Laudatio for Yoshiko Ogata

Laureate of the Henri Poincaré Prize 2021

It is my great pleasure and honor to briefly discuss Yoshiko Ogata's research accomplishments on this occasion of her Poincaré prize. The citation of the prize reads: "For groundbreaking work on the mathematical theory of quantum spin systems, ranging from the formulation of Onsager reciprocity relations to innovative contributions to the theory of matrix product states and of symmetry-protected topological phases of infinite quantum spin chains."

Yoshiko received her PhD from the University of Tokyo, where she was a physics major. She was a posdoc at University of Marseille and UC Davis, and then joined Kyushu University as a faculty member. In 2009, she moved to the department of mathematics of the University of Tokyo, where she is now a full professor.

Yoshiko has been working on problems in quantum many-body systems by using the operator algebraic formulation. She has solved, and is solving, a variety of the most difficult problems in physics that involve infinite degrees of freedom by developing precise, sometimes deep, mathematical tools. Let me discuss some examples.

With Vojkan Jaksic and Claude-Alain Pillet, Yoshiko studied the general problem of non-equilibrium steady states, and justified the linear response theory, especially the Onsager reciprocal relations. The Onsager relations are still among the most essential results in non-equilibrium physics, and I would say that this is a fundamental contribution to a traditional problem in physics.

In the field of quantum spin systems, Yoshiko has made several fundamental contributions on problems that are fashionable even in the physics community.

To explain her contributions, I would like to recall Duncan Haldane's famous discovery, which brought him the 2016 Nobel prize in physics, about low energy properties of the antiferromagnetic Heisenberg chain, whose Hamiltonian is

$$H = \sum_{j\in\mathbb{Z}} \boldsymbol{S}_j \cdot \boldsymbol{S}_{j+1},$$

where $(\boldsymbol{S}_j)^2 = S(S+1)$ with the spin quantum number $S = 1/2, 1/3/2, \ldots$. Haldane conjectured that when, and only when, S is an integer this model has a unique gapped ground state, namely, a unique ground state accompanied by a nonzero energy gap immediately above the ground state energy.

This conjecture has not yet been solved, but it was proved that a similar Hamiltonian

$$H_1 = \sum_{j\in\mathbb{Z}} \boldsymbol{S}_j \cdot \boldsymbol{S}_{j+1} + \frac{1}{3}(\boldsymbol{S}_j \cdot \boldsymbol{S}_{j+1})^2,$$

with $S = 1$ has a unique gapped ground state which is believed to be qualitatively similar to the ground state of the original Heisenberg chain. But it is also easy to write down a model that has a unique gapped ground state for a trivial reason. For example the $S = 1$ chain with the Hamiltonian

$$H_0 = \sum_{j \in \mathbb{Z}} (S_j^{\mathrm{z}})^2$$

clearly has a unique gapped ground state, which is the tensor product of the eigenstate $|0\rangle_j$ of S_j^{z}. It is then natural to ask whether these two ground states are "smoothly connected".

To be precise we say that the models with H_0 and H_1 are smoothly connected if there exists a family of Hamiltonians H_s, where $s \in [0, 1]$, with a unique gapped ground state that smoothly interpolates between H_0 and H_1. It was conjectured by Chen, Gu, and Wen in 2011 that H_0 and H_1 are indeed smoothly connected if one is allowed to use any short ranged Hamiltonian H_s to interpolate between them. This fact is now known rigorously. It follows, e.g., from Yoshiko's extensive classification theory of matrix product states published in 2016 and 2017 as a trilogy in Communications in Mathematical Physics.

But this is not the end of the story. Recall that both H_0 and H_1 have time-reversal symmetry, i.e., invariant under the transformation $\boldsymbol{S}_j \to -\boldsymbol{S}_j$ for all $j \in \mathbb{Z}$. It was conjectured by Gu and Wen in 2009 that if we require interpolating Hamiltonians H_s to also possess time-reversal symmetry then H_0 and H_1 are never connected smoothly. In this case the models with H_0 and H_1 are said to belong to different symmetry protected topological phases. This is indeed the fact that Yoshiko proved in her groundbreaking paper appeared in 2018 and published in CMP last year. In this and the following paper published this year in CMP, Yoshiko defined indices for a unique gapped ground state of a spin chain with certain symmetry. The indices take value in the second group cohomology $H^2(G, U(1))$ of the symmetry group, and are proved to provide classifications of symmetry protected topological phases. We should note that such indices were already defined by Pollmann, Turner, Berg, and Oshikawa back in 2010, but only for a limited class of states, namely, injective matrix product states, while Yoshiko's index theories cover an arbitrary unique gapped ground state. In this sense we can say that Yoshiko has completed the theory of symmetry protected topological phases in quantum spin chains. It is simply amazing that fully rigorous and general mathematical theory was developed only nine years after the original heuristic proposal. But this is not yet the end of the story. Yoshiko never stops. She has already completed the theory of symmetry protected topological phases of two-dimensional quantum spin systems, as we can hear from her in the next session!

I cannot help discussing one more work of Yoshiko's which is my favorite (and Yoshiko's favorite too, I hear). Suppose that there are n sequences of hermitian matrices $H_i^{(1)}, \ldots, H_i^{(n)}$ with $i \in \mathbb{N}$ which commute with each other asymptotically, i.e.,

$$\lim_{i\uparrow\infty} \|[H_i^{(\alpha)}, H_i^{(\beta)}]\| = 0,$$

for any $\alpha, \beta = 1, \ldots, n$. We then ask whether the sequences of matrices can be approximated by sequences of mutually commuting hermitian matrices, more precisely, whether there exist n sequences of hermitian matrices $Y_i^{(1)}, \ldots, Y_i^{(n)}$ such that

$$[Y_i^{(\alpha)}, Y_i^{(\beta)}] = 0,$$

for all $\alpha, \beta = 1, \ldots, n$ and $i \in \mathbb{N}$, and

$$\lim_{i\uparrow\infty} \|H_i^{(\alpha)} - Y_i^{(\alpha)}\| = 0,$$

for all $\alpha = 1, \ldots, n$.

This is indeed a famous classical problem, and it is well known that such commuting approximations do not exist in general if $n \geq 3$. In her paper in 2013 published in Journal of Functional Analysis, Yoshiko proved that *commuting approximations always exist* if the original non-commuting matrices are taken as the densities of extensive quantities of a quantum spin system. This result is natural for physicists since thermodynamics is a classical theory where all quantities commute, and these densities are precisely thermodynamic objects.

To prove the theorem, Yoshiko studies projections onto the spaces where these extensive quantities take almost constant values, and then estimates the ranks of the projections by means of the entropy functions. This estimate, with an operator algebraic technique, enables her to construct the desired set of commuting matrices. I would say that the proof is an example of ideal combination of ideas from statistical mechanics and techniques from operator algebra.

For me , It was a truly exciting experience to witness rapid progress in mathematical physics made by Yoshiko. But I am sure that this is far from the end. I am looking forward to many more new beautiful insights from Yoshiko.

I would like to end by congratulating Yoshiko on this occasion of her winning the Henri Poincaré Prize. 緒方さん、おめでとうございます。

Hal Tasaki, August 2021