International Association of Mathematical Physics

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Contents

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Cover photo: The audience at the "LUMS 2nd International Conference on Mathematics and its Applications in Information Technology", GC University - Lahore, Pakistan (March 2008).

Far from the shining cities

by Pavel Exner (IAMP President)

Talent, as it is well known, is a commodity with a pretty uneven distribution, and to flourish properly, it has to be nurtured and cultivated. In established centres where most of us function the system is set to recognize gifted people and help them to reveal their potential. It may not work always as we would wish it to do but this is not the topic I want to address here today.

Doing mathematical physics in other places is much more difficult and it is an easy conjecture that there are people who could enrich our discipline but due to the circumstances they miss the opportunity to do so. It is understandable that for most developing countries the top-level science is not the first priority, but if a truly bright person from such an environment succeeds it can have a strong positive feedback; if you want one example, think of what Abdus Salam has meant for physics in the developing world.

If our Association thinks of itself as a worldwide representative of our discipline, this aspect must not be abandoned. Guided by such considerations, the Executive Committee established a working group for developing countries headed by Tsou Sheung Tsun, who has a rich experience in this area, having done a lot of such work for the European Mathematical Society.

As its first action the group has prepared a paper published below in which they discuss the situation and possible actions. I am not going to summarize their conclusions here, rather I want to share with you general observations concerning the problem.

We know well that our Association is not large and even if we manage to bring in our colleagues who stay outside the situation will still not change substantially. Consequently, large actions supported by the IAMP budget are out of the question. On the other hand, many of our members have their own means to help. Some do indeed help and deserve sincere thanks for that; I encourage the others to think whether there is a way to contribute. Two natural possibilities could be a support of conference participation or of graduate studies. The question about the latter I want to address particularly to our institute Associate Members; some have historical ties to places outside the developed world and would know best how to use them.

There is also ample room for individual initiatives, for instance, voluntary lecture courses. Such things can be arranged through established schemes of our partner societies; members of our working group can help you to find the way. Those who did such a thing would probably agree that it is a rewarding experience. The success is by far not guaranteed, of course, but one has to try.

A first survey and report from the IAMP working group for developing countries

by *IAMP Working Group on Developing Countries*:
Bindu A. Bambah
Rafael Benguria
Norbert Hounkonnou
Tsou Sheung Tsun *(Chair)*

Preamble

The President and Executive Committee asked us to form a Working Group on Developing Countries in order to consider what IAMP can do to promote mathematical physics and to help mathematical physicists in the developing world. Our charge is to plan and initiate projects that do not involve a big budget.

For this purpose, the developing world can very roughly be divided into 5 regions, with distinct characteristics and needs, and also noted exceptions of much higher standard (so singularities really). As a rough guide, we can say: Subsaharian Africa (with the exception of South Africa), Mediterranean Africa, Southeast Asia (China and India are something of a singularity), Central Asia, and Latin America and the Caribean.

Furthermore we should note that, as in the developed world, there are various degrees of development in the developing world. In particular, we should take into account the recent emerging economies and perhaps try to build bridges with them to the benefit of all.

The first point perhaps to bear in mind is that, mathematical physics is not perceived by most developing countries and their development agencies as a priority topic, so that in the least developed areas this subject is all but non-existent. It would take a lot of resources to build up any mathematical physics in these areas, and it may actually not be a useful thing for them before they become more developed overall. So if the aim of IAMP is to promote mathematical physics (we shall come back to what individual members could do as individual mathematicians and physicists), then it may make better sense to concentrate on areas where there is already some research on the subject. Broadly speaking, we could say most of these are located in Southeast and Central Asia, parts of South America and Africa. Below we shall give a more detailed description of the state of affairs in these regions, and we hope to give a fuller picture of other regions in future.

Possible actions

One of the most useful actions would be to give grants to mathematical physicists from target countries to attend conferences and schools to be held in the developed and developing worlds. This hardly needs explanation: to be in actual real contact with other colleagues is something that can be precious to workers in isolated centres. The means the IAMP has to support conferences are limited, of course, and it would not be wise to divide this money formally; we simply encourage organizers of IAMP supported conferences to keep this point in mind and help when they find it possible and efficient - even at a small scale this could be useful.

IAMP can act as a liaison, in several ways.

Firstly, we should not forget that IAMP is not only individual members. Among our Associate Member one finds powerful mathematical-physics institutes and centres; recall that the list contains ESI (we hope that the common sense will win in Austria), Microsoft, Max Planck Leipzig, Steklov, and newly PIMS and CMTP in Rome. We believe that they can help substantially, through conference support and also through participating in scholarships given to young people to do a Ph.D. in a centre in the developed world and in existing good centres in developing world. There is again no need to stress how important it is for places where there are too few established mathematical physicist to keep the subject alive.

We encourage our Associate Members to think about ways they could help noting that things are easier when expenses are shared, as for example if there is a colleague in Europe who is willing to take such as student using his or her own grant. What we can do is the liaison in the first place.

A second way in which IAMP can act as liaison is to sponsor lectures to be given in a developing country. These lectures, if given by good and eminent mathematical physicists, can sometimes fire up young people to do the subject. They can also increase the visibility and prestige of the local mathematical physicists, particularly in the eyes of University decision makers. In this case we bring the possibility to the attention of our publisher Associate Members since a support to such courses can produce a publication which would be attractive beyond the orinal purpose of the lecture.

Lastly, there are actions which individual members can take to help. Not so much for mathematical physics, but for mathematics and physics at a more basic level. There is a very well subscribed voluntary lecturer scheme run by the IMU

http://www.math.ohio-state.edu/~imu.cdc/dcsg/activities.php

a mentor scheme run by the London Mathematical Society

http://www.lms.ac.uk/content/international-grants#Africa

and research schools run by CIMPA

http://www.cimpa-icpam.org/

etc.

What IAMP can do to guide its members might be to provide links to these organizations on its own website. Members of the working group are of course willing to advise colleagues who are interested in giving such a course.

Situation in Latin America

Argentina has the longest tradition in mathematics in Latin America. Names like Beppo Levi, Luis Santaló, Julio Rey Pastor from the first half of the XXth century are known worldwide, as are Alberto Calderón and others from the second half. That long tradition continues with the younger generation of Argentinian mathematicians. In mathematical physics there are active groups at the Universidad de Córdoba, and at the Universidad Nacional de La Plata. Both groups have ties with groups in Chile, and participate in joint events. Given the long and good tradition of mathematics in Argentina, it would be desirable to promote mathematical physics in Argentina to an even higher level.

Brazil has the best scientific organization in South America, with many people working in many areas. This is true also in mathematics. There are world class institutions like IMPA in Rio. In mathematical physics Brazil has for example the group of Dynamical Systems at IMPA and the Institute of Mathematical Physics at the University of Sao Paulo. They have organized many events, including the ICMP in Rio in 2006.

There are a few groups in mathematical physics in Chile. The largest group is at the Universidad Católica de Chile (PUC) (in the Departments of Physics and in Mathematics), in subjects like spectral theory, Schrödinger operators, disordered systems, and dynamical systems. This is the group with more formal ties with the IAMP, going back to the 1970's. The group at PUC has also been involved in the organization of conferences abroad, and in the organization of thematic semesters in Europe. There are smaller but active groups at the Universidad de Chile (School of Sciences) in Santiago, and at the University of Talca (250 km south of Santiago). There are close ties among mathematical physicists in Chile with many in Mexico, Argentina and Brazil. Several Chilean mathematical physicists belong to IAMP.

There are several groups working in mathematical physics in Mexico. The largest group is the group at UNAM (Ricardo Weder, Rafael del Río and others). But there other very active groups in Guanajuato (CIM), Cuernavaca (UNAM), Sonora, and others.

In the rest of Latin America, there are some groups in Colombia, Venezuela, Peru and Costa Rica.

The overall picture is one of activity in many centers with close ties and interactions among them.

A more detailed analysis can be found in

www.fis.puc.cl/~iamp

Situation in Africa

The African Institute of Mathematical Sciences (AIMS) is one of the most active centres for training mathematicians and physicists in Africa. The goals of AIMS are to promote mathematics and science in Africa, to recruit and train talented students and teachers and to build capacity for African initiatives in education, research and technology. It is a very well funded institution, and the role of IAMP is probably one of collaboration and liaison rather than any aid action.

AIMS is focussed around nine-month, postgraduate courses covering many of the most exciting areas of modern science, taught by outstanding Africans and other international lecturers. The courses are formally accredited by the three partner South African Universities. Students with good mathematics, science or engineering degrees are invited to apply and are supported on bursaries where needed.

The AIMS Next Einstein Initiative (NEI) seeks to unlock and nurture scientific talent across Africa. The dream is to produce an African Einstein within our lifetime. The plan is to create a pan-African network of mathematical sciences centres working together to train outstanding graduates, and to connect mathematicians and scientists in Africa to each other and to the world. The first of these centres are planned to open in Senegal (AIMS-Senegal), in Ghana and in Nigeria.

Elsewhere in South Africa, the National Institute for Theoretical Physics (NiTheP) leads and coordinates research programmes and fosters education in theoretical physics. It aims to provide a stimulating national and continental user facility for theoretical physics, and its role in research and education is well recognized, especially in under-represented communities.

In Benin, the International Chair in Mathematical Physics and Applications (ICMPA-UNESCO Chair; http://www.uac.bj/cipma) is one of the most important networks in the field of mathematical physics in Africa. The ICMPA welcome at least 30 invited lecturers from universities in the developed and emerging world (France, Belgium, Canada, USA, India, etc.) per year. In collaboration with the Catholic University of Louvain and Clark Atlanta university, it currently manages the implementation of the UNESCO/IBSP project 5-BJ-01, Development of Mathematical Physics in Africa, in particular the organization of international workshops and meetings. These provide young African researchers an opportunity to get to know one another and to initiate scientific collaboration. So far ICMPA has welcomed students from Senegal, Nigeria, Niger, Togo, Ghana, Democratic Republic of Congo, Cameroon, Ivory Coast and Benin in its Master degree and doctoral training programmes.

We also wish to mention the great initiative of ICTP (Italy, Trieste) to create and provide financial support to a number of centers in mathematics and physics in Africa with the objectives to initiate good master degree and PhD programs in these fields. These centers include the Institut de Mathématiques et Sciences Physiques (IMSP) in Benin (established in 1988), the National Mathematical Centre in Nigeria (created in 1989) and the Institute of Mathematical Sciences in Ghana (established in 1995). The doctoral theses are in topics such as general relativity, cosmology, operator theory, scattering and spectral theories.

A more detailed analysis can be found in

http://www.imsp-uac.org
http://www.nmcabuja.org
http://imsghana.org/

Situation in South Asia: India

Since the ICM 2010 took place in Hyderabad, we shall concentrate for this first report on a session about IAMP in that congress.

This session on the role of IAMP in promoting mathematical physics in developing countries was convened by Prof. Pavel Exner, coordinated by Prof. Bindu A. Bambah and chaired by Prof. H. Araki. The session was attended by many young mathematical physicists, primarily from India, but also from Africa, the Middle East, and South America. Prof. Exner highlighted the aims of the IAMP and encouraged participation from developing countries. He also gave an overview of the various programs, and first of all in Europe, for promoting research in mathematical physics, and invited young researchers of the developing world to apply. He also expressed satisfaction that the recent Fields Medals were given to two members of the mathematical physics community.

Prof. Bindu A. Bambah pointed out that in developing countries, even in India, which has an otherwise thriving mathematical community, mathematical physics is still at the crossroads. Mathematical physicists in India still face an identity crisis, being fully accepted neither by the mathematics nor the physics communities. Many of researchers have a mixed background, with a masters in physics and a doctorate in mathematics or vice-versa. The dogmatic and archaic hiring rules of the universities often disqualify them from both physics and mathematics departments, and thus they have often to settle for hostile job environments, which are detrimental to their research. IAMP can be a marvellous support system for this hybrid communitiy of mathematical physicists from developing countries.

After the session, a host of young mathematical physicists enquired about the IAMP and welcomed knowledge of an association which would sympathize with their situation and provide a forum in which they could benefit from research at the forefront of their field. They expressed the view that in developing countries like India, applied mathematics has meant only subjects such as fluid and continuum mechanics, whereas fields like general relativity (except string theory), mathematics of quantum systems and statistical mechanics have been ignored both by the mathematical and physics communities. Associations like IAMP can help rectify this situation.

International Congress of Mathematicians (2010),
the last big event in mathematical and mathematical physics communities,
took place in India

An Appeal

We do not need to emphasize how important it is to support our colleagues in the developing world. We ask everybody for collaboration, sharing information and whatever ideas you may have. Whatever we can do, it is only possible with everybody's help.

Bindu Bambah got her PhD from the University of Chicago under the supervision of Yoichiro Nambu. She is interested in quantum gravity, particle physics, non-linear algebras and the philosophy of science. She works at the University of Hyderabad.

Rafael Benguria got his PhD from Princeton University, and now works at Pontificia Universidad Católica de Chile. He is interested in Schrödinger operators, spectral theory and nonlinear elliptic equations.

Norbert Hounkonnou got his PhD from Catholic University of Louvain-la-Neuve (Belgium), and currently holds the International Chair of Mathematical Physics and Applications (ICMPA-UNESCO Chair), Université d'Abomey-Calavi, Benin. He is interested in algebraic methods and solvable models in quantum theory, special functions, orthogonal polynomials and ordinary differential equations.

Tsou Sheung Tsun got her doctorate in Geneva, and now works in the Mathematical Institute of the University of Oxford. She is interested in gauge field theory with applications to particle physics. In addition to that, she has a substantial record of assisting scientists in developing countries: she chairs the Committee for Developing Countries of the European Mathematical Society, and since 2009 she is the President of CIMPA. (Photo by James F. Hunkin.)

Andreï N.Kolmogorov - mathematician and physicist

by Yakov G. Sinai (Princeton/Moscow)

Yakov Grigor'evich Sinai received his Ph.D. (advisor Andreï N. Kolmogorov) from Moscow State University in 1960. In 1971 he became a Professor at Moscow State University and a senior researcher at the Landau Institute of Theoretical Physics. Since 1993 he has been a Professor of Mathematics at Princeton University. Sinai is a member of IAMP, the Russian Academy of Sciences, the United States National Academy of Sciences, Royal Society and others. Among his awards are the Boltzmann Medal (1986), the Dannie Heineman Prize for Mathematical Physics (1990), Dirac Medal (1992), the Wolf Prize in Mathematics (1997), the Nemmers Prize (2002), and the Henri Poincaré Prize (2009), see interview in the IAMP News Bulletin - October 2009. Sinai is highly respected in the physics community, where, as well as in mathematics, Kolmogorov-Sinai entropy, Sinai's billiards, Sinai's random walk, Sinai-Ruelle-Bowen measures, Pirogov-Sinai theory and his other achievements are basic notions that have shaped the understanding of many fundamental physical phenomena.

Andreï Nikolaevich Kolmogorov (1903-1987) was one of the most outstanding scientists of the twentieth century. First of all known as a brilliant mathematician he became very famous due to several important results in physics. In particular, it became standard for physicists to quote his name during the theoretical and experimental Seminars, Conferences and Workshops on turbulence. Indeed, turbulence is one of the important contributions of Kolmogorov in physics, dating from 1940. Since the time when his two short papers published in Dok.Akad.Nauk SSSR (1940) became known to the world, the approach and the methods of the study of turbulence phenomena in hydrodynamics, behaviour of atmosphere, oceanography etc, have been drastically changed. The excellent book: Turbulence. The legacy of A.N.Kolmogorov (Cambridge University Press, 1996) by Uriel Frisch, is an example of detailed and transparent presentation of fundamental revolutionary ideas of the Kolmogorov theory.

Another example of Kolmogorov's impact in physics is the renormalization group method. Nowadays it is a standard tool in Statistical Physics and Quantum Field Theory. In fact it is based on the idea of scale invariance, which has its origin in the Kolmogorov approach to turbulence. In my many discussions of the theory of turbulence with Kolmogorov, I always had the impression of talking to a physicist. His way of arguing made forget that I was talking to a great mathematician. For example, he was able to switch very easily from mathematics to such specific physical questions as equations of states for real gases or liquids, and then to corresponding recent analysis of the experimental data etc. He was almost 80 years old when I asked him how he discovered scale invariance. His answer surprised me a lot: he explained that it was a result of careful six-month studies of experimental data! To keep contact with exploration of the natural phenomena and to profit as he was saying from "the direct participation, where possible, in the experiments together with the physicists" Kolmogorov took part in oceanographic expeditions on board the research vessel "Dmitri Mendeleev" in 1969 and 1971. The program of this expedition was to study ocean scale turbulence. In fact the pure mathematical problem of existence and uniqueness of the solution of the Navier-Stokes equations never interested him seriously. It might be surprising, but he considered his theory of turbulence as purely phenomenological close to the theoretical physics, which explains a keen interest in his theory manifested by physicists like L.D.Landau. Kolmogorov believed that turbulence, similar to the majority of physics theories, cannot be transformed into a purely mathematical discipline.

Another direction developed by Kolmogorov, which is partially related to turbulence and which has important impact in physics is the theory of deterministic chaos. He was always stressing that to scrutinize any dynamical system one has first to elucidate the corresponding differential equations governing the evolution. There are two great discoveries in non-linear dynamics due to Kolmogorov.

The first one is KAM theory, where the letter K abbreviates Kolmogorov, and the letters AM stand respectively for V.I.Arnold and J.Moser. The second one is the concept of entropy and Kolmogorov (quasi-regular) dynamical systems. These two discoveries made a breakthrough in the analysis of non-linear dynamical systems. As concerns the KAM theory, one of my physicist friend told me that it is so natural that for him it is strange that this theory was not invented as a part of theoretical physics. On the other hand, the impact of Kolmogorov's ideas concerning the entropy as a characteristic of dynamical and complex systems is essential for physics, no less than for mathematics. One of the important consequences was the beginning of the theories of chaos, "deterministic chaos", quasi-regular systems etc. In 1959, which is usually dated as a time of the birth of these ideas, Kolmogorov apparently believed that dynamical systems generated by deterministic equations are different from dynamical systems arising in the probability even from the metrical point of view. His vision was that the entropy is an invariant that would be able to distinguish these two types of dynamics. Moreover, at that time Kolmogorov apparently believed in the exceptional importance of dynamical systems with the zero entropy. In his unpublished notes he constructed an invariant of dynamical systems expressed in terms of the growth of entropies of partitions over big intervals of time. Later he changed his mind and in 1959 introduced an important notion of quasi-regular dynamical systems (K-systems), for which his entropy was invariant. At that time I was also interested in definition of the entropy, which one could use for an arbitrary dynamical system. Since I already understood how finite partitions generate stationary random processes, together with other general facts this gives a certain solution. The next step was due to the V.A.Rohlin example of a dynamical system, for which Kolmogorov's definition did not provide an invariant. Then Rohlin suggested using my definition, and after calculations that I did, encouraged by him and by Kolmogorov, it was justified and published in the same 1959.

Soon after many examples of deterministic dynamical systems with positive entropy were discovered. After these examples appeared, a question about the properties of dynamics producing those systems with positive entropy arose. Again it is a physical intuition that was behind the notion of "deterministic chaos", which led to the widespread popularity of the entropy theory of dynamical system among physicists, who first learned this notion from the Boltzmann theory.

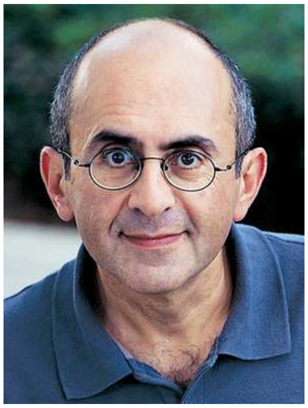
Another example of Kolmogorov's physical intuition can be seen from his works on diffusion processes. The story is that one of his University classmates was M.A.Leontovich, who became a well-known physicist and a specialist in thermo-nuclear fusion. In 1933 Kolmogorov and Leontovich wrote a paper on the subject, later known as the "Wiener Sausage". Many years after that Kolmogorov, based on his physical intuition due to this paper, proposed the answer to the problem of chasing Brownian particle, which was studied at that time by mathematicians E.Mischenko and L.Pontrijagin. The joint paper by three authors gave a complete solution of the problem.

My personal impression is that Kolmogorov never divided mathematics and physics, although he clearly distinguished their domains and methods. One of the best illustrations of this is the theory of turbulence, which is the subject of the next article of this Bulletin: "Kolmogorov and turbulence theory" by Gregory Falkovich.

I am grateful to Valentin Zagrebnov for assistance in preparation of these notes for the present issue of IAMP News Bulletin and in particular for his help in translations from the book "L'Héritage de Kolmogorov en Physique".

Kolmogorov and turbulence theory

by Gregory Falkovich (Rehovot, Israel)



Gregory Falkovich received his PhD from the Nuclear Physics Institute Novosibirsk in 1984, worked in the Russian Academy of Sciences, and since 1991 at the Weizmann Institute Science, from post-doc to professor and department head. Section Editor of J. Phys. A, he has also served on editorial boards of J. Stat. Mechanics and J. Stat. Physics. Falkovich got 4 awards from the Russian Academy of Sciences, one in Israel, and was elected Fellow of the Institute of Physics, London. He authored a textbook on Fluid Mechanics and a monograph on Turbulence.

Kolmogorov and his students put such a powerful spell on turbulence theory, that for over half a century most of the work in the area continued along the directions they developed. They first developed a practical and fruitful mean-field approach and then were the first to account for strong fluctuations.

Kolmogorov's works on stochastic processes and random functions immediately predate his work on turbulence. Turbulence presents a natural step from stochastic processes, as functions of a single variable, to stochastic fields, as functions of several variables.

Kolmogorov started by considering turbulent decay at large Reynolds number $Re = vl/\nu$, where v is the typical fluid velocity, l is the size of the turbulence region and ν is the kinematic viscosity. Assuming conservation of the Loitsyansky integral (squared angular momentum), $\Lambda = \int r_{12}^2 \langle (\mathbf{v}_1 \cdot \mathbf{v}_2) \rangle d\mathbf{r}_{12} \simeq v^2 l^5$, and estimating $dv^2/dt \simeq v^3/l$, Kolmogorov obtained $l(t) \propto \Lambda^{1/7} t^{2/7}$ [7]. He then attempted to find the second and the third moment of velocity exactly. Some time in 1940, Andrei Nikolaevich invited his student, Alexander Obukhov, and suggested to think about the energy distribution in developed turbulence. Obukhov later recalled that they met in two weeks, compared notes and found that the exponent was the same — the first Kolmogorov-Obukhov theory (KO41) came into being.

Kolmogorov and Obukhov published separately. The first Kolmogorov paper describes a multi-step energy cascade (an idea suggested previously by Richardson) as a "chaotic mechanism of momentum transfer" to pulsations of smaller scales [6]. Kolmogorov then argues that the statistics of velocity differences for small distances and time differences is determined by small-scale pulsations which must be homogeneous and isotropic (far from boundaries). He then makes a very strong assumption (later found to be incorrect) that the statistics of $\mathbf{v}_{12}$ at the distances r_{12} much less than the excitation scale L is completely determined by the mean energy dissipation rate, defined as

$$\bar{\epsilon} = \left\langle \frac{\partial v^2}{2\partial t} \right\rangle = \frac{\nu}{2} \sum_{ij} \left\langle \left(\frac{\partial v^i}{\partial x_j} + \frac{\partial v^j}{\partial x_i} \right)^2 \right\rangle .$$

That allows him to define the viscous (now called Kolmogorov) scale as $\eta = (\nu^3/\bar{\epsilon})^{1/4}$ and make the second (correct) assumption that for $r_{12} \gg \eta$ the statistics of velocity differences is independent of the kinematic viscosity ν. For $\eta \ll r_{12} \ll L$, one uses both assumptions and immediately finds from dimensional reasoning that $\langle v_{12}^2 \rangle = C(\bar{\epsilon} r_{12})^{2/3}$, where the dimensionless C is called the Kolmogorov constant (even though it is not, strictly speaking, a constant, as will be clear later).

What inspired a mathematician to hypothesize so boldly? "I soon understood that there was little hope of developing a pure, closed theory, and because of the absence of such a theory the investigation must be based on hypotheses obtained from processing experimental data. While I didn't do experiments, I spent much energy on numerical and graphical representation of the experimental data obtained by others" (A. N. Kolmogorov, 1985). The data apparently were from a wind tunnel [1]; they were used in the third 1941 paper [8] to estimate C.

More importantly, in that third paper, Kolmogorov implicitly *assumes* that, although proportional to ν, the dissipation rate $\bar{\epsilon}$ has a finite limit at $\nu \to 0$, and derives the elusive third moment. Schematically, one takes the equation of motion, $d\mathbf{v}/dt = \partial \mathbf{v}/\partial t + (\mathbf{v}\nabla)\mathbf{v} =$ force, at some point 1, multiplies it by $\mathbf{v}_2$ and subtracts the result of the same procedure taken at the point 2. All three forces acting on the fluid give no contribution in the interval $\eta \ll r_{12} \ll L$: viscous friction because $r_{12} \gg \eta$, external force because $r_{12} \ll L$ and the pressure term because of local isotropy. This is why that interval is called inertial, a term so suggestive as to be almost misleading, as we will see later. In this interval the cubic (inertial) term, which is the energy flux through the scale r_{12}, is equal to the time derivative term, which is a constant rate of energy dissipation: $\langle (\mathbf{v}_{12} \cdot \nabla) v_{12}^2 \rangle = -2\langle \partial v^2/\partial t \rangle = -4\bar{\epsilon}$. Integrating this one gets:

$$\langle (\mathbf{v}_{12} \cdot \mathbf{r}_{12}/r_{12})^3 \rangle = -4\bar{\epsilon} r_{12}/5 \ . \tag{1}$$

For many years, the so called 4/5-law (1) was the only exact result in the theory of incompressible turbulence. It is the first derivation of an "anomaly" in physics in the sense that the effect of breaking the symmetry (time-reversibility) remains finite while the symmetry-breaking factor (viscosity) goes to zero; the next example, the axial anomaly in quantum electrodynamics, was derived by Schwinger ten years later [13].

Obukhov's approach was based on the equation for the energy spectral density and essentially gave the same results. One can imagine the elation the authors felt upon discovering such beautiful simplicity in such a complicated phenomenon: the universality hypothesis was supported by the exact derivation of the third moment (1) and by the experimental data. One is tempted to conclude that the statistics of the velocity differences in the inertial interval is determined solely by the mean energy dissipation rate. What could possibly go wrong? The answer came from a physicist. In 1944, Landau wrote: "It might be thought that the possibility exists in principle of obtaining a universal formula, applicable to any turbulent flow, which should give $\langle v_{12}^2 \rangle$ for all distances r_{12} small compared with L. In fact, however, there can be no such formula, as follows from the following argument. The instantaneous value of v_{12}^2 might in principle be expressed in a universal way via the energy dissipation ϵ in that very moment. However, averaging these expressions is dependent on the variation of ϵ over times of large-scale motions (scale L), and this variation is different for different specific flows. Therefore, the result of the averaging cannot be universal" [12]. To put it a bit differently: the third moment (1) is linearly proportional to the dissipation rate ϵ and is then related in a universal way to the mean dissipation rate $\bar{\epsilon}$. Yet other moments $\langle v_{12}^n \rangle$ are averages of nonlinear functions of the instantaneous value ϵ, so that their expressions via the mean value $\bar{\epsilon}$ depend on the statistics of the input rate determined by the motions at the scale L. The question now is whether such influence of large scales changes order-unity factors (say making C non-universal) or changes the whole scale dependence of the moments, since now one cannot rule out the appearance of the factor (L/r_{12}) raised to some power. Kolmogorov and Obukhov themselves found the answers twenty years later.

Note however, that it is reasonable to expect that the moments depend in some regular way on the order n of the moment. KO41 is exact for $n = 3$ and must work reasonably well for $n = 2$ i.e. for the energy spectrum, which is indeed what measurements show. This is why this flawed theory turned out to be very useful in numerous geophysical and astrophysical applications as long as one is interested in the energy spectrum and not high moments or strong fluctuations. For twenty years, Kolmogorov and Obukhov developed the applications and generalizations of KO41 instead of looking for a better theory. In retrospect, that seems to be a right decision. Its implementation involved the creation of a scientific school (Monin, Yaglom, Barenblatt and many others).

¿From 1946, Kolmogorov arranged a bi-weekly seminar on turbulence which was a springboard for the explosive development of KO41 and applications. In 1949, Kolmogorov applied KO41 to the problem of deformation and break-up of droplets of one liquid in a turbulent flow of another fluid: a flow can break a droplet of a size a if the pressure difference due to the flow $\rho(\delta v)^2 \simeq \rho(\bar{\epsilon} a)^{2/3}$ exceeds the surface tension stress σ/a [10].

Around 1960-61, systematic measurements of wind velocity fluctuations had shown fluctuations much stronger than the theoretical estimates. Looking for an appropriate model for the statistics of ϵ, Obukhov turned to another seminal Kolmogorov 1941 paper [9] on a seemingly different subject: ore pulverization. Breaking stones into smaller and smaller pieces presents a cascade of matter from large to small scales. A stone that appears after m steps has a size ϵ_m, which is a product of the size ϵ of an initial large stone and m random factors of fragmentation: $\epsilon_m = \epsilon e_1 \dots e_m$, where $e_i < 1$. If those factors are assumed to be independent, then $\log \epsilon_m$ is a sum of independent random numbers. As m increases, the statistics of the sum tends to a normal distribution with the variance proportional to m. In other words, multiplicative randomness leads to log-normality. Since the number of steps of the cascade from L to r is proportional to $\ln(L/r)$, Obukhov then assumed that the energy dissipation rate coarse-grained on a scale r has such a log-normal statistics with variance $\langle \ln^2(\epsilon_r/\bar{\epsilon}) \rangle = B + \mu \ln(L/r)$, where B is a non-universal constant determined by the statistics at large scales. Note that the variance grows when r decreases and so other (not very high) moments: $\langle \epsilon_r^q \rangle \propto (L/r)^{\mu q(q-1)/2}$. Obukhov then formulated a refined similarity hypothesis: KO41 is true locally, that is, the velocity difference at the distance r is determined by the dissipation rate coarse-grained on that scale: $\delta v(r) \simeq (\epsilon_r r)^{1/3}$. Averaging this expression over the log-normal statistics of ϵ_r one obtains new expressions for the structure functions, which contain non-universal factors

C_n and universal exponents: $\langle v_{12}^n \rangle = C_n r^{n/3}(L/r)^{\mu n(n-3)/18}$. That general formula was actually derived by Kolmogorov who was shown Obukhov's draft containing only $n = 2$.

The new theory KO62 gives the same linear scaling for the third moment. Attempts to estimate μ from the experimental data on the variance of dissipation or velocity structure functions give $\mu \simeq 0.2$, so that KO62 only slightly deviates from KO41 for the value of n being less than a number between 10 and 12. Its importance must be then mostly conceptual. The main point is understanding that the relative fluctuations of the dissipation rate grow unboundedly with the growth of the cascade extent, L/r (in his paper, Kolmogorov credits that to Landau even though his 1944 remark did not mention any scale-dependence of the fluctuations [4]). That understanding opened the way to the description of dissipation concentrated on a measure, which was later shown to be multifractal. Let us stress another conceptual point: the 5/3-law for the energy spectrum is incorrect despite being the most widely-known statement on turbulence (outside of the turbulence community). Still, KO62 does not seem to be as momentous achievement as KO41. First, it evidently does not make sense for sufficiently high n. Second and more importantly, it is still under the spell of two magic concepts of the Kolmogorov school: Gaussianity and self-similarity. Comparing to KO41, the new version KO62 somehow pushes these two further down the road: the new (refined) self-similarity is local and Gaussianity is transfered to logarithms, replacing additivity with multiplicativity. Still, KO62 is based on the belief that the single conservation law (of energy) explains the physics of turbulence and that the (local) energy transfer rate completely determines local statistics. As we now believe, direct turbulence cascades (from large to small scales) on a fundamental level have nothing to do with either Gaussianity or self-similarity, even though these concepts can help to design useful semi-empirical models for applications. There is more to turbulence than just cascading. Energy conservation determines only a single moment (third for incompressible turbulence). To understand the nature of turbulent statistics, one returns to the old remark of Friedman that the correlation functions are "moments of conservation". In this way, one discovers an infinite number of statistical conservation laws having a geometrical nature, each determining its own correlation function; add that the exponents are now measured with higher precision and they are neither KO41, nor KO62, see e.g. [2, 3].

N.B. *Excerpted from the book chapter written for the Cambridge University Press.*

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NESS in quantum statistical mechanics: Where are we after 10 years ?

by Vojkan Jakšić & Claude-Alain Pillet
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Vojkan Jakšić (PhD, Caltech, 1991) and Claude-Alain Pillet (PhD, ETH-Zürich, 1986) are two well-known experts in quantum dynamical systems and non-equilibrium statistical mechanics. Their collaboration goes back to 1995, when they published a few papers on a model of quantum friction. Several concepts and methods which appeared there, for example the Jakšić-Pillet Glued Representation, became standard basic tools in the theory of open systems and systems out of equilibrium. In a nicely written 3-volume Lecture Notes in Mathematics v.1880-1882 (Springer 2006) on "Open Quantum Systems" one can find essential results of the present authors up to 2001-03. In their notes below they summarize the state of the art in this theory, to which they add many important results, after the last ten years of its active development.

Non-equilibrium steady states (NESS) of classical dynamical systems were discussed by David Ruelle in the last issue of this Bulletin. Here, we want to add a few words about NESS from the point of view of quantum dynamics. We refer to Ruelle's article for historical and conceptual background.

To maintain a quantum system $\mathcal{S}$ in a steady state out of equilibrium one needs, as in the classical case, some kind of thermostat. However, to comply with the rules of quantum mechanics, the dissipative forces exerted by this thermostat on the system $\mathcal{S}$ must be of Hamiltonian origin (we shall not discuss here Markovian approximations of quantum dynamics, see e.g. [LS, DDM]). The standard way to introduce such Hamiltonian dissipation is to consider $\mathcal{S}$ as an open system, i.e., to couple it to some large reservoir(s).

Consider a small quantum system $\mathcal{S}$ coupled to several extended reservoirs $\mathcal{R}_1, \ldots, \mathcal{R}_M$. We shall work in the framework of C^*-dynamical systems and denote by $\mathcal{O}_0$ the C^*-algebra describing the observables of $\mathcal{S}$. We assume $\mathcal{O}_0$ to be finite dimensional. Each reservoir $\mathcal{R}_j$ is described by a C^*-algebra $\mathcal{O}_j$. For simplicity we assume that the algebra of the joint system $\mathcal{S} + \mathcal{R}_1 + \cdots + \mathcal{R}_M$ is the C^*-tensor product $\mathcal{O} = \mathcal{O}_\mathcal{S} \otimes \mathcal{O}_\mathcal{R} = \otimes_{0 \le a \le M} \mathcal{O}_a$. The following is easily adapted to more general cases [AJPP1].

For $0 \le a \le M$ let τ_a^t be a continuous group of $*$-automorphisms of $\mathcal{O}_a$. The C^*-dynamical system $(\mathcal{O}_a, \tau_a)$ describes the isolated subsystem a. The dynamics of the decoupled joint system is $\tau = \otimes_{0 \le a \le M} \tau_a$. The dynamics τ_V of the coupled joint system is the local perturbation of τ induced by

$$V = \sum_{1 \le j \le M} V_j, \quad V_j = V_j^* \in \mathcal{O}_0 \otimes \mathcal{O}_j,$$

where V_j describes the interaction between $\mathcal{S}$ and $\mathcal{R}_j$.

Let ω be a state on $\mathcal{O}$. We say that ω_+ is a NESS of τ_V associated to the reference state ω if there exists a net $t_\alpha \to \infty$ such that

$$\omega_+(A) = \lim_\alpha \frac{1}{t_\alpha} \int_0^{t_\alpha} \omega \circ \tau_V^t(A)\, \underset{.}{t},$$

for all $A \in \mathcal{O}$. We denote by $\Sigma_+(\tau_V, \omega)$ the set of these NESS. A few remarks are in order:

1. By definition the elements of $\Sigma_+(\tau_V, \omega)$ are τ_V-invariant states on $\mathcal{O}$.

2. Strictly speaking, one should exclude the cases where the limit ω_+ turns out to be a KMS state for τ_V. This is expected (and called Return to Equilibrium) when ω is (normal relative to) a KMS state for the decoupled dynamics τ. In this case ω_+ will be ω-normal. In genuine non-equilibrium cases, ω_+ is singular with respect to ω.

3. Entropy production plays a central role in non-equilibrium statistical mechanics. We just mention that NESS have a non-negative entropy production rate [Ru2, JP1].

4. Since the set of all states on $\mathcal{O}$ is weak-$*$ compact $\Sigma_+(\tau_V, \omega)$ is not empty.

5. If the perturbation V is time dependent then natural non-equilibrium states (NNES) are defined in a similar way as limit points

   $$\omega_+^t(A) = \lim_\alpha \frac{1}{t_\alpha} \int_{-t_\alpha}^{t} \omega \circ \tau_V^{s\to t}(A)\, \underset{.}{s}.$$

   They satisfy $\omega_+^t \circ \tau_V^{t\to r} = \omega_+^r$ (see [Ru1]).

The structural theory of NESS has been developed in [Ru1, Ru2, JP1, JP3, O, AJPP1]. We mention a few results. One expects that NESS are insensitive to local perturbations of the initial state ω. The following result, proved in [AJPP1] (see also [JP3]), shows that this is indeed the case under a rather weak ergodic hypothesis.

Theorem 1 *Assume that ω is a factor state on $\mathcal{O}$ and that, for any ω-normal state η,*

$$\lim_{t\to\infty} \frac{1}{t} \int_0^t \eta([\tau_V^s(A), B])\, \underset{.}{s} = 0,$$

holds for all A, B in a dense subset of $\mathcal{O}$ (weak asymptotic Abelianness in mean). Then $\Sigma_+(\tau_V, \eta) = \Sigma_+(\tau_V, \omega)$ holds for all ω-normal states η.

In typical applications the reference state ω will be specified by the requirement that its restrictions to the subalgebras $\mathcal{O}_a$ are thermal equilibrium states at inverse temperature β_a (i.e., β_a-KMS states) for the corresponding dynamics τ_a[1]. This means that ω is a

[1]Chemical potentials μ_a can also be prescribed by appropriate definition of τ_a

KMS state at inverse temperature -1 for the dynamics $\sigma_\omega^t = \otimes_a \tau_a^{-\beta_a t}$. In particular, ω is modular and σ_ω is its modular group. The group σ_ω plays an important and somewhat unexpected role in the mathematical theory of linear response [JOP1].

Accordingly, we shall assume in the remainder of this paragraph that ω is modular and denote by $(\mathcal{H}_\omega, \pi_\omega, \Omega_\omega)$ the corresponding GNS representation of $\mathcal{O}$. The enveloping von Neumann algebra $\pi_\omega(\mathcal{O})''$ is in standard form and we denote by J the modular conjugation. For any dynamical group τ^t on $\mathcal{O}$ there exists a unique self-adjoint operator L on $\mathcal{H}_\omega$, the standard Liouvillean of τ, such that $\pi_\omega(\tau^t(A)) = \mathrm{e}^{\mathrm{i}tL}\pi_\omega(A)\mathrm{e}^{-\mathrm{i}tL}$ and $JLJ + L = 0$. If L is the standard Liouvillean of τ then $L_V = L + \pi_\omega(V) + J\pi_\omega(V)J$ is the standard Liouvillean of τ_V. The spectral analysis of L_V yields interesting information on the structure of $\Sigma_+(\tau_V, \omega)$ (see [AJPP1]).

Theorem 2 *Assume that the state ω is modular.*

1. *If* $\mathrm{Ker}\, L_V = \{0\}$ *then there is no ω-normal τ_V-invariant state. In particular, any NESS in $\Sigma_+(\tau_V, \omega)$ is purely ω-singular.*

2. *If the assumptions of Theorem 1 hold and if* $\mathrm{Ker}\, L_V \neq \{0\}$ *then it is one dimensional and there exists a unique ω-normal τ_V-invariant state ω_V. Moreover, $\Sigma_+(\tau_V, \omega) = \{\omega_V\}$.*

As already mentioned, case 1 in the above theorem is the expected behavior out of equilibrium while case 2 describes a typical equilibrium situation.

To our knowledge, there are two approaches to the construction of NESS which we now describe.

The scattering approach

The first approach was proposed by Ruelle in [Ru1] and relies on the scattering theory of C^*-dynamical systems (see [Ro]). The scattering approach assumes the existence of the strong limit

$$\alpha_V = \mathrm{s} - \lim_{t\to\infty} \tau^{-t} \circ \tau_V^t. \tag{2}$$

If it exists, this limit defines an isometric $*$-endomorphism of $\mathcal{O}$ such that $\alpha_V \circ \tau_V^t = \tau^t \circ \alpha_V$, the so called Møller morphism. α_V is injective but its range $\mathcal{O}_+$, a τ-invariant C^*-subalgebra of $\mathcal{O}$, can be strictly smaller than $\mathcal{O}$. One immediately obtains

Theorem 3 *Assume that the Møller morphism (2) exists and that ω is τ-invariant. It follows that, for all $A \in \mathcal{O}$,*

$$\lim_{t\to\infty} \omega \circ \tau_V^t(A) = \omega_+(A),$$

where $\omega_+ = \omega \circ \alpha_V$. In particular, one has $\Sigma_+(\tau_V, \omega) = \{\omega_+\}$.

If the previous theorem applies then α_V provides an isomorphism between the coupled dynamical system $(\mathcal{O}, \tau_V, \omega_+)$ and the decoupled one $(\mathcal{O}_+, \tau|_{\mathcal{O}_+}, \omega|_{\mathcal{O}_+})$. Ergodic properties of the latter are therefore inherited by the former. The following result is a simple consequence of this fact (see [AJPP1]).

Theorem 4 *Assume that the assumptions of Theorem 3 hold.*

1. *If $\omega|_{\mathcal{O}_+}$ is ergodic for $\tau|_{\mathcal{O}_+}$ then $\Sigma_+(\tau_V, \eta) = \{\omega_+\}$ for any ω-normal state η.*
2. *If $\omega|_{\mathcal{O}_+}$ is mixing for $\tau|_{\mathcal{O}_+}$ then*
$$\lim_{t\to\infty} \eta \circ \tau_V^t(A) = \omega_+(A)$$
holds for all $A \in \mathcal{O}$ and any ω-normal state η.

For a finite system coupled to infinite reservoirs we expect $\mathcal{O}_+ = \mathcal{O}_{\mathcal{R}}$ so that the coupled system out of equilibrium inherits the ergodic properties of the reservoirs.

C^*-scattering is much more difficult than Hilbert-space scattering and the only known technique to deal with it is the basic Cook's method. Consequently, its application is limited to quasi-free and locally interacting quantum gases. In the former case, it reduces to scattering in the one-particle Hilbert space and can be shown to coincide with the formal Landauer-Büttiker approach to ballistic quantum transport [AJPP2, N, CDNP, CNZ]. Genuine C^*-scattering was used in [FMU, JOP2] to derive linear response theory for locally interacting gases.

The Liouvillean approach

This alternative to the scattering approach has been proposed in [JP2] where the NESS of a N-level quantum system coupled to ideal Fermi reservoirs is constructed. For this kind of system it has not yet been possible to obtain the propagation estimates needed to construct the Møller morphism. In fact it is not clear that the scattering approach applies in this case.

In the Liouvillean approach, NESS are related to resonances of a new kind of generator of the dynamics in the GNS representation: The C-Liouvillean. The main advantage of this method is that the required analysis can be performed in a Hilbert space setting. The technical difficulties are related to the fact that the C-Liouvillean is not self-adjoint on the GNS Hilbert space. We shall only describe the strategy here and refer the reader to [JP2] for detailed implementation.

We assume that ω is modular and work directly in the GNS representation $(\mathcal{H}_\omega, \pi_\omega, \Omega_\omega)$, identifying $\mathcal{O}$ with $\pi_\omega(\mathcal{O})$. Recall that σ_ω is the modular group of ω, J the modular conjugation and L, L_V the standard Liouvilleans of τ, τ_V. Denote by Δ_ω the modular operator. If $t \mapsto \sigma_\omega^t(V)$ is analytic in the strip $\{z \in \mathbb{C} \,|\, |\mathrm{Im}\, z| < 1/2\}$ and bounded continuous in its closure, then the C-Liouvillean of τ_V is the closed operator defined on the domain of L by
$$K_V = L + V - J\sigma_\omega^{-\mathrm{i}/2}(V)J.$$
Since $J\sigma_\omega^{-\mathrm{i}/2}(V)J \in \pi_\omega(\mathcal{O})'$ one easily checks that $\mathrm{e}^{\mathrm{i}tK_V} A \mathrm{e}^{-\mathrm{i}tK_V} = \tau_V^t(A)$. Moreover, since $L\Omega_\omega = 0$ it follows from modular theory that
$$K_V\Omega_\omega = V\Omega_\omega - J\Delta_\omega^{1/2} V \Delta_\omega^{-1/2} J\Omega_\omega = V\Omega_\omega - J\Delta_\omega^{1/2} V\Omega_\omega = (V - V^*)\Omega_\omega = 0.$$

Hence $\omega\circ\tau_V^t(A) = (\Omega_\omega|\mathrm{e}^{\mathrm{i}tK_V}A\Omega_\omega) = (\mathrm{e}^{-\mathrm{i}tK_V^*}\Omega_\omega|A\Omega_\omega)$ where $K_V^* = L + V - J\sigma_\omega^{\mathrm{i}/2}(V)J$.

Suppose there exists a Gelfand triplet $\mathcal{K}\subset\mathcal{H}_\omega\subset\mathcal{K}'$ and a dense subalgebra $\tilde{\mathcal{O}}\subset\mathcal{O}$ such that $\tilde{\mathcal{O}}\Omega_\omega\subset\mathcal{K}$ and

$$w^*\!-\!\lim_{t\to\infty}\frac{1}{t}\int_0^t \mathrm{e}^{-\mathrm{i}sK_V^*}\Omega_\omega\,\mathrm{d}s = \Psi\in\mathcal{K}'$$

holds in $\mathcal{K}'$. Then the functional $\tilde{\mathcal{O}}\ni A\mapsto(\Psi|A\Omega_\omega)$ extends by continuity to a state ω_+ on $\mathcal{O}$, and we can conclude that $\Sigma_+(\tau_V,\omega)=\{\omega_+\}$. Note that if $\Psi\in\mathcal{H}_\omega$ then ω_+ is ω-normal. Thus, we expect that $\Psi\notin\mathcal{H}_\omega$ in genuine non-equilibrium situations. Under appropriate conditions one can show that Ψ is a zero-resonance vector of K_V^* i.e., that there exists an extension of K_V^* to $\mathcal{K}'$ of which Ψ is a zero eigenvector. In [JP2] and more recently in [MMS] spectral deformation techniques have been used to gain perturbative control on the resonances of K_V^*. This yields a convergent expansion for the NESS ω_+ in powers of the coupling V which, to lowest order, coincides with the weak coupling (van Hove) limit studied in [LS]. It also gives the convergence $\nu\circ\tau_V^t(A)\to\omega_+(A)$ for all ω-normal states ν and all $A\in\mathcal{O}$ with a precise estimates on the exponential rate of convergence for dense sets of such ν and A.

Recent developments

Recent research developments in the study of NESS concern fluctuations, that is, the Quantum Central Limit Theorem for entropic observables [JPP], Quantum Large Deviation Principle(s), and extensions of Evans-Searles (ES) and Gallavotti-Cohen (GC) Fluctuation Theorems to quantum statistical mechanics. (For pioneering work on this subject we refer the reader to [TM], see also [Ku, Roe].) One difficulty this research program faced is that a natural dynamical system framework for the study of entropic fluctuations was already lacking on the classical level. Even the basic examples of thermostated systems and open systems were studied in the literature in an unrelated way and it was far from obvious which aspects of the theory are model dependent and which are universal. A general axiomatic dynamical system framework for the study of entropic fluctuations in classical statistical mechanics has been recently proposed in [JPR]. This framework has a direct extension to quantum statistical mechanics [JOPP]. It turns out that the quantum theory of entropic fluctuations is much richer then its classical counterpart, with a family of ES and GC Quantum Fluctuation Theorems indexed by a deformation parameter $q\in[0,1]$. The Fluctuation Theorems at $q=1$ directly link to the Large Deviation Principle for the Full Counting Statistics of a repeated quantum measurement of the energy/entropy transfer across the system (see [Roe] for references). From the mathematical point of view, the $q=1$ Fluctuation Theorems are naturally linked with the Araki-Masuda theory of non-commutative L^p-spaces [AM] with large deviation parameter equal to $1/2p$. The Fluctuation Theorems at $q=0$ are of the variational origin and reduce to the Onsager Reciprocity Relations and Green-Kubo fluctuation-dissipation formulae in the linear regime near equilibrium. The Fluctuation Theorems at $q=0$ and $q=1$ have a very different physical interpretation. The ones for $q\in]0,1[$ interpolate between these two extreme cases. Another surprise is a link between the $q = 1$ Fluctuation Theorems with Quantum Hypothesis Testing and Quantum Information Theory [HMO, JOPS].

The dynamical assumptions introduced in [JPR, JOPP] are ergodic in nature and in a certain sense minimal (that is, necessary) to have a meaningful theory. They are also typically very difficult to verify in physically interesting models. Nevertheless, the Quantum Fluctuation Theorems have been recently proven [AJOPP] for all models for which the existence of NESS has been established in the last decade (spin-fermion systems [JP2], directly coupled fermionic reservoirs [FMU, JOP2], electronic black-box models [AJPP2]).

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Herbert Spohn the Dannie Heineman laureate in mathematical physics

by Hans-Otto Georgii (Munich, Germany)

Hans-Otto Georgii is Emeritus Professor of Probability Theory at the Ludwig-Maximilian University of Munich. His main research interests are models of classical Statistical Mechanics and applications of Large Deviation Theory.

It is a pleasure to congratulate Herbert Spohn on receiving the 2011 Dannie Heineman Prize for Mathematical Physics, donated by the American Physical Society. The citation says "for his seminal contributions to nonequilibrium statistical mechanics as exemplified by his exact solutions of growth models and stationary states of open systems. Combining mathematical rigor with physical insight his work elucidates the transition from microscopic to macroscopic behavior." He follows on Michael Aizenman who received the last year's prize. In fact, Michael and Herbert were the first two PhD students of Joel Lebowitz, in the same year 1975.

As its former president in the period 2000–2002, Herbert Spohn is well-known to the IAMP community. He is also well-known as a member of the editorial boards of numerous journals, including the Communications in Mathematical Physics and the Journal of Statistical Physics. A further sign of his international reputation is the fact that in 1993 he was awarded with the Max Planck Research Award, which the German Max Planck Society confers on pairs of researchers, one from Germany and the other working abroad; his partner was Joel Lebowitz. Further stages of his career can be found on the prize's web page www.aps.org/programs/honors/prizes/heineman.cfm. Here I want to emphasize his constant commitment to the numerous PhD students and postdocs in his group, to whom he was, and is, not only an excellent advisor but also a friend. The number of 70 coauthors from many different nations, as listed in MathSciNet, demonstrates his worldwide scientific influence.

I consider Herbert as a prototype of the rather rare species of researchers who live for, and represent, the classical unity of Mathematics and Physics. He received his education in Physics, but now (since 1998) holds a chair for Applied Probability at the Mathematics Department of the Technical University Munich (TUM). This universality of mind, which soon made him well-known in the Mathematical Physics community, did not well fit into the traditional German system of separate disciplines. So it took some time until the chair

Herbert Spohn, Oberwolfach 2007

at TUM provided him with the adequate possibilities of building up a large research group and to attract numerous visitors. His activities at the forefront of research contribute much to making Munich an attractive place not only for me and the Mathematical Physics community in Munich, but in fact for many colleagues from all over the world. The Dannie Heineman Prize reflects our high esteem of these activities. Congratulations, Herbert!

We are happy to add to above the following (last minute) announcement.

AMS News Releases, January 2011.
http://www.ams.org/news/ams-news-releases/ams-news-releases

Providence, RI - **Herbert Spohn**, professor and chair of mathematical physics at the Technical University of Munich, has received the 2011 AMS Leonard Eisenbud Prize for Mathematics and Physics. Presented every three years by the American Mathematical Society, the prize honours a work or group of works that brings mathematics and physics closer together and that appeared within the preceding six years. The prize was awarded on January 7, 2011, at the Joint Mathematics Meetings in New Orleans.

Spohn is honoured for his group of works on stochastic growth processes:

- "Exact height distribution for the KPZ equation with narrow wedge initial condition" (with T. Sasamoto, Nuclear Phys. B 834 (2010), no. 3, 523-542);
- "One-dimensional Kardar-Parisi-Zhang equation: An exact solution and its universality" (with T. Sasamoto, Phys. Rev. Lett. 104, 230602 (2010)); and
- "Scaling limit for the space-time covariance of the stationary totally asymmetric simple exclusion process" (with P. L. Ferrari, Comm. Math. Phys. 265 (2006), no. 1, 1-44).

Our congratulations, Herbert !

The Editorial Board

QMath11 – the series continues

Keeping a conference series alive is not an easy thing. On one hand there has to be a community with the interest to meet from time to time, usually at different places, to share the joy of new results, on the other hand some members must be willing to accept the burden of organizing the gathering. The last factor kills some series after a few first meetings, and none has its existence guaranteed forever.

The conference I am writing here about belongs to a series which survived its teens and entered the third decade of its life. It started in 1987 in Dubna, with the author and Petr Šeba as midwives, and after two repetitions at the same place it commuted over various sites: Liblice, Blossin, Ascona, Prague, Taxco, Presqu'île de Giens, and Moeciu; the eleventh issue under the traditional title "Mathematical Results in Quantum Physics" found its place in the historical town Hradec Králové in Eastern Bohemia on September 6–10, 2010, having attracted about 140 participants from all corners of the world. Rather complete information about the meeting can be found at the conference page, http://qmath11.uhk.cz/

University of Hradec Králové

Twelve plenary lectures were read in the large lecture hall of the University of Hradec Králové new building. *Matthieu Lewin* spoke on renormalization of Dirac's polarized vacuum, *Elliott Lieb* on multi-polaron systems, and *Alessandro Giuliani* on renormalization group for interacting electrons on the honeycomb lattice. *Robert Sims* presented a talk on Lieb-Robinson bounds in many-body systems. Concerning random systems, *Simone Warzel* described a surprising result about phase diagram of the Anderson model on trees, and *Benjamin Schlein* spoke on bulk universality for Wigner matrices. With respect to quantum mechanics and spectral theory, *Olaf Post* discussed convergence results for "thick" graphs, *Rupert Frank* spoke on sharp constants in inequalities on the Heisenberg group, *Timo Weidl* on semiclassical estimates and their modifications, and *Vadim Kostrykin* on quadratic forms which are not semibounded. Finally, there were talks on quantum information, *Nilanjana Datta* discussed relative entropies and entanglement monotones, and *Karol Zyczkowski* spoke on entanglement in random systems.

In addition to that there were topical sessions on *Spectral theory* (organized by Michael Loss), *Many body quantum systems* (Marcel Griesemer), *Quantum chaos* (Tomaz Prosen), *Quantum field theory organized* (Dirk Kreimer), *Quantum information* (Julia Kempe), and finally, *Physics of social systems* (Petr Šeba). They typically had three invited talks and a number of contributed ones; the most heavily subscribed were spectral theory with nineteen contributions and quantum information with fourteen.

There was one more session, a special one devoted to the 60th birthday of *Ari Laptev* with the invited talks of Rafael Benguria, Leander Geisinger, Bernard Helffer, Arne Jensen, and Michael Loss; also plenaries of Elliott Lieb, Timo Weidl and Rupert Frank as well as some other contributions were dedicated to that occasion. The speakers praised not only Ari's mathematical results and his widely known and acknowledged service to the community, but recalled also his unusual life path demonstrating his character.

The conference had an inspirative atmosphere with numerous discussions, both in the talks and in coffee breaks, and during relaxed time reserved for social program such as a concert by students of a local musical school and a trip to a picturesque rock town of Adršpach and a brewery in Náchod. There were also discussions about a continuation of the series giving hope that the eleventh issue of the QMath series will not be its last.

Pavel Exner

Regensburg Conference 2010 "Quantum Field Theory and Gravity", September 28 – October 1, 2010

In 2003 Jürgen Tolksdorf founded a series of conferences on quantum field theory and gravity: conceptual and mathematical advances in the search for a unified framework. In the tradition of the meetings in Blaubeuren/South Germany (2003 and 2005) and Leipzig (2007), it was the goal of the Regensburg conference 2010 to bring together physicists and mathematicians working in quantum field theory and general relativity and to encourage scientific discussions on fundamental and conceptual issues. The Regensburg conference comprehended 22 plenary talks and two one-hour plenary discussions on research strategies. Given by invited speakers, the selected talks introduced different directions of research, in a way as non-technical and easily accessible as possible. The conference was also intended for young researchers on the graduate and post-graduate level. About 100 physicists and mathematicians attended the conference; 27 young scientists received financial support by several sources including IAMP and the German National Academy of Sciences Leopoldina. The plenary speakers were:

- Claus Kiefer (Köln),
- Klaus Fredenhagen (Hamburg),
- Christian Fleischhack (Paderborn),
- Blake Temple (University of California, Davis),
- Mu Tao-Wang (Columbia University, New York),
- Renate Loll (Utrecht),
- Daniele Oriti (Max-Planck Institute for Gravitational Physics Albert Einstein, Golm),
- Andreas Döring (Oxford),
- Thomas Elze (Pisa),
- Miguel Sanchez (Granada),
- Andreas Grotz (Regensburg),
- Robert Oeckl (Morella, Mexico),
- Christian Bär (Potsdam),
- Chris Fewster (York),
- Rainer Verch (Leipzig),
- Dieter Lüst (Max-Planck Institute for Physics Werner Heisenberg, Munich),
- Michael Dütsch (Göttingen),
- Michael Kiessling (Rutgers University, New Jersey),
- Stefan Hollands (Cardiff),
- Julian Barbour (Oxford),
- Jerzy Kijowski (Warsaw),
- Domenico Giulini (Hannover).

The lectures given at the conference will be published by Birkhäuser in a volume, which will continue the following two volumes: B. Fauser, J. Tolksdorf, E. Zeidler (eds.), Quantum Gravitation: Mathematical Models and Experimental Bounds, Birkhäuser, 2006, and Quantum Field Theory - Competitive Methods, Birkhäuser, 2008.

Regensburg

The conference was financially supported by IAMP, and the organizers are very grateful for this support.

Felix Finster
Jürgen Tolksdorf
Olaf Müller
Marc Nardmann
Eberhard Zeidler

News from the IAMP Executive Committee

New individual members

IAMP welcomes the following new members

1. Gerardo Adesso, School of Mathematical Sciences, University of Nottingham, Nottingham, UK
2. Stephan De Bievre, Department de Mathematiques Universite Lille 1 - Sciences et Technologies, Villeneuve d'Ascq, France
3. Sihon Crutcher, U.S. Army Research, Development, and Engineering Center, Meridianville, Alabama, USA
4. Franois Germinet, Department of Mathematics, University of Cergy-Pontoise, Cergy-Pontoise, France
5. Natalie Gilka, Department of Mathematics & Statistics, The University of Melbourne, Melbourne, Australia
6. H. Hogreve, IFISR - International Foundation for Independent Scientific Research, New York, New York, USA
7. Jan Kříž, Department of Physics, Faculty of Science, University of Hradec Kralove, Hradec Kralove, Czech Republic
8. Terry A. Loring, Department of Mathematics and Statistics, University of New Mexico, Albuquerque, New Mexico, USA
9. Maria Clara Nucci, Department of Mathematics and Informatics, University of Perugia, Perugia, Italy
10. Annalisa Panati, Centre de Physique Theorique, Universite de Toulon, Marseille France
11. Konstantin Pankrashkin, Laboratory of mathematics, University Paris Sud, Orsay France
12. G. Janardhana Reddy, Department of Mathematics, National Institute of Technology, Warangal, India
13. Constanza Rojas-Molina, Department of Mathematics, Laboratoire AGM University of Cergy-Pontoise, Cergy-Pontoise, France

Open positions

- Deadline for application March 31, 2011: Post-Doctoral position.
  - Duration: 1 or 2 years, starting in Fall 2011
  - Location: Mathematics department of the university of Cergy-Pontoise, France

  The candidate should have a strong background in the study of quantum systems, with tools from spectral theory, nonlinear analysis and PDEs, or numerical analysis. She/he is expected to work in the team of the ERC Starting Grant project MNIQS (Mathematics and Numerics of Infinite Quantum Systems)

  http://mniqs.math.cnrs.fr

and to interact with the other members of the project.

To apply, send a CV and a short description of your research to Mathieu LEWIN: Mathieu.Lewin@math.cnrs.fr

- Deadline for application February 28, 2011: Postdoctoral position in Random Matrix Theory and Related Fields

  http://www.hit.ac.il/ac/files/Eugene.Kanzieper/research/research.jobs.htm

  at the Department of Applied Mathematics, Holon Institute of Technology, Israel.

- Deadline for application Jan 31, 2011: Post-doctoral position in mathematical physics. The Mathematical Physics group at the Helsinki University is seeking a post-doctoral researcher in the field of non-equilibrium statistical mechanics. The position is funded through a European Research Council (ERC) Advanced Grant and will be up to 3 years. The researcher will be working in an active research environment including the Center of Excellence in Analysis and Dynamics http://mathstat.helsinki.fi/huippu.

  Applicants should send a CV and three letters of recommendation by email to antti.kupiainen@helsinki.fi. Deadline for application is January 31 2011 and position will be available September 1 2011. Earlier start date is also possible.

Recent conference announcements

With support from IAMP:

- June 30 – July 3, 2011, Signatures of Quantumness in Complex Systems, the University of Nottingham, School of Mathematical Sciences, Nottingham, UK

- February 14–19, 2011, School and Workshop on Mathematical Methods in Quantum Mechanics in Bressanone, Italy

- January 24–27, 2011, Workshop on Classical and Quantum Integrable Systems (CQIS-2011) http://sites.google.com/site/cqisihep/

Other conferences:

- May 25 –27, 2011, Operator theory and boundary value problems, Université Paris-sud, Orsay, France http://orsay2011.info

- April 24 – 29, 2011, Graphene Week 2011: Fundamental Science of Graphene and Applications of Graphene-Based Devices

  http://www.esf.org/index.php?id=7278

  Universitätszentrum Obergurgl (Ötz Valley, near Innsbruck), Austria. A conference of The European Science Foundation (ESF) in partnership with Leopold-Franzens-Universität Innsbruck. Chaired by Prof. Vladimir Falko, Lancaster University, UK;

Prof. Andre Geim, University of Manchester, UK (2010 Nobel Prize in Physics); Prof. Karsten Horn, Fritz-Haber-Institut Berlin, DE & Prof. Sankar Das Sarma, University of Maryland, US.

Announcement of the International Scientific Committee for ICMP12

The 12th International Congress in Mathematical Physics

http://www.icmp12.com/

will be held in Aalborg, August 6-11, 2012. The international Scientific Committee has been selected: Pavel Exner (President IAMP), Krzysztof Gawedzi, Arne Jensen (local organizer), Herbert Spohn, Reinhardt Werner, Horng-Tzer Yau, Lai-Sang Young.

Jan Philip Solovej (IAMP secretary)

Claudio D'Antoni

Claudio D'Antoni passed away on October 29, 2010, at the age of sixty, after a sudden and rare illness. Claudio was Professor of Analysis at the University of Roma "Tor Vergata".

Claudio initially took a degree in physics but was afterwards attracted to mathematics, too. He always bore physical motivation in mind when choosing a research problem. His interaction with the German school in Local Quantum Field Theory during his von Humboldt Fellowship that started in 1988 proved particularly fruitful.

In the course of his scientific career, Claudio, with his insights, made several contributions, mainly to mathematical physics and related problems in operator algebras. We would mention his study of nuclearity conditions and of the scaling limit structure. He also made contributions of a different nature, as testified by his papers on multiplier C*-algebras.

Claudio had an incredible culture, in particular a scientific culture, that he shared with pleasure with collaborators, friends and colleagues.

He was reserved and discreet but still open to and with an interest in others. The picture here shows him in a relaxed moment at a conference in Timişoara in 2004.

Claudio was a pleasant person and a helpful colleague, open-minded in science and in scientific collaboration. He leaves his wife, two daughters and many friends. We shall all remember him with affection.

Sergio Doplicher
Francesco Fidaleo
Daniele Guido
Tommaso Isola
Roberto Longo
Gerardo Morsella
Nicola Pinamonti
Claudia Pinzari
Florin Radulescu
John Roberts
Giuseppe Ruzzi
Laszlo Zsido