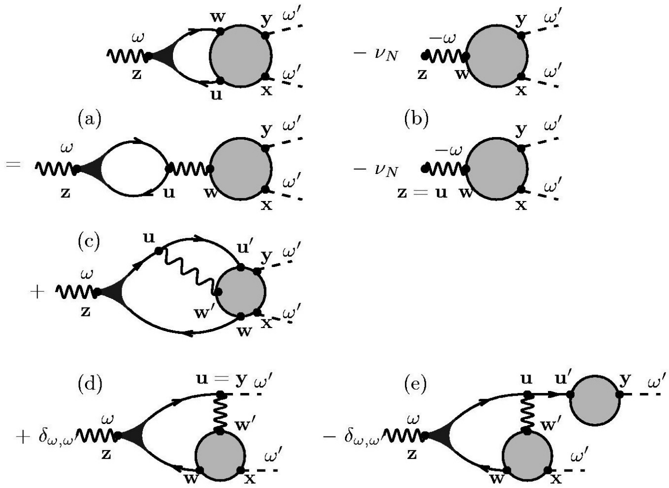

Figure 7: Decomposition of the correction terms to the Ward identities.

J_μ^i are its density $\mu = 0$ and current $\mu = 1$. Moreover $\hat{R}_{\mu,\nu}$ is continuous in $\underline{p}$ while the first term is not continuous, and $Z_\mu(x_2)$ is exponentially decaying. The functions $\lambda_H, Z_\mu, \hat{R}$ are expressed by complicated series in U, depending on all microscopic details.

The reason why the above equation, representing the correlations of the interacting Kane-Mele model as the correlations of the helical model plus corrections, is useful is that the helical model satisfies extra symmetries with respect to the Kane-Mele model; in addition to a global phase symmetry a chiral phase symmetry is valid. There is therefore an extra set of Ward identities for the helical model, allowing closed expressions to be obtained, from which exact expression for the correlations can be derived. Ward identities in lattice models are simple consequences of the conservation of the currents. Things are more subtle in a QFT like the helical model; Ward identities may not coincide with the ones one would naively guess from conservation laws, given the possible presence of extra terms known as anomalies. In particular, if $J_0^c = J_1^s = \rho_+ + \rho_-$ and $J_1^c = J_0^s = \rho_+ - \rho_-$, $\rho_\pm$ being the fermionic densities, one gets, if $\varepsilon_c = +$, $\varepsilon_s = -$,

$$-ip_0 \langle \hat{J}^i_{0,\underline{p}} ; \hat{\psi}^-_{\underline{k}+\underline{p},\sigma} \hat{\psi}^+_{\underline{k},\sigma} \rangle_H + p_1 v \langle J^i_{1,\underline{p}} ; \hat{\psi}^-_{\underline{k}+\underline{p},\sigma} \hat{\psi}^+_{\underline{k},\sigma} \rangle_H = \frac{\sigma}{Z(1-\varepsilon_i \tau)} [G_2(\underline{k}) - G_2(\underline{k}+\underline{p})],$$

where $G_2(k)$ is the average of two Fermi fields, $D_\sigma(\underline{p}) = -ip_0 + \sigma v p$ and $\tau = \frac{\lambda_H}{4\pi v}$; similar Ward identities hold for $< \hat{\rho}_{\underline{p},\sigma} ; \hat{\rho}_{-\underline{p},\sigma} >_H$. In the above identity τ is the anomaly, and its origin can be traced to the presence of the regularization necessary to define the functional integrals defining a QFT like the helical model. Remarkably, the anomaly τ is linear in the coupling λ_H; all possible interaction corrections at higher orders vanish. This fact is analogous to a well-known property of QED in $3+1$ dimensions known as the Adler-Bardeen nonrenormalization theorem [19]; for $d = 1+1$ models it has been proved at a nonperturbative level in [40]. Some ideas of the proof can be grasped from Fig.7: the corrections to the Ward identity due to cut-offs can be written as truncated expectations written as sum of terms as in the r.h.s.; the contributions (c),(d),(e) are vanishing in the limit of removed cut-off and only (a) survive.

The vertex functions of the interacting Kane-Mele model are proportional to those of the helical model, $\langle \hat{J}^i_{\mu,\underline{p}}\,; \hat{\psi}^-_{\underline{k}+\underline{p},\sigma}\hat{\psi}^+_{\underline{k},\sigma}\rangle_H$ times a multiplicative factor Z^i_μ; by comparing the Ward identities for the helical model with those of the Kane-Mele vertex functions, one gets the following relations, if $Z^i_\mu = \sum_{x_2} Z^i_\mu(x_2)$:

$$\frac{vZ^i_0}{Z^i_1} = 1 \qquad \frac{Z^i_0}{Z(1-\varepsilon^i\tau)} = 1.$$

In addition, the current correlations of the helical model can be exactly computed by Ward identities and one gets, up to terms vanishing as $p \to 0$ or $a \to \infty$,

$$G_{\rho^c,j^s_1}(\underline{p}) = -\frac{Z^c_0 Z^s_1}{Z^2(1-\tau^2)}\frac{1}{\pi v}\frac{p_0^2}{p_0^2+v^2p_1^2} \quad G_{j^i_1,j^i_1}(\underline{p}) = -\frac{Z^i_1 Z^i_1}{Z^2(1-\tau^2)}\frac{1}{\pi v}\frac{p_0^2}{p_0^2+v^2p_1^2}.$$

By noting that

$$\frac{Z^c_0 Z^s_1}{Z^2(1-\tau^2)v} = 1,$$

the universality of the spin conductance follows. In addition $\frac{Z^c_0 Z^\sharp_1}{Z^2(1-\tau^2)v} = K$, with $K = \frac{1+\tau}{1-\tau}$, so that the other relations likewise follow.

7 Conclusions

We have seen examples of properties of matter which are universal, that is, essentially independent from microscopic details and common to a large class of systems with different microscopic structure. A mathematical theory is quite well developed in absence of interaction, where a single-particle description is valid; much less is known in presence of interaction, where collective effects due to the enormous number of particles involved play a crucial role. Interaction is always present in real systems, so one cannot neglect it to explain universality properties. We have reviewed some recent results proving universality in a number of paradigmatic models; the results are based on the rigorous control of functional integrals combined with subtle cancellations coming from Ward identities. Functional integrals appear to be similar to the ones appearing in QFT, and universality follows in this approach via exact conservation laws and properties of the emerging QFT description.

The theoretical understanding of universal phenomena in quantum matter, and its relation with QFT, is surely at its beginning. The main limitations at the moment are the restriction to weak and short-range interactions, while in real materials strong and long-range interactions are often present. The fractional Hall effect or the perhaps simpler problems related to universal conductivity properties in graphene with Coulomb interactions or in Weyl semimetals are major problems that still need a full theoretical understanding.

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CIMPA RESEARCH SCHOOLS 2020

CALL FOR PROJECTS

The organization of Research Schools in developing countries is the historical activity of CIMPA. It aims to foster the development of research in mathematics in deprived areas where there is a real drive and a scope for a research project. Each Research School offers an introduction to recent research in some field of mathematics and is specifically dedicated to the students and research teaching staff from developing countries. The budget allocated by CIMPA to the school is essentially used to cover the participation costs of those participants.

The Call for Projects of CIMPA Research Schools 2020 is open. **The project proposals must be submitted by September 10, 2018.** The selection of the projects will be completed by the end of January 2019.

ELIGIBILITY CRITERIA

The Research School project must be set up jointly by two administrative and scientific coordinators, a "local" one and an "external" one:

- The local coordinator must hold a position at the host institution of the school.
- The external coordinator must hold a position in an institution located in one of the partner countries of CIMPA as of 1 January 2019 (France, Norway, Spain and Switzerland presently, other countries may join CIMPA by 1 January 2019 thus becoming eligible).

The Research School project must necessarily meet the following criteria:

- The Research School must take place within an academic environment (university or equivalent).
- The topic of the Research School must correspond to prior on-site research work and be part of an active research area in mathematics, in pure and applied mathematics as well as related subjects such as computer sciences and theoretical physics.
- The Research School is intended first and foremost for young mathematicians of the country in which it is held and for those of countries nearby, its level must be between pre-doc and post-doc.
- The Research School must last about ten days (between 8 and 15 days) and offer a scientific program including several series of courses at introductory and advanced levels (from 4 to 8 courses) and training sessions. Research talks at upper scientific level can be scheduled but not more than one hour per day. By no means, should the school resemble a conference or a workshop.

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- The provisional budget of the school (resources and expenses) must be clearly described. The financial support provided by CIMPA shall not represent more than 1/3 of the total budget of the Research School. At least 2/3 of the CIMPA financial support needs to be used for travel and/or living expenses of CIMPA participants.
- With regard to gender balance, the scientific and organization committees as well as the list of lecturers must comprise at least 30% women and at least 30% men.

More generally, the project of the Research School must comply with the guidelines described in the document CIMPA_RS_INSTR_en.pdf.

EVALUATION CRITERIA

Research School projects will be evaluated according to the following criteria:

- Scientific and pedagogical quality.
- Scientific relevance of the project with respect to the local context.
- Impact of the project on local and regional development.
- Reliability of the provisional budget.
- Financial commitment of local institutions.
- Level of involvement of women in the project.

FUNDING

The selected project proposals for CIMPA Research School 2019 will receive financial support from CIMPA in the range of €8,000 to €12,000. Since CIMPA's financial support should not represent more than 1/3 of the total budget, the coordinators must seek to secure diversified sources of funding at the local and international level as early as possible (see document CIMPA_RS_FUNSOL_en.pdf).

HOW TO SUBMIT A PROJECT PROPOSAL

The project coordinators must download the document CIMPA_RS_APP_en.odt, fill it in, convert it to a pdf file and and send it to the email address provided, along with their CV's and a supporting letter from the host institution. Project proposals must be submitted **by September 10, 2018.**

DOCUMENTS TO DOWNLOAD ON THE CIMPA WEBSITE

- Proposal Template to complete and return: CIMPA_RS_APP_en.odt
- Call for Projects in pdf: CIMPA_CALL_RS_2020_en.pdf

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- Instructions for completing the Proposal Template: CIMPA_RS_INSTR_en.pdf
- Possible sources of funding: CIMPA_RS_FUNSOL_en.pdf
- Example of project proposal: CIMPA_RS_APP_EX_en.pdf

Écoles de Recherche **2019** Research Schools

Montevideo, Uruguay Feb. 11 > 22
Elliptic Curves: Arithmetic and Computation

Saida, Algeria Mar. 1 > 9
Analyse stochastique et applications

Hanoi, Vietnam Mar. 11 > 22
Hyperplane Arrangements: Recent Advances and Open Problems

Petion-Ville, Haiti Mar. 11 > 23
Cryptography, Theoretical and Computational Aspects of Algebraic Number Theory and Algebra

Isfahan, Iran Apr. 7 > 19
Isfahan School and Conference on Representations of Algebra

Dhaka, Bangladesh Jun. 10 > 21
Dynamical Systems and Applications to Biology

Havana, Cuba Jun. 11 > 22
Mathematical Models in Biology and Related Applications of Partial Differential Equations

Tabriz, Iran Jun. 15 > 22
Graphs, Algorithms and Randomness

Kathmandu, Nepal Jun. 17 > 26
Summer School in Mathematical Biology

Bogota, Colombia Jun. 24 > Jul. 5
Research School on Quantum Symmetries

Tunis, Tunisia Jun. 25 > Jul. 5
Science des données pour l'ingénierie et la technologie

Tehran, Iran Jun. 29 > Jul. 11
Control and Information Theory

Limbe, Cameroon Jul. 2 > 13
Algebraic Geometry, Number Theory and Applications in Cryptography and Robot Kinematics

Kenitra, Morocco Jul. 3 > 13
Modélisation, analyse mathématique et calcul scientifique dans la gestion des déchets ménagers

Ibarra, Ecuador Jul. 15 > 26
The Geometry of Mechanics

Antananarivo, Madagascar Jul. 16 > 25
Approches algorithmiques et statistiques de l'apprentissage

Cordoba, Argentina Jul. 29 > Aug. 9
Hopf Algebras and Tensor Categories

Buenos Aires, Argentina Oct. 28 > Nov. 8
Algebraic and Geometrical Methods in String Theory

Joao Pessoa, Brazil Nov. 4 > 13
Syzygies, from Theory to Applications

Talca, Chile Dec. 2 > 13
Fourth Latin American School on Algebraic Geometry and its Applications

Varanasi, India Dec. 5 > 15
Finsler Geometry and Applications

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Promoting Research in Math in Developing Countries

News from the IAMP Executive Committee

New individual members

IAMP welcomes the following new members

1. DR. JACK DYSON, Marche Polytechnic University, Ancona, Italy
2. DR. GONZALO BLEY, Aarhus University, Denmark
3. PROF. DAVID DAMANIK, Rice University, Houston, USA
4. MR. MAXIMILIAN DUELL, TU Munich, Germany
5. DR. ANGELO LUCIA, University of Copenhagen, Denmark
6. PROF. IAN MORRISON, West Chester University, USA
7. DR. JONATHAN KRESS, University of New South Wales, Sydney, Australia
8. PROF. YVAN SAINT-AUBIN, University of Montreal, Canada
9. PROF. ANTON GORODETSKI, University of California at Irvine, USA
10. PROF. EUNGHYUN LEE, Nazarbayev University, Astana, Kazakhstan
11. PROF. DOROTHEA BAHNS, University of Göttingen, Germany

Recent conference announcements

Random physical systems

Dec. 11-15, 2018. Puerto Natales, Patagonia, Chile.
Organized by C. Sadel, A. Klein, A. Ramirez.

http://patagonia2018.christian-sadel.de/

Open positions

For more information on these positions and for an updated list of academic job announcements in mathematical physics and related fields visit

`http://www.iamp.org/page.php?page=page_positions`

Benjamin Schlein (IAMP Secretary)

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