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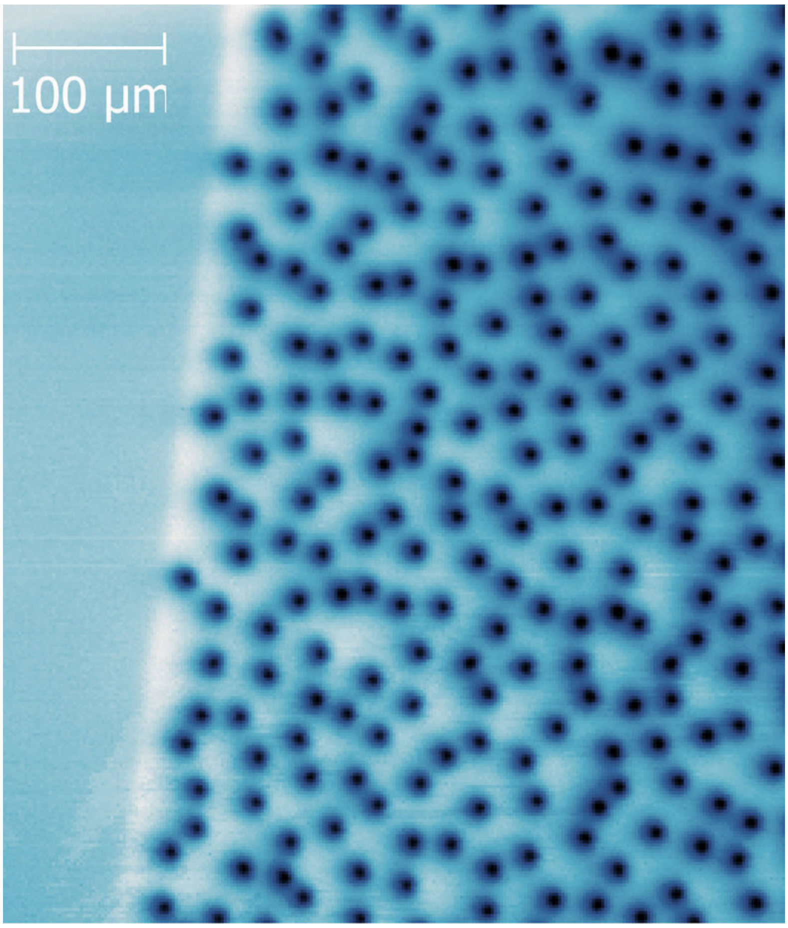

International Association of Mathematical Physics

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Cover picture: Images of vortices in 200 nm thick YBCO film taken by Scanning SQUID Microscopy.This file is licensed under the Creative Commons Attribution 4.0 International license. See `https://commons.wikimedia.org/wiki/File:YBCO_vortices.jpg`

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Where will your next student find a job?

When a student gets a PhD in mathematical physics, various academic careers beckon, whether in a physics department or a mathematics department, whether a postdoc or a teaching position, and these could take a young scientist to any part of the world. Many in our community understand the academic job market and are highly successful in launching our students' academic careers.

We are, on the whole, much less familiar with the job market outside academia and less capable of helping our students set out on careers in business, industry, or government (BIG), but statistically a lot of our students are headed this way. In this issue of the *News Bulletin* we will begin a series of articles on career alternatives, starting with some perspectives by Philippe Tondeur about an effort supported by mathemaics organizations in the US, called the BIG Math Network and a related recent publication called the *BIG Jobs Guide*, as well as a reprinted interview from SIAM with the authors of the *BIG Jobs Guide*. Future issues will include some first-person accounts of mathematical physicists who have found rewarding careers outside academia.

We at the *News Bulletin* would like to hear from our readership if you have insights to offer, especially with perspectives complementary to those in this issue, which come from mathematicians in North America. Please write to us with any suggestions of people who could either provide resources for pursuing careers outside academia or additional first-person accounts of their journeys to careers in government or the private sector. You may send suggestions to the editor at bulletin@iamp.org.

Evans Harrell (Chief Editor)

Systems of points with Coulomb interactions

by SYLVIA SERFATY (New York)

Large ensembles of points with Coulomb interactions arise in various settings of condensed matter physics, classical and quantum mechanics, and even approximation theory, and give rise to a variety of questions pertaining to calculus of variations, partial differential equations, and probability. We will review motivations from these fields as well as "mean-field limit" results that allow us to derive effective models and equations describing those systems at the macroscopic scale. We then explain how to analyze the next-order beyond the mean-field limit, obtaining information about systems at the microscopic level. In the setting of statistical mechanics this allows, for instance, to observe the effect of temperature and to connect with crystallization questions.

1 General setup

The 18th century physicist Charles-Augustin de Coulomb was the first to postulate that electrically charged particles interact with one another by a force proportional to the inverse square of their distances, in a way similar to Newton's gravitational force. In this paper we are interested in large systems of points (or particles) interacting by such forces, having as motivation, besides the case of classical mechanics, numerous other situations that we will detail below.

Recalling that the force is the gradient of the energy, we consider a system of N particles with energy of the form

$$\mathcal{H}_N(x_1,\dots,x_N)=\frac{1}{2}\sum_{1\le i\neq j\le N}\mathsf{g}(x_i-x_j)+N\sum_{i=1}^N V(x_i). \tag{1.1}$$

Here the points x_i belong to the Euclidean space $\mathbb{R}^{\mathsf{d}}$, although it is also interesting to consider points on manifolds. The interaction kernel $\mathsf{g}(x)$ is taken to be

$$\mathsf{g}(x)=-\log|x|, \quad \text{in dimension } \mathsf{d}=2, \tag{1.2}$$

$$\mathsf{g}(x)=\frac{1}{|x|^{\mathsf{d}-2}}, \quad \text{in dimension } \mathsf{d}\ge 3. \tag{1.3}$$

Up to a multiplicative constant, this is the Coulomb kernel in dimension $\mathsf{d}\ge 2$, i.e. the fundamental solution to the Laplace operator, solving

$$-\Delta \mathsf{g}=\mathsf{c_d}\delta_0\ , \tag{1.4}$$

where δ_0 is the Dirac mass at the origin, and $\mathsf{c_d}$ is an explicit constant depending only on the dimension.

It is also interesting to broaden the study to the one-dimensional logarithmic case

$$\mathsf{g}(x) = -\log|x|, \quad \text{in dimension } \mathsf{d} = 1, \tag{1.5}$$

which is not Coulombian, and to more general Riesz interaction kernels of the form

$$\mathsf{g}(x) = \frac{1}{|x|^{\mathsf{s}}} \quad \mathsf{s} > 0. \tag{1.6}$$

The one-dimensional Coulomb interaction with kernel $-|x|$ is also of interest, but has been extensively studied and is well understood.

We also include a possible external field or confining potential V, which is assumed to be sufficiently smooth and tending to infinity fast enough at infinity. The factor N in front of V makes the total confinement energy of the same order as the total repulsion energy, effectively balancing them and confining the system to a subset of $\mathbb{R}^{\mathsf{d}}$ of fixed size.

The Coulomb interaction and the Laplace operator are obviously extremely important and ubiquitous in physics as the fundamental interactions of nature (gravitational and electromagnetic) are Coulombic. Below we will further review the reasons for studying this type of systems.

There are several mathematical problems that are interesting to study, all in the asymptotic limit of $N \to \infty$:

(1) understand the minimizers and possibly critical points of (1.1) ;

(2) understand the statistical mechanics of systems with energy $\mathcal{H}_N$ and inverse temperature $\beta > 0$, governed by the so-called Gibbs measure

$$d\mathbb{P}_{N,\beta}(x_1, \dots, x_N) = \frac{1}{Z_{N,\beta}} e^{-\beta \mathcal{H}_N(x_1,\dots,x_N)} dx_1 \dots dx_N. \tag{1.7}$$

Here $\mathbb{P}_{N,\beta}$ is the probability density of observing the system in the configuration $(x_1, \dots, x_N)$ if the inverse of the temperature is $1/\beta$. The constant $Z_{N,\beta}$, which is called the "partition function" in physics, is the normalization constant that makes $\mathbb{P}_{N,\beta}$ a probability measure[1] i.e.

$$Z_{N,\beta} = \int_{(\mathbb{R}^{\mathsf{d}})^N} e^{-\beta \mathcal{H}_N(x_1,\dots,x_N)} dx_1 \dots dx_N; \tag{1.8}$$

(3) understand the dynamic evolutions associated to (1.1), such as the gradient flow of $\mathcal{H}_N$ given by the system of coupled ODEs

$$\dot{x}_i = -\frac{1}{N} \nabla_i \mathcal{H}_N(x_1, \dots, x_N), \tag{1.9}$$

the conservative dynamics given by the systems of ODEs

$$\dot{x}_i = \frac{1}{N} \mathbb{J} \nabla_i \mathcal{H}_N(x_1, \dots, x_N), \tag{1.10}$$

[1] One does not know how to explicitly compute the integrals (1.8) except in the particular case of (1.5) for specific cases of V where they are called Selberg integrals (cf. [Fo])

where $\mathbb{J}$ is an antisymmetric matrix, or the Hamiltonian dynamics given by Newton's law

$$\ddot{x}_i = -\frac{1}{N}\nabla_i \mathcal{H}_N(x_1, \ldots, x_N); \tag{1.11}$$

and we can also be interested in these dynamics with an added noise.

From a mathematical point of view, the study of such systems touches on several fields of mathematical analysis (partial differential equations and calculus of variations, approximation theory), probability theory, mathematical physics, and even geometry (when one considers such systems on manifolds). Some of the crystallization questions they lead to also overlap with number theory, as we will see below.

2 Motivations

There is a large number of motivations for the study of the above questions. We briefly describe some of them:

1. In superconductors, superfluids, and Bose-Einstein condensates, one observes the occurrence of quantized "vortices" which behave mathematically like interacting particles with two-dimensional Coulomb interactions. In these systems the vortices repel each other logarithmically, while being confined together by the effect of the magnetic field or rotation, and the result of the competition between these two effects is that, as predicted by Abrikosov, the vortices arrange themselves in a perfect triangular lattice pattern, called *Abrikosov lattice*, cf. Figure 2.1 (for more pictures, see www.fys.uio.no/super/vortex).

   These systems are, in fact, described by an energy (the Ginzburg-Landau energy) and the associated PDEs, but we can show rigorously (in a study started by Bethuel-Brezis-Hélein and continued by [SS], see also [Se1]) that, in the case (1.2), the analysis of the vortices is reduced to the discrete problems described above.

   Another motivation is the analysis of vortices in classical fluids, such as initiated by Onsager, see [MP], or in plasma physics.

2. Fekete points in approximation theory: these points arise in interpolation theory as the points minimizing interpolation errors for numerical integration. They are defined as those points maximizing the quantity

   $$\prod_{i \neq j} |x_i - x_j|$$

   or, equivalently, minimizing

   $$-\sum_{i \neq j} \log |x_i - x_j|.$$

   They are often studied on the sphere, or on other manifolds. In approximation theory [SK] we are also interested in the minimization of Riesz energies

   $$\sum_{i \neq j} \frac{1}{|x_i - x_j|^{\mathsf{s}}} \tag{2.1}$$

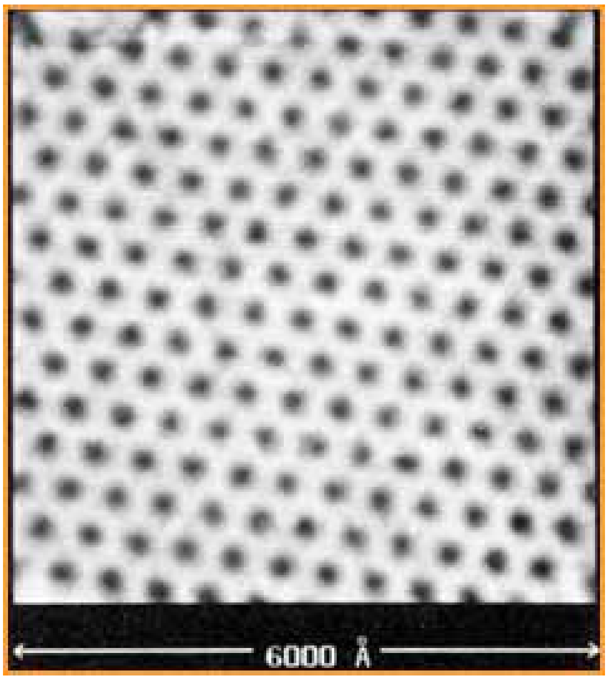

Figure 2.1: Vortices (in black) forming an Abrikosov lattice. H. F. Hess et al. Bell Labs *Phys. Rev. Lett.* 62, 214 (1989).

for all values of s. One can show that, by letting $s \to 0$, the minimizers of Riesz energies converge to those of the logarithmic energy, whereas when $s \to \infty$ they converge to the minimizers of the optimal sphere-packing problem (whose solution in dimension 2 is known, from Fejes Tóth, to be also the triangular lattice represented in Figure 2.2). It has been proved by Hales that the solution of the same packing problem in dimension 3 is an FCC (face-centered cubic) lattice, as was conjectured by Kepler. In higher dimensions, the solution is only known in dimensions 8 and 24, due to a recent breakthrough by Viazovska (see the presentation in [Coh] and the review [Sl]). In high dimensions, where the problem is important for error correcting codes, the solution is expected *not* to be a lattice.

3. Statistical mechanics and quantum mechanics: In physics the ensemble given by (1.7) in the Coulomb case is called a two-dimensional Coulomb gas or one-component plasma and is a classical ensemble of statistical mechanics whose analysis is considered difficult due to the long range of the interactions. The study of the two-dimensional Coulomb gas, as well as the one-dimensional log gas, is also motivated by the analysis of certain quantum wave-functions (fractional quantum Hall effect, free fermions in a magnetic field, . . .), and well as by several stochastic models in probability, cf. [Fo]. The variant of the two-dimensional Coulomb case with coexisting positive and negative charges is interesting in certain theoretical physics models (XY-model, sine-Gordon) which exhibit a Kosterlitz-Thouless phase transition (see [Spe]).

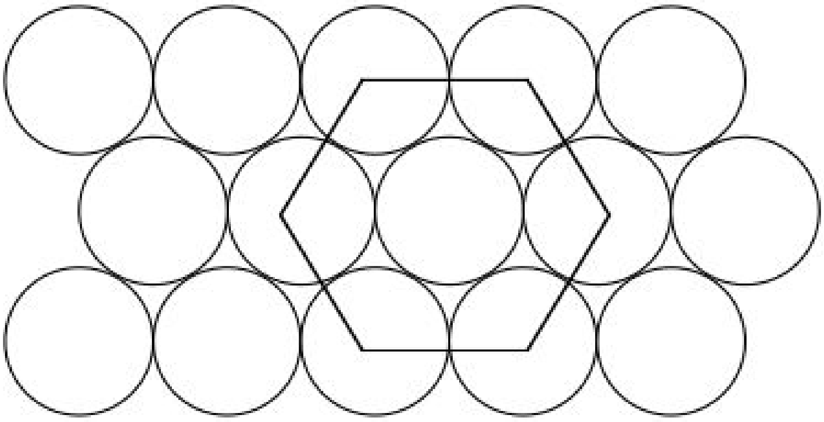

Figure 2.2: Solution of the sphere-packing problem in dimension 2.

4. Random matrices (see [Fo]): in the particular cases (1.5) and (1.2), the Gibbs measure (1.7) corresponds, in certain instances, to the distribution law of the eigenvalues of certain well known ensembles:
   - the law of the complex eigenvalues of an $N \times N$ matrix where the entries are Gaussian i.i.d. is (1.7) with (1.2), $\beta = 2$, and $V(x) = |x|^2$. This is called the Ginibre ensemble.
   - the law of the real eigenvalues of an $N \times N$ Hermitean matrix with complex Gaussian i.i.d. entries is (1.7) with (1.5), $\beta = 2$ and $V(x) = x^2/2$. This is called the GUE (unitary Gaussian) ensemble.
   - the law of the real eigenvalues of an $N \times N$ symmetric matrix with Gaussian i.i.d. entries is (1.7) with (1.5), $\beta = 1$ and $V(x) = x^2/2$. This is called the GOE (orthogonal Gaussian) ensemble.
5. Complex geometry provides other examples of motivations. See, for instance, the works of Robert Berman and co-authors.

3 The mean-field limit and theoretical physics

3.1 Questions

The first question that naturally arises is to understand the limit as $N \to \infty$ of the *empirical measure* defined by

$$\mu_N := \frac{1}{N} \sum_{i=1}^{N} \delta_{x_i} \tag{3.1}$$

for configurations of points that minimize the energy (1.1), critical points, solutions of the evolution problems presented above, or typical configurations under the Gibbs measure (1.7), thus hoping to derive effective equations or minimization problems that describe the average or mean-field behavior of the system. The term mean-field refers to the fact that, from the physics point of view, each particle feels the collective (mean) field $g * \mu_N$ generated by all other particles. Convergence in the mean-field sense is, thus, equivalent, in some sense, to the "propagation of molecular chaos" (see [Go]). From the statistical mechanics point of view, we also try to understand the temperature dependence of the behavior of the system and the eventual occurrence of phase transitions.

3.2 The equilibrium measure

The energy (1.1) can be written as

$$\mathcal{H}_N(x_1,\ldots,x_N) = N^2\left(\frac{1}{2}\iint_{\mathbb{R}^\mathsf{d}\times\mathbb{R}^\mathsf{d}\setminus\triangle} \mathsf{g}(x-y)d\mu_N(x)d\mu_N(y) + \int_{\mathbb{R}^\mathsf{d}} V(x)d\mu_N(x)\right),$$

where $\triangle$ denotes the diagonal of $\mathbb{R}^\mathsf{d}\times\mathbb{R}^\mathsf{d}$. Thus, it is natural to consider the "continuum version" of the energy, namely:

$$\mathcal{I}_V(\mu) := \frac{1}{2}\iint_{\mathbb{R}^\mathsf{d}\times\mathbb{R}^\mathsf{d}} \mathsf{g}(x-y)d\mu(x)d\mu(y) + \int_{\mathbb{R}^\mathsf{d}} V(x)d\mu(x).$$

It is well known from potential theory that, in the space of probability measures, $\mathcal{I}_V$ admits a unique minimizer, μ_V, which is called the *equilibrium measure*, and is characterized by the fact that there exists a constant c such that

$$\begin{cases} h^{\mu_V} + V \geq c & \text{in } \mathbb{R}^\mathsf{d} \\ h^{\mu_V} + V = c & \text{in the support of } \mu_V \end{cases} \tag{3.2}$$

where

$$h^{\mu_V}(x) := \int_{\mathbb{R}^\mathsf{d}} \mathsf{g}(x-y)d\mu_V(y) \tag{3.3}$$

is the (electric) potential generated by μ_V. This is true for Coulomb and for Riesz interactions, as well as for more general kernels. In the Coulomb case, the equilibrium measure can be interpreted with the help of an obstacle problem (in the Riesz case with a fractional obstacle problem), cf. [Se1, Chap. 2]. An example is provided by Coulomb interaction (in any dimension) with confinement potential $V = c|x|^2$: in this case we can verify that the equilibrium measure is always (a multiple of the) characteristic function of a ball. In the context of the Ginibre ensemble in Random Matrix Theory, this is known as the "circle law". Another important example is that of the logarithm interaction in dimension 1 with quadratic potential V: the equilibrium measure has density $\sqrt{x^2-a^2}\mathbf{1}_{|x|<a}$, known, in Random Matrix Theory, as the (Wigner) *semi-circle law* for the ensembles GOE and GUE.

The energy $\mathcal{I}_V$ is the "mean-field limit" of the energy $\mathcal{H}_N$, and one can show without much difficulty that, for the minimizers of $\mathcal{H}_N$, the empirical measure converges to μ_V, and $\frac{1}{N^2}\min\mathcal{H}_N$ converges to $\mathcal{I}_V(\mu_V)$.

We can interpret $\nabla(h^\mu + V)$ as the total mean-force felt by a distribution with density μ. Therefore, in view of (3.2) it is null for the minimizers. More generally, we expect that the critical points of $\mathcal{H}_N$ have a limiting empirical distribution satisfying

$$\nabla(h^\mu + V)\mu = 0. \tag{3.4}$$

For the dynamics **(3)**, the formal limit of (1.9) or (1.10) is

$$\partial_t \mu = -\mathrm{div}\,(\nabla(h^\mu + V)\mu), \tag{3.5}$$

or

$$\partial_t \mu = -\mathrm{div}\,(\mathbb{J}\nabla(h^\mu + V)\mu), \tag{3.6}$$

again with $h^\mu = \mathsf{g} * \mu$. In the case (1.2), (3.6) with $V = 0$ is also well known as the vorticity form of Euler's equation.

The difficulty in rigorously proving the convergence towards solutions of these equations (whose well posedness also needs to be proved) consists in passing to the limit in the products of the type $\nabla h^\mu \mu$, which are nonlinear and, a priori, ill defined in the energy space. In the case of (1.2) we can overcome these difficulties by the reformulation of these terms introduced by Delort in the context of his works in fluid mechanics, but this approach does not work in higher dimensions.

Until recently, all convergence results were limited to sub-Coulomb singularities ($\mathsf{s} < \mathsf{d}-2$) or to dimension 1. Recently, a modulated energy method developed in [Se2] for the mean-field limit of the Ginzburg-Landau equations, based on the stability of solutions of the limiting equations for the "Coulomb norm" (or "Riesz norm")

$$\|\mu\|^2 = \iint \mathsf{g}(x-y)d\mu(x)d\mu(y),$$

allowed for the treatment of Coulomb interactions and for more singular Riesz cases:

Theorem 1 ([Se3])**.** *For the dynamics* (1.9) *and* (1.10)*, for all* d*, and all* $\mathsf{s} \in [\mathsf{d}-2, \mathsf{d})$ *in* (1.6)*, or* (1.5) *or* (1.2)*, the empirical measures converge to the solutions of* (3.5) *or* (3.6)*, when* $N \to +\infty$*, provided these are sufficiently smooth and the initial data energies converge to those of their limits.*

This result was preceded by one by Duerinckx in dimension 1 and 2 for $\mathsf{s} < 1$ and followed by some generalizations by Bresch-Jabin-Wang [BJW].

As far as (1.11) is concerned, the limiting equation is formally found to be the Vlasov-Poisson equation

$$\partial_t \rho + v \cdot \nabla_x \rho + \nabla(h^\mu + V) \cdot \nabla_v \rho = 0, \tag{3.7}$$

where $\rho(t,x,v)$ is the density of particles at time t with position x and velocity v, and $\mu(t,x) = \int \rho(t,x,v)dv$ is the density of particles. Notwithstanding recent progresses, we do not yet know how to prove convergence of (1.11) to (3.7) when the interaction is Coulomb or has a stronger singularity. About this topic, one can consult the reviews [Jab, Go].

3.3 With temperature: statistical mechanics

Let us now consider (1.8) and turn our attention to problem (2). It is known that even with temperature the behavior of the system is still governed by the equilibrium measure. The result can be phrased using the language of Large Deviations Principles, and states, essentially, that if E is a subset of the space of probability measures, after identifying the configurations $(x_1, \dots, x_N)$ in $(\mathbb{R}^{\mathsf{d}})^N$ with their empirical measures, we have

$$\mathbb{P}_{N,\beta}(E) \approx e^{-\beta N^2(\min_E \mathcal{I}_V - \min \mathcal{I}_V)}, \tag{3.8}$$

which implies, due to the uniqueness of the minimizer μ_V of $\mathcal{I}_V$, that the configurations for which the empirical measure do not converge to μ_V have a very small probability. For example, in the case of matrices in GOE or GUE, for which the equilibrium measure is the semi-circle law, we deduce as an application a corollary of a result by Ben Arous and Guionnet: the probability that a GOE or GUE matrix is definite positive (and thus, that all their eigenvalues are positive, which is incompatible with the semi-circle law which is symmetric relative to 0) decreases like e^{-cN^2}.

In other words, at this leading order, temperature does not affect the mean-field behavior of the system. (This is not what happens if we replace β by β/N: in this case we have a modified equilibrium measure which spreads out with the temperature, minimizing $\beta\mathcal{I}_V(\mu) + \int \mu \log \mu$).

4 Beyond mean-field

In order to observe, for example, the effect of temperature (see Figure 4.1) it is interesting to go beyond the mean-field limit: expanding the energy $\mathcal{H}_N$ to next order we have, at the same time, access to information about the typical *microscopic* behavior of the configurations. Observe that, at the microscopic scale, the typical distance between nearest neighbors is $N^{-1/\mathsf{d}}$.

4.1 Rigidity and Gaussian fluctuations

For minimizers of the energy $\mathcal{H}_N$ or of typical configurations under (1.7), since one already knows that $\sum_{i=1}^N \delta_{x_i} - N\mu_V$ is small, one knows, for instance, that the so-called discrepancy in balls $B_r(x)$, defined by

$$D(x,r) := \int_{B_r(x)} \sum_{i=1}^N \delta_{x_i} - N\, d\mu_V\ ,$$

is of order $o(r^{\mathsf{d}}N)$, for fixed $r > 0$. It can be asked whether this estimation can be refined, and if it remains true at mesoscopic or microscopic scales, i.e., for r of order $N^{-\alpha}$ with $\alpha < 1/\mathsf{d}$ or $N^{-\frac{1}{\mathsf{d}}}$, and for all temperatures. This corresponds to a *rigidity result.*

In [AS] we obtain the best-to-date result on this question, for the case of Coulomb interactions, and for all regimes of temperature: to properly quantify temperature effects, we must replace the inverse temperature by $\beta_N = \beta N^{\frac{2}{\mathsf{d}}-1}$. There is then a minimal length scale $\sim \max(\beta^{-\frac{1}{2}}N^{-\frac{1}{\mathsf{d}}}, 1)$ above which configurations acquire rigidity: the energy per unit volume and number of points in cubes of such sizes is bounded (independently of β and N) and the

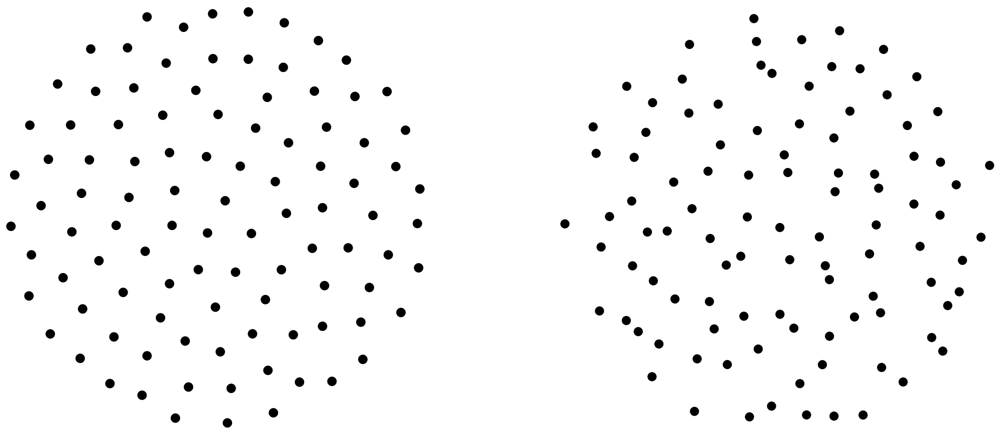

Figure 4.1: Case (1.2) with $N = 100$ and $V(x) = |x|^2$, for $\beta = 400$ (left) and $\beta = 5$ (right).

discrepancy is much smaller than the volume. When β is fixed independently of N, this result shows rigidity down to the microscale $N^{-\frac{1}{d}}$, and the bound on the numbers of points implies for the first time the existence of limiting point processes for any fixed β (and in all dimension), up to subsequences.

In the context of two-dimensional Coulomb interactions, we can prove a stronger rigidity result, which is on linear statistics obtained by integrating $\sum_{i=1}^N \delta_{x_i} - N\mu_V$ not over a ball, but against a sufficiently smooth test function. In this way we get an even more precise result, since we can prove that these quantities converge to a Gaussian with explicitly known mean and variance:

Theorem 2 ([LS2]). *In case (1.2), let us assume that $V \in C^4$ and μ_V has connected support Σ with a regular boundary. Let $f \in C^3_c(\Sigma)$. Then*

$$\sum_{i=1}^N f(x_i) - N \int_\Sigma f \, d\mu_V$$

converges in law to a Gaussian with

$$\text{mean} = \frac{1}{2\pi}\left(\frac{1}{\beta} - \frac{1}{4}\right) \int_{\mathbb{R}^2} \Delta f \log \Delta V \qquad \text{variance} = \frac{1}{2\pi\beta} \int_{\mathbb{R}^2} |\nabla f|^2.$$

This result can be localized with test-functions f supported on any mesoscale $N^{-\alpha}$, $\alpha < \frac{1}{2}$. It is also true for energy minimizers, taking formally $\beta = \infty$.

For an idea of the proof we suggest the lecture notes [Se4].

This result can be interpreted in terms of the convergence to a suitable *Gaussian Free Field*, a sort of two-dimensional analogue of Brownian motion. Note that a similar result was obtained by Bauerschmidt-Bourgade-Nikula-Yau, and it was previously known for $\beta = 2$, and in the unidimensional logarithm case for all values of β.

If f is sufficiently smooth, the associated fluctuations are typically of order 1, i.e. much smaller than we could expect, for example comparing with the standard Central Limit Theorem where the fluctuation of the sum of N i.i.d. random variables is typically of order $\sqrt{N}$. Proving this result in higher dimension or for more general interactions remains an open problem.

4.2 Next order in the energy

As we pointed out above, the approach we employ (initiated with Etienne Sandier, and continued with Nicolas Rougerie, Mircea Petrache, Thomas Leblé and Scott Armstrong) consists in studying the next order of the expansion of the energy about the measure $N\mu_V$, which is formally the minimizer. Expanding and using the characterization (3.2), the "order 1" terms in $\sum_{i=1}^N \delta_{x_i} - N\mu_V$ vanish and we obtain

$$\mathcal{H}_N(x_1, \ldots, x_N) = N^2 \mathcal{I}_V(\mu_V) + F_N^{\mu_V}(x_1, \ldots, x_N) \tag{4.1}$$

where

$$F_N^{\mu_V}(x_1, \ldots, x_N) = \frac{1}{2} \iint_{\triangle^c} \mathsf{g}(x-y) d\Big(\sum_{i=1}^N \delta_{x_i} - N\mu_V\Big)(x) d\Big(\sum_{i=1}^N \delta_{x_i} - N\mu_V\Big)(y), \tag{4.2}$$

and again $\triangle$ denotes the diagonal $\mathbb{R}^\mathsf{d} \times \mathbb{R}^\mathsf{d}$. This is a next-order expansion of $\mathcal{H}_N$ valid for arbitrary configurations.

The "next-order energy" $F_N^{\mu_V}$ can be seen as the total Coulomb energy of the neutral system formed by the N positive point charges at the points x_i and the diffuse negative charge $-N\mu_V$ with the same mass. The goal is now to define a limit of this energy when $N \to \infty$, that will be the total Coulomb energy (per unit volume) of an infinite system of positive charges and a (let us say) uniformly distributed negative charge. In physics such a system is called a *jellium*. The precise definition of this limiting energy is a bit complex, but it uses, in a crucial way, the Coulomb nature of the interaction. In fact, since g is the kernel of the Laplacian, we observe that if $h^\mu = \mathsf{g} * \mu$ is the electrostatic potential generated by a charge distribution μ (with zero integral), then h^μ solves the Poisson equation

$$-\Delta h^\mu = \mathsf{c_d} \mu,$$

which is a local elliptic PDE, and, additionally, using the Gauss-Green formula, we can write

$$\iint_{\mathbb{R}^\mathsf{d} \times \mathbb{R}^\mathsf{d}} \mathsf{g}(x-y) d\mu(x) d\mu(y) = -\frac{1}{\mathsf{c_d}} \int_{\mathbb{R}^\mathsf{d}} h^\mu \Delta h^\mu = \frac{1}{\mathsf{c_d}} \int_{\mathbb{R}^\mathsf{d}} |\nabla h^\mu|^2.$$

In another way, we can rewrite the interaction energy (which involves a double integral) in the form of a single integral of a local function of the electrostatic (or Coulomb) potential generated by this distribution, itself a solution of a local equation. In Riesz's case, these manipulations can be replaced by similar ones using the fact that g is the kernel of an elliptic operator in divergence form, which is still local.

With the help of this observation we succeed in defining an infinite volume energy for an infinite configuration of points $\mathcal{C}$ neutralized by a distributed charge (let us say -1), via the solutions of

$$-\Delta H = \mathsf{c_d} \left(\sum_{p \in \mathcal{C}} \delta_p - 1 \right) .$$

We shall denote this energy by $\mathbb{W}(\mathcal{C})$. When the configuration of points $\mathcal{C}$ is periodic with respect to a lattice Λ, the energy $\mathbb{W}(\mathcal{C})$ has an explicit form: if there are M points a_i in the fundamental cell we have (up to constants)

$$\mathbb{W}(\mathcal{C}) = \sum_{1 \le i \ne j \le M} G_{\mathbb{T}}(a_i, a_j) ,$$

where $G_{\mathbb{T}}$ is the Green function of the torus $\mathbb{T} := \mathbb{R}^{\mathsf{d}}/\Lambda$.

We can show that $\mathbb{W}$ can be obtained as the limit (in a certain sense) of the functional $F_N^{\mu_V}$ in (4.1). It also follows from an expansion to the next order of the minimum of the energy $\mathcal{H}_N$ and from the fact that, after dilation, the minimizers of $\mathcal{H}_N$ must converge (almost everywhere with respect to the origin of the dilation) to a minimizer of $\mathbb{W}$ (see, for example, [Se1]).

We are therefore led to try to determine the minimizers of $\mathbb{W}$. This problem is extremely difficult, with the exception of the one dimensional case, where we can prove that the minimum of $\mathbb{W}$ is attained by the lattice $\mathbb{Z}$. In dimension 2 the only positive result is the following:

Theorem 3. *The minimum of $\mathbb{W}$ over lattices of volume 1 in dimension 2 is achieved uniquely by the triangular lattice.*

Here the triangular lattice means $\mathbb{Z}+\mathbb{Z}e^{i\pi/3}$, properly scaled, i.e., what is called the Abrikosov lattice in the context of superconductivity. This partial result is, in fact, a result from number theory, known since the 1950s, about the minimization of Epstein's zeta function (cf. [Mont] and references therein). It corresponds to minimizing the height of a flat torus in Arakelov geometry.

Since the triangular lattice is observed in experiments with superconductors, and since we have proved that the minimization of the Ginzburg-Landau energy of the superconductor reduces to that of $\mathbb{W}$ [SS], it is natural to conjecture that the triangular lattice is a global minimizer of the energy.

According to a conjecture of Cohn-Kumar, the triangular lattice should be a universal minimizer in dimension 2 (i.e., should minimize a large class of interaction energies). An analogous role is played in dimensions 8 by the lattice E_8, and in dimension 24 by the Leech lattice, for which the Cohn-Kumar conjecture was proven very recently [CKMRV].

In dimensions $\mathsf{d} \ge 3$ (except for $\mathsf{d} = 8$ and $\mathsf{d} = 24$), the minimization of $\mathbb{W}$, even among lattices, is an open problem. As before, we can think that this relative minimum is global, but we expect this to be true only in low dimensions since computer simulations provide clear indications that in dimensions $\mathsf{d} \ge 9$ the minimizers are not lattices.

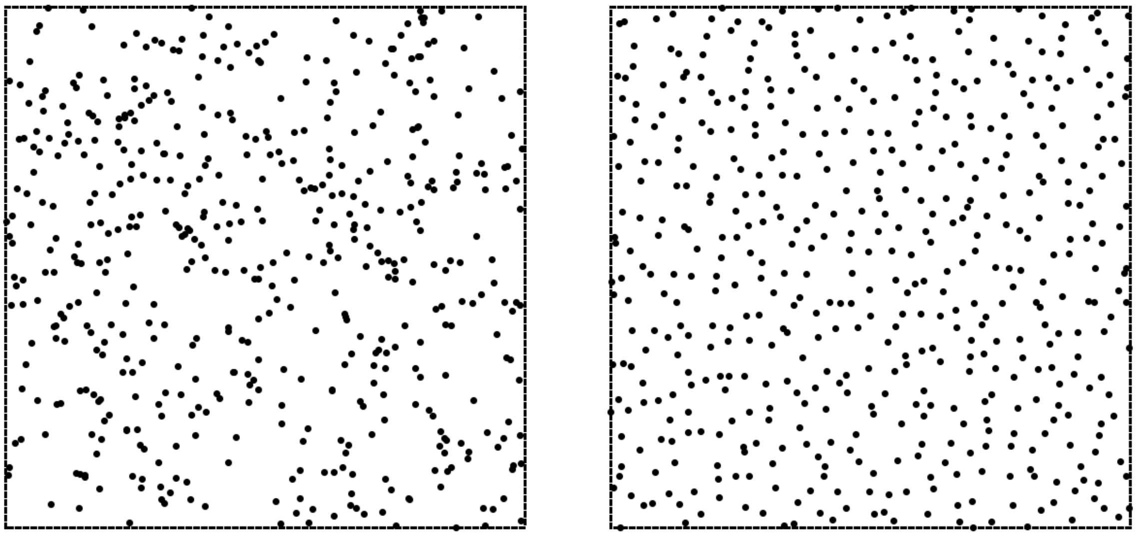

Figure 4.2: Simulation of the Poisson point process with intensity 1 (left), and of the Ginibre process (right)

These questions belong to the more general family of crystallization problems for which very few positive results are known once the dimension is larger than or equal to 2 (cf. the review [BLe]).

4.3 Next order LDP with temperature

As we saw above, the macroscopic (or "mean-field") behavior of the system does not depend upon the temperature and is given by the equilibrium measure. On the other hand, one shows that the microscopic behavior depends on the temperature and is governed by a weighted sum of the energy $\mathbb{W}$ in the previous paragraph and of a relative entropy. To observe this interesting interplay between energy and entropy, we need to consider again the temperature rescaling $\beta_N = \beta N^{\frac{2}{\mathsf{d}}-1}$ for some fixed β. To formulate the result, one dilates the configurations by $N^{-1/\mathsf{d}}$, as in the previous paragraph, and considers the limiting point process P^x obtained by averaging near each point x. Here a point process is a law on infinite configurations of points. For instance, the Ginibre process is obtained by passing to the limit $N \to \infty$ (after dilation) of the Ginibre ensemble; the Poisson process Π with intensity 1 corresponds to points thrown independently of each other in such a way that the probability of having $N(B)$ points in a set B is

$$\Pi\left(N(B) = n\right) = \frac{|B|^n}{n!} e^{-|B|}.$$

Thus, one defines a "specific relative entropy" with respect to the Poisson process, denoted by $\mathsf{ent}[\cdot|\Pi]$, that we can think of as measuring how much the process P is close to Poisson.

For all $\beta > 0$, we define the functional $\mathcal{F}_\beta$

$$\mathcal{F}_\beta(P) := \int_\Sigma \frac{\beta}{2}\mathbb{W}(P^x) + \mathsf{ent}[P^x|\Pi]dx, \tag{4.3}$$

with $P = \int_\Sigma P^x dx$. We can now formulate a large deviations result.

Theorem 4 ([LS1])**.** *For all cases* (1.5)*,* (1.2) *and* (1.3) *with* $\mathsf{d} - 2 \le \mathsf{s} < \mathsf{d}$*, with smooth assumptions on* V *and* μ_V*, and for all* $\beta > 0$*, we have a Large Deviations Principle at speed* N *with rate function* $\mathcal{F}_\beta - \inf \mathcal{F}_\beta$*, in the sense that*

$$\mathbb{P}_{N,\beta}(P_N \simeq P) \simeq e^{-N(\mathcal{F}_\beta(P) - \inf \mathcal{F}_\beta)}.$$

In this way, the Gibbs measure $\mathbb{P}_{N,\beta}$ concentrates on microscopic point processes which minimize $\mathcal{F}_\beta$. This minimization is due to a competition between energy and entropy. When $\beta \to 0$ the entropy dominates and we can prove that the limit processes converge to a Poisson process. When $\beta \to \infty$, the energy $\mathbb{W}$ dominates, which, heuristically, forces the configurations to be more "ordered" and to converge to the minimizers of $\mathbb{W}$. Between these two extremes we have intermediate situations and to know if there is a critical β corresponding to a crystalization, or to a liquid-solid phase transition (which is conjectured to take place for (1.2) in some physics papers), is a problem that remains open. In dimension 1, on the other hand, due to the fact that we can identify the minimizers of $\mathbb{W}$, it can be concluded that a true crystallization result holds when temperature tends to 0.

One consequence of this result is to provide a variational interpretation for the few known limiting processes: the so-called "sine-β" process, limit in the one-dimensional case (1.5), and the Ginibre's process: they must minimize $\beta\mathbb{W} + \mathsf{ent}$.

We would like to obtain more information about the limiting point processes, namely the behavior and decay of the "two-point correlation functions", which would shed light on the existence of phase transitions and crystallization. Unfortunately, this theorem does not seem to provide much help for those problems.

As we have seen, many questions remain open, notably those of crystallization, of identification of the minimizers and of the minima of $\mathbb{W}$ and $\mathcal{F}_\beta$, and of the generalization of Theorem 2 to dimensions $\mathsf{d} \ge 3$, to Riesz interactions, and even to more general interactions.

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Business, Industry, and Government Careers in the Mathematical Sciences

by PHILIPPE TONDEUR (Urbana-Champaign)

Career preparation for graduate students in the mathematical sciences ideally provides superb research and education development for aspiring mathematical scientists. An aspect of career development not yet routinely part of graduate studies in the mathematical sciences are research experiences related to business, industry and government (BIG). For the purposes of this discussion, you can think of this acronym as standing in for mathematically oriented research in a non-academic setting. It is not yet a standard term used outside the mathematical sciences.

BIG career opportunities have substantially expanded over the last decades, as has the number of graduating PhD's in the mathematical sciences. This is not the case for the number of academic tenure track and tenured positions to be filled. For a discussion of this discrepancy in a larger context, see [4]. Yet in the mathematical sciences context personally and financially rewarding BIG careers are continually opening up in well-established as well as unexpected fields, appearing under a bewildering number of labels, attesting to the forever spreading of mathematical tools across an equally bewildering number of fields. All this is an expression of the fantastic expansion of the impact of the mathematical sciences. Vital skills looked for in these positions are mathematical thinking and problem solving skills.

There are many approaches to prepare graduate students for BIG careers. In the *BIG Jobs Guide* [2], the authors give practical and compelling advice how to navigate that territory, based on their deep and very successful experiences in advisory roles. They concentrate on students trained in Mathematics, Statistics and Operations Research. Mathematical Physics students will equally benefit from the fantastic career advice outlined by the authors. The main transitions considered are undergraduates envisioning BIG Jobs, and graduate students as well as postdocs contemplating BIG Jobs. But the job seeking principles provided will be helpful for other transitions.

This Job Guide was a developed by the BIG Math Network [1], which is dedicated to the development of partnerships between academia, business, industry, and government. It was launched in 2016, and is supported by most of the professional societies focused on the mathematical sciences. Such partnerships have developed over time in manifold ways, and the network [1] seeks to spread and amplify these developments.

To achieve its goals, activity plans for the BIG Math Network are described in [3] as follows:

- Communicate the career value of mathematical science training to students, faculty, and BIG employers (case studies, interviews with BIG career scientists)
- Facilitate and create interactions between students, faculty, and BIG employers
- Share knowledge on how to prepare for BIG internships and jobs (webinars, panels, presentations by previous BIG interns and BIG job holders)
- Collect and create best practices and training material for preparing students for BIG jobs

- Promote BIG job opportunities via professional societies and at society meetings (career fairs, panels)

It is a pleasure to report that such activities have increasingly gained active support by mathematically implicated professional societies, in particular the American Mathematical Society, the Society for Industrial and Applied Mathematics, and the Mathematical Association of America.

References

[1] BIG Math Network (https://bigmathnetwork.org) is an organization to foster networking between academia, business, industry, and government.

[2] BIG Jobs Guide, by Rachel Levy, Richard Laugesen, and Fadil Santosa, Society for Industrial and Applied Mathematics, Philadelphia, 2018, xii + 141pages. Invaluable advice for aspiring mathematical scientists thinking about careers in BIG.

[3] BIG Career Developments for Mathematics Graduate Students, by Richard Laugesen, Rachel Levy, and Fadil Santosa, Notices of the American Mathematical Society, April 2019, 523-524.

[4] Private sector nears rank of top PhD employer, Science March 15, 2019, Vol. 363 Issue 6432, p. 1135.

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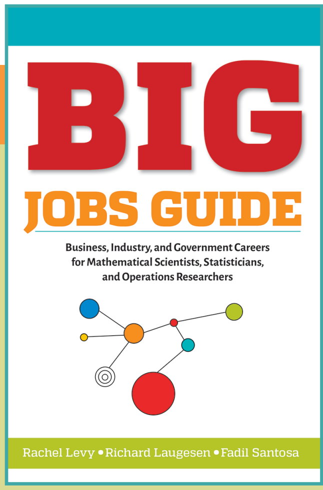
The authors talk about



(and other stuff too)

SIAM: What inspired you to write *BIG Jobs Guide*?

Fadil Santosa: I have been helping graduate students and postdocs start their careers in industry for over 20 years. In fact, of the 18 PhD students that did their dissertations with me, 16 of them are in industry. So I've gained some insights on what gets them hired by companies, and I thought it was time to put together a "how-to" book.

Rick Laugesen: It was "who: inspired me, rather than "what." It was my graduate students at the University of Illinois. As graduate director from 2012 to 2017, I met annually with each PhD student to discuss their progress, plan their professional development, and explore career options. Students would ask, "What kind of employer hires a math PhD?" and, more specifically, "Are they going to be interested in me?" Then they'd want to know, "How should I prepare for a career in industry or government?" *BIG Jobs Guide* answers all these questions!

Rachel Levy: Access to useful information about how to get jobs is not available to a majority of students pursuing degrees in the mathematical sciences. Many faculty are not sure how best to mentor students who want to pursue BIG jobs because it hasn't been part of their own experience or these jobs haven't been seen as desirable in their departments.

What are some of the hottest areas for recent graduates with degrees in math, statistics, operations research, and related areas in the business world?

Fadil: One recent PhD got a job at Ford Research in Palo Alto working on autonomous vehicles. Another is at a start-up in San Francisco that makes prototype circuit boards. In fact, some of the more interesting jobs are with start-ups. You can make a much bigger impact but you have to put in long hours and there are risks. My advice is to find a job where the work is exciting and challenging and the societal impact is aligned with your own values.

Rick: Machine learning and, more generally, data science are certainly hot right now, but career options abound for mathematical scientists, statisticians, and operations researchers all over the country, in many different fields, and at all different kinds of organizations. The best way to find a satisfying career is to start by reflecting on your own values and priorities. If you know what matters to you, then you can tune out the hype and concentrate on finding an organization that provides interesting challenges, good compensation, and opportunities for growth.

To throw out a few examples of the variety of employers that job seekers can consider, graduate students from Illinois have done internships at a beverage conglomerate's research group, agricultural and construction machinery manufacturers, the customer analytics section of a utility company, an advertising group within a well-known internet giant, a government communications research lab, a public health department, various top Wall Street firms, and several federally funded national labs. These students came from a cross section of mathematical areas—combinatorics, mathematical biology, representation theory, harmonic analysis, algebraic topology, number theory, and so on. Employers are not particular about your thesis area and like to hire mathematical talent, provided you're flexible and willing to tackle new projects.

Rachel: The pipeline into jobs depends on your degree, school, and networking connections. If you're able to honestly position yourself as a data scientist, you're likely to be paid more than a statistician, operations researcher, or mathematician in the same company.

What kinds of government jobs are good prospects for these same folks?

Rick: The national labs hire a lot of master and doctoral students for internships and full-time positions. Many governmental organizations deal with complex logistical issues where mathematical and statistical thinking comes into play, including the Social Security Administration, the Department of Defense, government contractors such as the Center for Naval Analyses and Institute for Defense Analyses, the Centers for Disease Control, NASA, the Environmental Protection Agency, and the Congressional Budget Office. The list goes on.

Fadil: I don't think enough students consider jobs that are related to policy. When they think of government jobs, they think of the labs, but it's important that scientifically strong people be engaged in governmental places like the EPA, DOE, DOD. People with mathematical sciences backgrounds have the skills to analyze and solve complex problems and support decision makers in government.

Why are business and industry listed separately? Aren't these terms synonymous?

Rachel: We might think of industry as making something tangible and business as possibly dealing in finance or software. Another important sector is nonprofits or NGOs, which you might think of as a subset of businesses. We adopted the term "BIG" from a Mathematical Association of America special interest group, which had been in existence for a long time.

Fadil: We used to think of business as banks, insurance companies, and service providers. Industry used to mean companies that manufacture products, such as auto makers, computer companies, and appliance makers, but now they're interchangeable. I don't think B-G or I-G sounds as good as B-I-G, however, so the B is here to stay!

Rick: We needed that extra letter because SIAM balked at "IG Jobs Guide." Some people make a distinction between "businessy" companies, such as financial firms, and "industrial" companies, such as medical equipment manufacturers. Both types of companies employ mathematicians, statisticians, and operations researchers.

What is operations research?

Rachel: Operations research (OR) is a field that develops solutions for processes, decisions, or systems that balance feasibility and optimality. Sometimes it is also called systems engineering or management science. While it can be considered as a type of applied mathematics, OR departments are also found in business and engineering schools. Some cool OR stories are here *https://www.informs.org/Impact*.

What lead you to become not only a mentor but also an advocate for mentorship?

Rachel: As an educator, mentoring has always been important to me.

Fadil: It's important that students and postdocs have someone with knowledge of industry to mentor them, and I hope this book will be useful to those stepping into that role. Making students aware of what they have to offer and being a cheerleader as they start their journeys is a rewarding role.

Rick: It is really satisfying to help students chart the future course of their life.

How do people a few years into their careers transition from, for example, an academic job to one in the business world?

Rick: Transitioning from one career—say, a teaching position at a university—to an entirely different one in the business world is overwhelming to many people. What are the first steps in undertaking such a big change in careers? Take a deep breath and remember to believe in yourself.

Rachel: Identify who you are and what you want. We talk in our book about how to do this.

Fadil: This is a great time for people trained in the mathematical sciences. There is a real unmet need for talents with strong analytical skills. Transitioning is a matter of presenting a portfolio of experiences and demonstrating abilities that companies find valuable—translating your knowledge and experience in academia to competencies that recruiters recognize. You've probably done significant projects, so translate one into something that hiring managers will understand and acknowledge as important for the position. Do a few informational interviews. And, most importantly, you need to network.

If you could only give one piece of advice to job seekers, what would it be?

Rachel: I really like the advice in our book from Nicole Morgan that you need to know who you are and then search for a job that aligns with your values and with the skills you want to use on the job.

Fadil: You will need to network like crazy and you need to be properly prepared: have a good resume ready and study ahead of the interviews. Be persistent. Your dream job is waiting for you.

Rick: Did I mention buying ***BIG Jobs Guide***? Only $15 for students online at the SIAM bookstore.

2018 / xii + 141 pages
Softcover / 978-1-611975-28-4
List $25.00 / SIAM Member $17.50
SIAM Student $15.00 / Code OT158

New members elected to the US National Academy

The *News Bulletin* and the IAMP are pleased to announce that two mathematical physicists in our organization have been elected as members of the National Academy of Sciences of the United States, and a third as a Foreign Associate. They are:

1. ARTUR ÁVILA, Institut für Mathematik, Universität Zürich
2. JENNIFER CHAYES, Microsoft Research
3. BARRY SIMON, Division of Physics, Astronomy, and Mathematics, California Institute of Technology

An announcement of this year's nominations is to be found at http://www.nasonline.org/news-and-multimedia/news/2019-nas-election.html.

Please join us in extending our heartfelt congratulations to Artur, Jennifer, and Barry!

Evans Harrell (Chief Editor)

News from the IAMP Executive Committee

New individual members

IAMP welcomes the following new members

1. PROF. IRINA RADINSCHI, “Gheorghe Asachi” Technical University, Romania
2. DR. AMANDA YOUNG, University of Arizona, USA.

Recent conference announcements

Union College Mathematics Conference 2019

Sept. 13-15, 2019. Schenectady, NY, USA.

http://www.math.union.edu/ hatleyj/UnionConference2019/UCMC2019index.html

Scaling limits in kinetic theory

Sept. 30 - Oct. 4, 2019. ENS Lyon, France.

http://homepages.ulb.ac.be/ mduerinc/summerschool’19.html

Aspect 19: Asymptotic analysis and spectral theory

Sept. 30 - Oct. 4, 2019. University Paris Sud, Orsay, France.

http://aspect19.blogspot.com/

Spring School in Analysis and Mathematical Physics

October 14-22, 2019. Pontifical Catholic University of Chile, Santiago.

This conference is partially supported by the IAMP

http://escueladoc.mat.uc.cl/en/

Mathematical Challenges in Quantum Mechanics.

June 8-13, 2020. Como, Italy.

https://mcqm.it/

Open positions

Postdoc position in theoretical quantum physics at LMU Munich

We are seeking a highly motivated postdoc to join our Emmy-Noether research group in Theoretical Quantum Physics, starting off on 1. August at the LMU Munich. The candidates are expected to have strong analytic skills, ideally with a background in fermionic quantum systems and reduced density matrices. Mathematical Physicists are encouraged to apply as well. Our projects are concerning the interface of Quantum Information Theory and Quantum Many-Body Physics. We resort to analytic approaches partly complemented/guided by computational studies to gain universal insights into interacting quantum many-body systems. A particular emphasis lies on the concept of reduced density matrices and the ground state problem.

For more details see “PhD/Postdoc opportunities” on our (previous) website at
https://www2.physics.ox.ac.uk/contacts/people/schilling

Required documents: CV including a list of publications, contact details of two referees, a brief outline of research plans, (link to) PhD thesis.

Inquiries and applications to: christian.schilling@physics.ox.ac.uk (please state “PhD” in the subject). Review of applications will begin on 15 July, and continue until the position is filled. Starting date: 1 August or later in 2019.

PhD position in theoretical quantum physics at LMU Munich

We are seeking a highly motivated PhD student to join our Emmy-Noether research group in Theoretical Quantum Physics, starting off on 1.August at the LMU Munich. The candidates are expected to have strong analytic skills, ideally with a background in fermionic quantum systems and reduced density matrices. Mathematical Physicists are encouraged to apply as well. Our projects are concerning the interface of Quantum Information Theory and Quantum Many-Body Physics. We resort to analytic approaches partly complemented/guided by computational studies to gain universal insights into interacting quantum many-body systems. A particular emphasis lies on the concept of reduced density matrices and the ground state problem.

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Required documents: CV including a list of publications, contact details of two referees, (link to) Bachelor/Master thesis, academic transcript.

Inquiries and applications to: christian.schilling@physics.ox.ac.uk (please state “PhD” in the subject). Review of applications will begin on 15 July, and continue until the position is filled. Starting date: 1 August or later in 2019.

Postdoctoral Position in mathematical physics/probability/pdes at Helsinki University

The Mathematical Physics and PDE groups at the University of Helsinki, see

http://mathstat.helsinki.fi/mathphys/

and

https://www.helsinki.fi/en/researchgroups/geometric-analysis-and-partial-differential-equations,

are looking for postdocs in the following fields: Rigorous Statistical Mechanics and Renormalization Group, Singular Stochastic PDEs, Conformal Field Theory and Liouville Quantum Gravity, Stochastic Homogenization.

The positions are funded through the European Research Council (ERC) Advanced Grant "Quantum Fields and Probability" and the ERC Consolidator Grant "Quantitative Stochastic Homogenization of Variational Problems". Further information: antti.kupiainen@helsinki.fi.

Postdoctoral Researcher (3 years) in mathematics/mathematical physics at University of Helsinki

The Mathematical Physics group, see

https://wiki.helsinki.fi/display/mathphys,

at the Department of Mathematics and Statistics, see

https://www.helsinki.fi/en/faculty-of-science/faculty/mathematics-and-statistics,

of the University of Helsinki invites applications for a Postdoctoral Researcher for a full-time three-year position. The employment is scheduled to commence in September 2019 but upon mutual agreement the start can also be later in 2019.

The work will be conducted together with Professor Jani Lukkarinen, see

https://researchportal.helsinki.fi/en/persons/jani-lukkarinen,

as part of the Matter and Materials, see

https://www.helsinki.fi/en/matter-and-materials,

profiling action funded by the Academy of Finland. A successful candidate will have experience in one or more of the following mathematical topics related to the project: analysis of evolution equations involving nonlinearities, perturbation theory or related graph methods, stochastic particle systems, mathematical kinetic theory.

In addition, familiarity with physics of such transport phenomena will be beneficial: such topics include equilibrium and nonequilibrium statistical mechanics, kinetic theory, and solid state physics.

The detailed announcement may be found at

https://www.helsinki.fi/en/open-positions/postdoctoral-researcher-mathematicsmathematical-physics

Please submit your application using the University of Helsinki Recruitment System following the instructions found on the above web page by August 15, 2019.

For any further information not available on the above web page please contact prof. Jani Lukkarinen by email: Jani.Lukkarinen@helsinki.fi.

Lecturer in Pure Mathematics or Mathematical Physics, at Cardiff School of Mathematics

Applications are invited for a Lectureship in Pure Mathematics or Mathematical Physics in the School of Mathematics at Cardiff University. This is a full-time, open-ended post starting on 1 January 2020 or as soon as possible thereafter.

Applicants for the post should have a record of or potential to produce research that reaches the highest standards of excellence in terms of originality, significance and rigour in Mathematics. S/he should have the ability to deliver high-quality teaching at undergraduate and postgraduate levels. S/he will also be expected to take a full part in the life of the School, including administrative duties. Candidates must have a PhD in Mathematics or a closely related discipline. The appointment will be in any area of Geometry, Algebra, Mathematical Physics or Topology (GAPT). The GAPT group currently consists of 15 members (staff, Post Docs, and PhD students) whose research interests span a variety of related subjects in pure mathematics and mathematical quantum physics. These include (in alphabetical order):

- Algebraic topology with links to homotopy theory and operator algebras
- Combinatorics, including alternating sign matrices, graphs, lattice paths, plane partitions and polytopes
- Conformal field theory, including vertex (operator) algebras, Lie theory and its generalisations
- DG-categories and derived categories associated to algebraic varieties, with applications to algebraic geometry
- Mathematical quantum field theory, including operator-algebraic approaches and integrable field theories
- Operator algebras and noncommutative geometry, including braided subfactors and modular tensor categories

Ideally, we are looking for someone with links to more than one of these subjects.

The group holds weekly seminars with speakers from the UK and abroad, and frequently organises international conferences and programmes. Recent examples include a 6-month programme Operator Algebras: Subfactors and Applications at the Isaac Newton Institute Cambridge (2017), a 1-month programme Operator Algebras and Quantum Physics at the Simons Center for Geometry and Physics (2019), workshops on Post-rational Conformal Field Theory (2019) and Algebraic Combinatorics (2019), and many others. Moreover, the COW seminar in Algebraic Geometry meets regularly at Cardiff.

Deadline for applications is August 8, 2019. More information can be found at

https://krb-sjobs.brassring.com/TGnewUI/Search/home/HomeWithPreLoad?PageType=JobDetails&partnerid=30011&siteid=5460&jobid=1504381

For enquiries about this position contact: LechnerG@cardiff.ac.uk.

For more information on these positions and for an updated list of academic job announcements in mathematical physics and related fields visit

http://www.iamp.org/page.php?page=page_positions

Benjamin Schlein (IAMP Secretary)

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