

International Association of Mathematical Physics



# News Bulletin

October 2012



# International Association of Mathematical Physics News Bulletin, October 2012

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## Contents

Message from the President	3
The XVIIth International Congress on Mathematical Physics	6
The XVIIIth International Congress on Mathematical Physics in Santiago de Chile	8
Annales Henri Poincaré Prize Ceremony in Aalborg	14
2D Coulomb Gas, Abrikosov Lattice and Renormalized Energy	17
Phase transitions in $k$ -mer systems	27
Letter to the Editor	35
Rudolf Haag – Celebrating 90 Years	37
A celebration for Giovanni Gallavotti's 70th birthday	41
The Poincaré Seminar	43
Message from the Treasurer	47
News from the IAMP Executive Committee	52

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*Cover photo:* Laureates celebrated on the XVIIth ICMP in Aalborg (from left to right): Barry Simon, Sylvia Serfaty, Freeman Dyson, Artur Avila, Alessandro Giuliani, Wojciech de Roeck, Ivan Corwin.

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News Bulletin (International Association of Mathematical Physics)

## After Aalborg

by ANTTI KUPIAINEN (IAMP President)



The high point of the three year life cycle of our association is undoubtedly our Congress, which was held this year in Aalborg, Denmark. Together with the immediately preceding Young Researchers Symposium, the Congress was an eight day festival of mathematical physics with 16 plenary talks and over a hundred invited and contributed talks in up to four parallel sessions with topics ranging from string theory to the theory of computation and neuroscience. Its participants included a delightfully large number of young scientists, students and postdocs. Indeed, the Congress was more youthful than any ICMP I can remember. Mathematical physics continues to attract good young scientists in a wide spectrum of fields - a fact we should appreciate.

Different generations were present among our prize laureates as well. Two of the Henri Poincaré prizes went to scientists that have been prominent in our field for six and four decades respectively: Freeman Dyson and Barry Simon. The two other Poincaré prizes were received by young stars, Nalini Anantharaman and Sylvia Serfaty, and the IAMP Early Career Award was given now for the second time, to Artur Avila. The recipients of the three IUPAP Young Scientist Prizes in mathematical physics were Ivan Corwin, Alessandro Giuliani and Wojciech de Roeck.

As is customary, the IAMP General Assembly was held the first evening of the Congress. This is the only meeting during our three year cycle where members have a chance to participate directly in our decision making. In particular it includes the reports by the president and the treasurer on the activities during the past three years.

For my part I was reporting mostly about the activities of the previous Executive Committee that finished its work in 2011. This was a pleasant job indeed! During the past three years significant reforms and new initiatives were taken in our association. Here are the most important ones:

- A major effort was the revision of membership. This reduced the “official” membership by half, from 1386 to 681 but almost doubled the number of paying members from 373 to 599. In particular many “dormant” members were activated by the amnesty of past fees. There were also 5 new Associate members (total 11) each paying 270/year. All this has led to a considerable improvement of our finances, as is discussed in the treasurer’s report in this bulletin.
- Another very important development was the reform of the IAMP Bulletin. It is now a very attractive journal coming out four times a year. Thanks go to chief editor Valentin Zagrebnov and the whole enthusiastic editorial board, Evans Harrell, Masao Hirokawa, David Krejčířík and Jan Philip Solovej. Valentin has agreed to

stay in charge for the next three year period. David Krejčířík will step down, and Rafael Benguria will replace him. Robert Sims will also join the board as well as the IAMP secretary Manfred Salmhofer ex officio.

- A similar change took place in the web page whose appearance and contents were thoroughly revised and extended by David Jörg and Manfred Salmhofer and which is maintained by Jan-Philip Solovej.
- A new web based database was set up by Volker Bach and Dietmar Kähler. This has among other things membership information, pages for payment of fees and donations.
- A joint conference series was established with EMS. The first one was held at the Erwin Schrödinger Institute in Vienna last summer.
- Conference support has increased: 13 conferences were supported in the last 3 years compared with 10 in the previous period. With improved finances, more expansion is ahead.
- IAMP has been active in the community, e.g. in the effort to help save ESI in Vienna.

So quite a lot was achieved during the past 3 years. Credit for this goes to the previous EC and especially to the previous president, Pavel Exner, who was a driving force behind many of these reforms.

However we should not rest on our laurels. In particular considering membership there are still many problems. Only one third of the Aalborg congress participants were IAMP members. Among speakers the percentage was a bit better - a little over 40. Thus many in our community haven't found it natural to join IAMP.

- In particular we need more young members. Our senior members should take it as their duty to recruit their students, postdocs and younger colleagues. Judging from the last Congress there is a large potential here.
- Our geographical spread is biased. Currently IAMP has a poor representation in Asia: we have a few Japanese members but almost no Chinese. Similarly Latin America is almost absent. There should be a big potential to expand here.
- There is a real danger that our scope is viewed as much too narrow, basically being dominated by quantum mechanics and statistical physics. Indeed, looking at the present EC all of us are essentially working in these fields. Also, among Aalborg speakers IAMP membership was 81% in the combined sections of Statistical Mechanics, Quantum Mechanics, Quantum Field Theory and Many Body Theory and only 13% in all the other sections!

However, there is a lot more in mathematical physics than our core fields as was evident in the congress. In particular Dynamical Systems and PDE's have for a long time been major fields in our community but there is no representation for them in our EC. Algebraic and geometric mathematical physics has been hugely influential for now more than 30 years. They have been present in our Congresses but not in our EC. Some of the most exciting mathematical physics during the past 15 years has been done by probabilists and analysts who have found our problems exciting and fruitful. None of this is represented in our EC. Ditto quantum information, . . . , the list is long.

We need to reach out to these and new emerging fields to ensure our continuity and relevance. We need to convince the people who find our field attractive that apart from being analysts or probabilists or string theorists they also are mathematical physicists. We are an interdisciplinary community and therefore we need constantly to balance between specialization and spreading out. Specialization improves sense of community, but may lead to dropping away from the most exciting developments. Spreading out has opposite effects. Today we are too specialized.

We should try to start rectifying this imbalance in the next EC ballot by recruiting attractive candidates in a wide spectrum of fields. We should pay careful attention when appointing the prize committees for the next round of HP and ECA prizes as well as when choosing the scientific committee for ICMP 2015.

The major decision made by the GA was of course the choice of Santiago, Chile as the site for ICMP 2015. The EC, the GA and the whole congress were impressed by Rafael Benguria's enthusiastic presentations of the venue and the local organizers. You can judge for yourself by reading his article in this bulletin. Only one out of six IAMP members showed up in Aalborg. I sincerely hope this percentage will be much higher in Santiago!

One final remark concerning our congress. Now is the time to start thinking about making bids for ICMP 2018. ICMP is the main activity of our association. There is no ICMP without IAMP but also conversely, without ICMP there is little rationale for IAMP. Organizing a congress of this scale is not a small job, but it is a great service to our community. For ICMP 2015 we had only one bid, which luckily turned out to be outstanding. It would be important to have several bids for 2018. I would especially like to encourage bids from Asia and North America, where ICMP has taken place too rarely.

## The XVIIth International Congress on Mathematical Physics

by ARNE JENSEN (Convenor of ICMP 2012, Aalborg)



The seventeenth Congress of the IAMP was held in Aalborg, Denmark, August 6–11, 2012. It was preceded by the Young Researcher Symposium (YRS) August 3 and 4.

Here are some of my observations and impressions from the two events. The YRS had app. 90 participants, most of them young researchers. There were three plenary talks of two hours each, by Klaus Mølmer (Aarhus), Robert Seiringer (Montreal), and Tom Spencer (Princeton). In the afternoons there were three parallel sessions with a total of 45 talks by the young participants. My impression was of a very active and attentive audience for the plenary lectures. All the contributed talks attracted a good audience. A lasting influence of the YRS will be the personal contacts established among the participants.

The main event, the Congress, had app. 350 participants. Of those app. 100 were IAMP members. Thus one has to realize that the majority of the participants were not members of the IAMP. But I hope they have become members by now.

The Congress started with opening remarks by the IAMP president, Antti Kupiainen, and a welcome address from the Dean of the Faculty of Engineering and Science at Aalborg University, Eskild Holm Nielsen. Then the prize ceremonies followed. You can read more about the eight prizes awarded in this issue of the Bulletin.

I want to tell you what happened during the ceremony. Aalborg was hit by torrential rain Monday morning, and half an hour into the ceremony the water penetrated the roof and cascades of water hit the floor. Fortunately few people were actually hit by the water. The efficient staff immediately started cleaning up, while the ceremony continued. Certainly a drama that will not be forgotten.

The plenary lectures were given in the morning and the invited and contributed topical session talks in the afternoon. There were 16 plenary talks, app. 55 invited talks, and app. 60 contributed talks. If you want to see the detailed program, go to the web site [www.icmp12.com](http://www.icmp12.com), which will continue to be available in the coming years. The slides from the plenary lectures are available on this site.

It was my impression that the lower attendance figure did not limit the very active exchange of ideas and discussions during the breaks.

One novelty at the Aalborg Congress was the e-posters. The posters were submitted as presentations and loaded onto computers with a large display. A menu system allowed one to view any presentation at any time. After a few days people got used to the system and it was used extensively. Certainly the presentations were of a much higher quality than traditional paper posters.

We are now in the process of editing the proceedings, and they will be available Spring 2013, published by World Scientific. The registered participants will receive a personal



copy.

I look forward to meeting many of the IAMP members at the next ICMP, in Santiago de Chile in 2015.



Dean Eskild Holm Nielsen addressing the Congress at the opening ceremony

## The XVIIIth International Congress on Mathematical Physics in Santiago de Chile

by RAFAEL BENGURIA (Convenor of ICMP2015, Santiago de Chile)

*At this moment, by an undeserved stroke of fortune,  
I am the direct voice of the poets of my race and the indirect  
voice for the noble Spanish and Portuguese tongues,*

**Gabriela Mistral** (Chilean Poet)

Nobel Banquet, Stockholm, Dec. 10, 1945.

The International Congress of Mathematical Physics (ICMP), on its three year cycle, is the most important event of the International Association of Mathematical Physics. The first time I participated in one of these meetings was in Lausanne, in August 1979, soon after I had obtained my Ph.D. It was the first time I had a chance to participate in a major international conference. It was a great experience, and since then, I have participated in most of the following ICMP's.

Following the statutes of the IAMP, the site of the next Congress, in this case ICMP 2015, was decided in the recent ICMP held in Aalborg. Thus, the XVIIIth ICMP will take place in Santiago de Chile, in August 3-8, 2015 and, as has been the tradition since ICMP 2000 (London), it will be preceded by the Young Researchers Symposium (YRS).

### Venue

We are planning to hold the meeting at the “Centro de Extensión” of the P. U. Católica de Chile, in downtown Santiago. This convention center is located near “Barrio Lastarria”, around the Santa Lucía Hill, where the city was founded in 1541. Nearby this Convention Center there are several hotels, as well as hostels, of many different prices. Along the narrow streets of Barrio Lastarria there are restaurants, coffee shops, museums and art galleries, and nearby there is the Santa Lucía park, and Parque Forestal. The Convention Center is conveniently located next to a subway station on Line 1.

### The Young Researchers Symposium

The YRS-2015 will take place on Friday, July 31st and Saturday, August 1st, at the Campus San Joaquín of the P. Universidad Católica de Chile (PUC). The Campus San Joaquín of PUC, is a nice university campus (of  $\approx 50$  Ha), located at approximately 15 minutes by subway from Plaza Baquedano (a central landmark in downtown Santiago). In fact, this campus is located immediately to the east of the San Joaquín subway station, on Line 5. It has a good infrastructure, with well equipped auditoriums, a mathematical



physics library (Biblioteca Gauss), and access to computer facilities and Wi-Fi connection. Both the Physics Department and the Mathematics Department at PUC (which are close in distance) have meeting rooms for discussions.



Park on the Santa Lucía Hill

## Congress Reception, “Museo de Arte Precolombino”

The Precolombian Museum, doubtless the city’s best museum, is located in the old palace of the “Real Aduana” (built in 1805), a block away from the main square (Plaza de Armas) in downtown Santiago. It exhibits an exquisite collection of pottery from major pre-Colombian cultures, intricately molded anthropomorphic vessels, a hefty Mayan stone stele, and a fascinating Andean textile display. The Museum has kindly agreed to offer its facilities to ICMP 2015, in order to hold the Official Reception and Cocktail on the evening of Monday, August 3, 2015.

## Local Organizing Committee

The Congress Chairs as well as the Scientific Committee are to be decided by Executive Committee of the IAMP. At present, the Local Organizing Committee consists of Rafael Benguria (Chair), Matías Coudurier, Alejandro Ramírez, Roberto Rodríguez, Mariel Saez

and Rafael Tiedra, from the P. Universidad Católica de Chile; Eduardo Friedman, Alejandro Jofré, and Axel Osses, from Universidad de Chile, Andrés Navas from Universidad de Santiago de Chile and Jorge Zanelli from CECS in Valdivia. This Local Organizing Committee has members of many different areas of mathematics (related to physics). Even though Gunther Uhlmann is not part of the local organizing committee (since he works in the US), he has agreed to collaborate with us in the organization of ICMP 2015.

## About Santiago de Chile

Santiago de Chile is the capital as well as the largest city of Chile. It is located in the country's central valley, at an elevation of 520 meters (in downtown Santiago) (i.e., approximately 1706 feet above sea level). Chile is very long (approximately 4,200 km from North to South) and very narrow (approximately 180 km in average from East to West). Hence, Santiago is close to both the Andes (less than one hour away to several ski resorts), and the Pacific Ocean (approximately one and a half hour away from Valparaíso). The highest mountain one can see from Santiago is Cerro El Plomo (5434 m, i.e., approximately 17,828 ft), with a permanent glacier on the top. Chile's steady economic growth has transformed Santiago into a modern metropolitan area, with extensive suburban development, dozens of shopping centers, and an impressive high-rise architecture. It has a modern transportation infrastructure, including the steadily-growing underground (METRO, with 5 existing lines and two more under construction). There are three main research universities in Santiago (Universidad de Chile, the oldest, was founded in 1842, P. U. Católica de Chile was founded in 1888, and U. de Santiago de Chile goes back to 1849), and several other research universities across the country. During the winter months there are ballet, concerts and opera in Santiago at the Municipal Theater (Teatro Municipal de Santiago), at the Teatro de la Universidad de Chile, at the Teatro Municipal de Las Condes, and others.

## Climate in Santiago in August

Recalling that Santiago is in the southern hemisphere (Latitude 33.45 degrees South, Longitude 70.67 degrees West), during August we are still in winter. The average high temperature in August is 16.7 C (62.1 F), and the average low is 4.8 C (40.6 F). The average precipitation in August is 51.8 mm (2.06 inches). On average there are approximately 140 hours of sunshine during that month.

## Connectivity

Santiago de Chile is well connected, with direct flights, to North America, Central and South America, Europe and Australia. Through Buenos Aires and Sao Paulo it is also well connected with Africa and the Middle East. Through Australia, the US or Europe, Santiago is well connected with the rest of Asia. Visas are not required for nationals

from most countries in Europe, South America, Japan and other countries in Asia. For nationals from other countries, getting visas is easy.



Zapallar, Chile. ©Michaël Lejeune, CC-BY-SA-2.5, Wikimedia Commons

## Activities for Accompanying Persons

Central Chile has many different attractions. Within an hour from Santiago there are world class ski resorts, like Valle Nevado, and others. Chile is famous for its wines, and near Santiago you can explore many vineyards in the Maipo Valley, in the Casablanca Valley, the Cachapoal Valley and many others, or you may want to visit the “Colchagua Museum” in Santa Cruz, on the Colchagua Valley. An hour and a half away from Santiago you can visit Valparaíso, Viña del Mar, as well as many resorts on the coast of the Pacific. You may also want to visit the house of the poet Pablo Neruda in Isla Negra.





Valle Nevado Ski Resort

## Invitation

Chile is a remote country, almost at the end of the world. In the distant past, coming from Europe to Valparaíso around Cape Horn was a traveler's dream. Non-fiction books, like the narrative of John Byron (the grandfather of the poet) written in 1746, or the "Journal of a Residence in Chile during the Year 1822" written by Mary Graham (1824), and others, inspired many. Also, famous fiction books like Daniel Defoe's "Robinson Crusoe" (1719) or Jules Verne's, "Les Enfants du Capitaine Grant" (1867), also served that purpose. The best known traveler's book about Chile, though, is Charles Darwin's "The voyage of the Beagle" (1837). Perhaps most people know Chile through its poets, its wine, its wonderful mediterranean climate, its diverse landscape, its many beautiful National Parks, or the large observatories in the north of the country. However, we would also like Chile to be known by the quality of its scientific research. On behalf of the Local Organizing Committee, we look forward to seeing you all in Chile in 2015 !



Torres del Paine, National Park

## Annales Henri Poincaré Prize Ceremony in Aalborg

by KRZYSZTOF GAWĘDZKI (Editor-in-Chief, Annales Henri Poincaré)

The journal **Annales Henri Poincaré** was created in the year 2000 by merging “Annales de l’Institut Henri Poincaré, physique théorique” and “Helvetica Physica Acta”, publications with long tradition, existing since 1930 and 1928, respectively. Managed jointly by Institut Henri Poincaré and by the Swiss Physical Society and published by Birkhäuser-Springer, under the direction of Vincent Rivasseau, its editor-in-chief from 2000 to 2012, AHP became one of the leading journals in mathematical and theoretical physics, printing annually about 1600 pages. In order to attract outstanding authors, in particular young ones, and to publicize their achievements, AHP awards each year the AHP Prize, funded by Birkhäuser, for the most remarkable articles published in the journal. The winners are chosen by the Editorial Board in a three-stage procedure: nominations-evaluation-voting. The prize-winning articles, from 2007 together with the Distinguished Papers that almost made it, are freely accessible at the website of AHP <http://www.springer.com/birkhauser/physics/journal/23>. The prizes are handed out at special events accompanied by a short scientific session where the winners present their results. That is usually done on the occasion of IAMP Congresses in order to profit from the affluence of mathematical physicists. The last of these events took place on August 8 in Aalborg at the margin of ICMP 12. Barbara Hellriegel, representing Birkhäuser, and the author of these words had the pleasure of handing out medals accompanying the 2008-2010 AHP Prizes to the following awardees:

- for the year 2008, jointly, to Péter Bálint and Imre Péter Tóth for the paper *Exponential decay of correlations in multi-dimensional dispersing billiards* and to Leonid Parnovski (absent in Aalborg) for the paper *Bethe-Sommerfeld conjecture*;
- for the year 2009, to Dmitry Dolgopyat and Bassam Fayad for the paper *Unbounded Orbits for Semicircular Outer Billiard*;
- for the year 2010, to Jean-Marie Barbaroux, Thomas Chen, Vitali Vougalter and Semjon Wugalter (represented in Aalborg by the latter) for the paper *Quantitative Estimates on the Binding Energy for Hydrogen in Non-Relativistic QED*.



AHP Prize medal



P. Bálint and I. P. Tóth





D. Dolgopyat, B. Fayad, B. Hellriegel and K. Gawędzki



S. Wugalter

The diplomas for Distinguished Papers were attributed:

- for the year 2008, to Jérémie Faupin for the paper *Resonances of the Confined Hydrogen Atom* and to Frédéric Hérau, Michael Hitrik and Johannes Sjöstrand for the paper *Tunnel Effect for Kramers-Fokker-Planck Type Operators*;
- for the year 2009, to Marcel Griesemer and David G. Hasler for the paper *Analytic Perturbation Theory and Renormalization Analysis of Matter Coupled to Quantized Radiation*;
- for the year 2010, to Gabriel Rivière for the paper *Entropy of Semiclassical Measures for Nonpositively Curved Surfaces* and to Wojciech De Roeck, Jürg Fröhlich and Alessandro Pizzo for the paper *Absence of Embedded Mass Shells: Cerenkov Radiation and Quantum Friction*.



M. Griesemer, D. G. Hasler and B. Hellriegel



W. De Roeck

The accompanying scientific session was composed of three 30 min. presentations by Imre Péter Tóth, Bassam Fayad and Semjon Wugalter. The talks gave an opportunity to Congress participants and to distinguished associates of AHP, that showed up in numbers despite of an already vast ICMP program, to get a view of the AHP laureates and of their work.

At the end of the session, Bertrand Duplantier, a long-date friend of AHP, took a few minutes to present the activity of **Séminaire Poincaré**, a remarkable biannual series of talks devoted to topics of great importance in physics, with written English accounts published in the collection “Progress in Mathematical Physics” of Birkhäuser.

Pictures courtesy of Birkhäuser and Georg Kopsky

## 2D Coulomb Gas, Abrikosov Lattice and Renormalized Energy

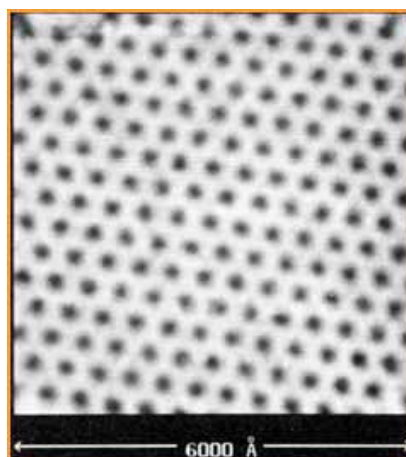
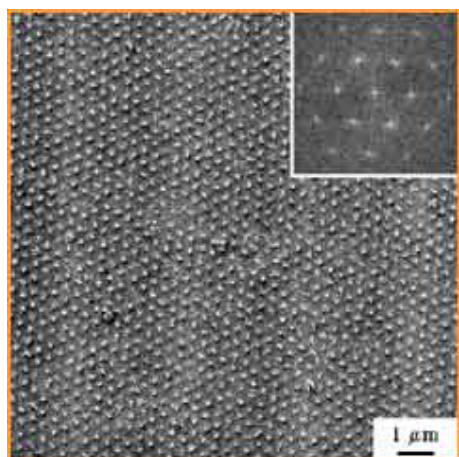
by SYLVIA SERFATY (Recipient of the Henri Poincaré Prize)



*Sylvia Serfaty is a Professor of Mathematics at the Université Pierre et Marie Curie - Paris 6, and a Global Distinguished Professor at the Courant Institute of Mathematical Sciences, New York University. She studied at the Ecole Normale Supérieure in Paris and earned her PhD in mathematics in 1999 at the University of Paris Sud -Orsay. She then was a CNRS researcher, and on the faculty at the Courant Institute of New York University from 2001 to 2008. She was an Invited Speaker at the International Congress of Mathematicians in 2006. She received an NSF Career award in 2003, a European Mathematical Society Prize in 2004, a EURYI award in 2007, and the Henri Poincaré Prize of the IAMP in 2012. Her work has addressed PDE and variational models coming mostly from physics, and particularly vortices and phase transitions in the Ginzburg-Landau model of superconductivity. More recently she has gotten interested in Coulomb systems.*

I would like to describe in this article some research done with Etienne Sandier, which has led us from the study of the Ginzburg-Landau equation of superconductivity – a rather involved system of PDE – to that of a well-known statistical mechanics system: namely the classical Coulomb gas.

A type-II superconductor, cooled down below its critical temperature, has a particular response in the presence of an applied magnetic field. Above a certain value of the external field called the *first critical field*, vortices appear. When the field is large enough, the experiments (dating from the 60's) show that they arrange themselves in (often) perfect triangular lattices, cf. <http://www.fys.uio.no/super/vortex/>.



Abrikosov lattices

These are named *Abrikosov lattices* after the physicist Abrikosov who had predicted, from the Ginzburg-Landau model, that vortex lattices should appear [A]. These vortices repel each other like Coulombic charges, while being confined inside the sample by the applied magnetic field. Their triangular lattice arrangement is the result of these two opposing effects.

The Ginzburg-Landau model was introduced on phenomenological grounds by Landau and Ginzburg in the 50's [GL] ; after some nondimensionalizing procedure, in a two-dimensional domain it takes the form

$$GL_\varepsilon(\psi, A) = \frac{1}{2} \int_{\Omega} |(\nabla - iA)\psi|^2 + |\nabla \times A - h_{\text{ex}}|^2 + \frac{1}{2\varepsilon^2} (1 - |\psi|^2)^2, \quad (1)$$

where  $\psi$  is a complex-valued “order parameter,”  $A$  is the gauge-field,  $h_{\text{ex}}$  the intensity of the applied magnetic field, and the parameter  $\varepsilon$  is the inverse of the “Ginzburg-Landau parameter”, a material constant equal to the ratio between the penetration length of the magnetic field, and the vortex core size. For full details on the model, we refer to [GL, DeG, FH, SS1].

The Ginzburg-Landau model has led to a large amount of theoretical physics literature — probably most relevant to us is the book by De Gennes [DeG]. However, a precise mathematical proof of the phase transition at the first critical field, and of the emergence of the Abrikosov lattice as the ground state for the arrangement of the vortices was still missing.

In the 90's, researchers coming from nonlinear analysis and PDEs became interested in the associated Ginzburg-Landau equation (precursors were Berger, Rubinstein, Schatzman, Baumann, Phillips...), with the notable contribution of Bethuel-Brezis-Hélein [BBH] who introduced systematic tools and asymptotic estimates to study vortices, but in the simplified Ginzburg-Landau equation not containing the magnetic gauge, and allowing only for a fixed number of vortices. It was however not clear that this approach could work to treat the case of the full magnetic model. It is only with the works of Sandier [Sa] and Jerrard [Je] that tools capable of handling unbounded numbers of vortices (as  $\varepsilon \rightarrow 0$ ) started to be developed. Relying on and expanding these tools, together with Etienne Sandier, in a series of works later revisited in a book [SS1], we started to analyze the full model and obtained the proof of the phase transition, and the computation of the asymptotics of the first critical field in the limit  $\varepsilon \rightarrow 0$ . We characterized the optimal number and distribution of the vortices and derived in particular a “mean-field regime” limiting distribution for the vortices, which I will describe just below. Note that this analysis and in particular the tools developed to understand the vortices have proven useful to study vortices in rotating superfluids like Bose-Einstein condensates (cf. e.g. [CPRY] and references therein), a problem which bears a large similarity to Ginzburg-Landau from the mathematical perspective, and of current interest for experiments.

In the Ginzburg-Landau model, an important quantity is the function  $h_\varepsilon(x) = \nabla \times A_\varepsilon(x)$  corresponding to the intensity of the *induced* magnetic field at the point  $x$  in the sample  $\Omega$  and which satisfies what is called in this context the *London equation*

$$\begin{cases} -\Delta h_\varepsilon + h_\varepsilon = 2\pi \sum_i d_i \delta_{a_i}^{(\varepsilon)} & \text{in } \Omega \\ h = h_{\text{ex}} & \text{on } \partial\Omega, \end{cases} \quad (2)$$

where  $\{a_i\}_i$  is the collection of the vortex centers,  $d_i \in \mathbb{Z}$  the topological *degrees* of these vortices, and  $h_{\text{ex}}$  the intensity of the external applied field. The London equation is written with true Dirac masses, but the exact right-hand side in (2) is a sum of quantized charges which should be thought of as somehow smeared out at a scale of order  $\varepsilon$ . Some computations (with the help of all the mathematical machinery developed to describe vortices) lead eventually to the conclusion that everything happens as if the Ginzburg-Landau energy  $GL_\varepsilon$  of a configuration were equal to

$$\begin{aligned} GL_\varepsilon(\psi_\varepsilon, A_\varepsilon) &\simeq \frac{1}{2} \int_{\Omega} |\nabla h_\varepsilon|^2 + |h_\varepsilon - h_{\text{ex}}|^2 \\ &= \frac{1}{2} \iint_{\Omega \times \Omega} G_\Omega(x, y) \left( 2\pi \sum_i d_i \delta_{a_i}^{(\varepsilon)} - h_{\text{ex}} \right)(x) \left( 2\pi \sum_i d_i \delta_{a_i}^{(\varepsilon)} - h_{\text{ex}} \right)(y), \end{aligned} \quad (3)$$

where  $G_\Omega$  is a Green kernel, solution to

$$\begin{cases} -\Delta G_\Omega + G_\Omega = \delta_y & \text{in } \Omega \\ G_\Omega = 0 & \text{on } \partial\Omega, \end{cases} \quad (4)$$

and  $h_\varepsilon$  solves (2). With this way of writing, and in view of the logarithmic nature of  $G_\Omega$ , one recognizes essentially a pairwise Coulomb interaction of positive charges in a constant background. It remains to understand for which value of  $h_{\text{ex}}$  vortices become favorable, and with which distribution. To really understand that, the effect of the “smearing out” of the Dirac charges needs to be more carefully accounted for. In the end, one finds that the mean-field limit energy arising from (3) is

$$\begin{aligned} \frac{GL_\varepsilon}{h_{\text{ex}}^2} &\simeq_{\varepsilon \rightarrow 0} \mathcal{E}^{MF}(h) = \frac{1}{2\lambda} \int_{\Omega} |-\Delta h + h| + \frac{1}{2} \int_{\Omega} |\nabla h|^2 + |h - 1|^2 \\ &= \frac{1}{2\lambda} \int_{\Omega} |\mu| + \frac{1}{2} \iint_{\Omega \times \Omega} G_\Omega(x, y) d(\mu - 1)(x) d(\mu - 1)(y), \end{aligned} \quad (5)$$

where  $\lambda = \lim_{\varepsilon} \frac{h_{\text{ex}}}{|\log \varepsilon|}$  is the suitably normalized intensity of the external field. Here  $h = \lim_{\varepsilon \rightarrow 0} \frac{h_\varepsilon}{h_{\text{ex}}}$  is here related to the limiting “vorticity”  $\mu = \lim_{\varepsilon \rightarrow 0} \frac{1}{h_{\text{ex}}} 2\pi \sum_i d_i \delta_{a_i}$  by  $-\Delta h + h = \mu$  in  $\Omega$  with  $h = 1$  on  $\partial\Omega$ .

In (5) the first contribution to the energy is the total self-interaction of the vortices in (3) (which would have been formally infinite if considering true Dirac masses), while the second one is the cross-interaction of the vortices. Minimizing  $\mathcal{E}^{MF}$  leads to finding an optimal distribution of vorticity which is uniform on a subdomain of  $\Omega$  depending on  $\lambda$  (and determined by a classical “obstacle problem”). For more details on all this, see [SS1, SSproc].

But this is still far from explaining the optimality of the Abrikosov lattice. To (begin to) explain it, one needs to look at the next order in the energy asymptotics (5), and at the blown-up of (2) at the inverse of the intervortex distance scale, which here is simply  $\sqrt{h_{\text{ex}}}$ . Once this blow-up is performed and the limit  $\varepsilon \rightarrow 0$  is taken, (2) becomes

$$-\Delta H + 1 = 2\pi \sum_a \delta_a \quad \text{in } \mathbb{R}^2, \quad (6)$$



where the limiting blown-up points  $a$  form an infinite configuration in the plane, and these are now true Diracs (one may in fact reduce to the case where all degrees are equal to  $+1$ , other situations being energetically too costly). We will see below how to compute the associated next order energy in the context of the Coulomb gas. Note that one may recognize here essentially a *jellium* of infinite size, and  $E = \nabla H$  the electric field generated by the points (its rotated vector field  $j = E^\perp$  corresponds to the superconducting current in superconductivity). A jellium denotes a set of point charges with identical charges with Coulomb interaction, *screened* by a uniform neutralizing background. It is also called a *one-component plasma*.

This connection with the jellium is what prompted us to examine in [SS3] the consequences that our study could have for 2D classical Coulomb gases. More precisely, let us consider a 2D Coulomb gas of  $n$  particles in a confining potential  $V$  (growing sufficiently fast at infinity), and let us take the mean-field regime of interaction where the Hamiltonian is given by

$$w_n(x_1, \dots, x_n) = - \sum_{i \neq j} \log |x_i - x_j| + n \sum_{i=1}^n V(x_i). \quad (7)$$

(At least if the potential  $V$  is homogeneous, one may always reduce to this case by scaling.) Note that ground states of this energy are also called “weighted Fekete sets”, they arise in interpolation, cf. [ST]. Among interacting particle systems, Coulomb gases have always been considered as particularly interesting but delicate, due to the long range nature of the interactions (which is particularly true in 1 and 2 dimensions). The case of one-dimensional Coulomb gases can be solved more explicitly [BL, AM], and crystallisation at zero temperature is established. In dimension 2, many studies rely on a rather “algebraic approach” with exact computations (e.g. [Ja2]), or require a finite system or a condition ensuring local charge balance [AJ, SM]. Our approach is strictly energy-based and this way valid for any temperature and general potentials  $V$ .

A simple analysis of (7) leads to the following mean-field limit for  $w_n/n^2$  as  $n \rightarrow \infty$ , analogous to (5):

$$\mathcal{F}(\mu) = \int_{\mathbb{R}^2 \times \mathbb{R}^2} -\log |x - y| d\mu(x) d\mu(y) + \int_{\mathbb{R}^2} V(x) d\mu(x) \quad (8)$$

defined for  $\mu$  a probability measure. The unique mean-field minimizer, which is also called the *equilibrium measure* in potential theory is a probability density  $\mu_0$  (it can also be viewed as the solution of an obstacle problem). For example if  $V(x) = |x|^2$ , it is a multiple of the characteristic function of a ball (the *circle law* for the Ginibre ensemble in the context of random matrices), and this is analogous to the obstacle problem distribution found for Ginzburg-Landau.

The connection with the Ginzburg-Landau situation is made by defining analogously the potential generated by the charge configuration using the average  $\mu_0$  as a neutralizing background; this yields the following equation playing the role of the analogue to (2):

$$H_n = -2\pi \Delta^{-1} \left( \sum_{i=1}^n \delta_{x_i} - n\mu_0 \right) \quad \text{in } \mathbb{R}^2.$$



The next step is again to express this in the blown-up coordinates at scale  $\sqrt{n}$  around  $x_0$ ,  $x' = \sqrt{n}(x - x_0)$ , via  $H'_n$  the solution to

$$H'_n(x') = -2\pi\Delta^{-1} \left( \sum_{i=1}^n \delta_{x'_i} - \mu_0(x_0 + \frac{x'}{\sqrt{n}}) \right). \quad (9)$$

When taking  $n \rightarrow \infty$ , the limit equation to (9) is

$$-\Delta H = 2\pi \left( \sum_a \delta_a - \mu_0(x_0) \right) \quad \text{in } \mathbb{R}^2, \quad (10)$$

analogous to (6), corresponding to another infinite jellium with uniform neutralizing background  $\mu_0(x_0)$ .

To understand the next order behavior, we split  $\nu_n := \sum_{i=1}^n \delta_{x_i}$  as  $n\mu_0 + (\nu_n - n\mu_0)$ . Noting that

$$w_n(x_1, \dots, x_n) = \iint_{\Delta^c} -\log|x-y| d\nu_n(x) d\nu_n(y) + \int V(x) d\nu_n(x),$$

where  $\Delta$  denotes the diagonal, inserting  $\nu_n = n\mu_0 + (\nu_n - n\mu_0)$ , we eventually find the exact decomposition

$$w_n(x_1, \dots, x_n) = n^2 \mathcal{F}(\mu_0) - \frac{n}{2} \log n + \frac{1}{\pi} W(\nabla H'_n, \mathbf{1}_{\mathbb{R}^2}) + 2n \sum_{i=1}^n \zeta(x_i), \quad (11)$$

where, for every function  $\chi$ , and for  $j = \nabla H'_n$ , we let

$$W(j, \chi) := \lim_{\eta \rightarrow 0} \int_{\mathbb{R}^2 \setminus \cup_{i=1}^n B(x_i, \eta)} \chi |j|^2 + \pi(\log \eta) \sum_i \chi(x_i). \quad (12)$$

The function  $\zeta$  in (9) plays no other role than confining the particles to  $\text{Supp}(\mu_0)$  (it is zero there, and positive elsewhere), so it remains to understand the limit  $n \rightarrow \infty$  of  $W(\nabla H'_n, \mathbf{1}_{\mathbb{R}^2})$  corresponding to (10). One of our main results (in (16) below) is that this term is of order  $n$ . Note that  $W(\nabla H'_n, \mathbf{1}_{\mathbb{R}^2})$  can be seen as the interaction of the distribution of charges and the blown-up background  $\mu_0$ , but computed in a “renormalized” fashion:  $H'_n$  has a logarithmic singularity near each  $a$ , and thus  $|\nabla H'_n|^2$  is not integrable; however, when removing small balls of radius  $\eta$  around each  $x_i$ , adding back  $\pi \log \eta$ , and letting  $\eta \rightarrow 0$ , this singularity can be “resolved”. Taking the limit  $n \rightarrow \infty$  thus allows to define the “renormalized” total self-interaction of an infinite jellium. There are technical difficulties in taking that limit, in particular due to the lack of local charge neutrality of the system. On the other hand, one important advantage of this formulation is that it transforms, via (11), the sum of pairwise interactions into an extensive quantity in space (12), which allows for localizing (via a screening procedure), cutting and pasting, ...

Let me now describe the limiting quantity defined for solutions of (10). Let  $m > 0$  be a given positive number (corresponding to the density of points). We say a vector field  $j$  belongs to the class  $\mathcal{A}_m$  if

$$j = \nabla H \quad -\Delta H = 2\pi(\nu - m) \quad \text{for some } \nu = \sum_{p \in \Lambda} \delta_p, \quad \text{where } \Lambda \text{ is a discrete set.} \quad (13)$$

As said above, the vector-field  $j$  physically corresponds to the electric field generated by the point charges. The use of the electric field ensemble as a tool to study Coulomb systems first appeared in [Le] for 1D Coulomb system and later in [AM], where translational symmetry-breaking was established.

We define the *renormalized energy*  $W$  for  $j \in \mathcal{A}_m$  by

$$W(j) := \limsup_{R \rightarrow \infty} \frac{W(j, \chi_{B_R})}{|B_R|}, \quad (14)$$

where  $\chi_{B_R}$  is any cutoff function supported in  $B_R$  with  $\chi_{B_R} = 1$  in  $B_{R-1}$  and  $|\nabla \chi_{B_R}| \leq C$ , and  $W(j, \chi)$  is defined as in (12).

In the particular case where the configuration of points  $\Lambda$  has some periodicity, i.e. if it can be seen as  $n$  points  $a_1, \dots, a_n$  living on a torus  $\mathbb{T}$  of appropriate size, then  $W$  can be expressed much more simply as a function of the points only:

$$W(a_1, \dots, a_n) = \frac{\pi}{|\mathbb{T}|} \sum_{j \neq k} G(a_j - a_k) + \pi \lim_{x \rightarrow 0} (G(x) + \log |x|), \quad (15)$$

where  $G$  is the Green function of the torus (i.e. solving  $-\Delta G = \delta_0 - 1/|\mathbb{T}|$ ). The Green function of the torus can itself be expressed explicitly in terms of some Eisenstein series and the Dedekind eta function. The definition (14) thus allows to generalize such a formula to any infinite system, without any periodicity assumption.

The results we obtained in [SS3] establish that minimizers of  $w_n$  converge as  $n \rightarrow \infty$  after blow up around almost any point  $x_0$  at scale  $\sqrt{n}$ , to minimizers of  $W$  (or rather the electric fields do), and we have:

$$\min w_n = n^2 \min \mathcal{F} - \frac{n}{2} \log n + \frac{n}{\pi} \int_{\mathcal{A}_{\mu_0}(x)} \min W \, dx + o(n). \quad (16)$$

The order  $n$  term in this equation corresponds to the average of  $W$  with respect to the blow up centers. It is the term that sees the difference between different microscopic patterns of points, beyond the macroscopic averaged behavior  $\mu_0$ . In [SS2] we had previously proven the same result for minimizers of the Ginzburg-Landau energy: they have currents which after blow-up, and after letting  $\varepsilon \rightarrow 0$ , minimize  $W$ .

Thus, our study reduces the problem to understanding the minimization of  $W$ . This belongs to the more general family of crystallization problems for which there are very few results (cf. [Th] for an exceptional example) and is known to be a very hard problem (even in the periodic context) due to the long-range nature of the logarithmic interaction. There are only very partial answers. One of them is that  $W$  can be minimized among

lattices with prescribed volume. We find that this minimization question is equivalent to minimizing the Epstein zeta function  $\zeta(\Lambda) = \sum_{p \in \Lambda} \frac{1}{|p|^s}$  with  $s > 2$  over lattices  $\Lambda$  of fixed volume. This question was in turn solved in the 60's (in works by Cassels, Rankin, Ennola, Diananda, see e.g. [Mo]) and it was established that the unique minimizer is the triangular “Abrikosov” lattice, hence the same is true for  $W$ . It is then natural to conjecture, in view of the above results and the experiments in superconductors, that this is also a global minimizer of  $W$ . If this last step of proving the conjecture were accomplished, it would completely justify the emergence of the Abrikosov lattice in superconductors and in zero temperature Coulomb gases.

Once the ground states of  $w_n$  had been understood, we were able to obtain information on the finite temperature states. The Gibbs measure for the same mean-field Coulomb gas at temperature  $1/\beta$  is

$$d\mathbb{P}_n^\beta(x_1, \dots, x_n) = \frac{1}{Z_n^\beta} e^{-\beta w_n(x_1, \dots, x_n)} dx_1 \cdots dx_n \quad (17)$$

where  $Z_n^\beta$  is the associated partition function, i.e. a normalization factor that makes  $d\mathbb{P}_n^\beta$  a probability measure. The particular case of  $\beta = 2$  and  $V(x) = |x|^2$  corresponds to the law of eigenvalue for random matrices with iid normal entries, the *Ginibre ensemble*. The connection between Coulomb gases and random matrices was first pointed out by Dyson [Dy]. For general background and references, we refer to [Fo].

Inserting the splitting (11) into this relation, and using the properties of  $W$  immediately yields new results on  $\mathbb{P}_n^\beta$ . For example it yields a next order expansion of the partition function,

$$n\beta f_1(\beta) \leq \log Z_n^\beta - \left( -\beta n^2 \mathcal{F}(\mu_0) + \frac{\beta n}{2} \log n \right) \leq n\beta f_2(\beta),$$

where  $f_1(\beta)$  and  $f_2(\beta)$  are independent of  $n$ , bounded, and

$$\lim_{\beta \rightarrow \infty} f_1(\beta) = \lim_{\beta \rightarrow \infty} f_2(\beta) = \frac{1}{\pi} \int \min_{\mathcal{A}_{\mu_0(x)}} W dx := \alpha_0.$$

Only the term in  $n^2$  of this expansion was previously known.

The final result is a large deviations type result, which says that events with average of  $W$  above  $\alpha_0 + o_\beta(1)$  have exponentially small probability as  $\beta \rightarrow \infty$ , which means crystallization as the temperature tends to 0 towards minimizers of  $W$  (the term crystallization is used here because these minimizers are believed to have a crystalline structure, as seen above). To our knowledge, this is the first time Coulomb gases are rigorously connected to triangular lattices (again modulo the solution to the conjecture on the minimum of  $W$ ), in agreement with predictions in the literature ([AJ] and references therein).

Let me finish by two remarks. The first one is about the quantum case (of the Coulomb gas), which turns out to be quite different: the next order term in the equivalent of (16) is no longer of order  $n \log n$  but rather it is a term of order  $n$  identified to be the ground state of the Bogoliubov Hamiltonian, see [LNSS]. The second one is that an adapted

definition of  $W$  can be computed for classical random point processes arising in random matrix theory and serve as a quantitative measure of disorder, or the statistical energy per particle, cf. work with A. Borodin [BS].

In conclusion, we believe that the renormalized energy  $W$  is a natural and useful object for describing large Coulomb systems and computing their effective interaction. It also allows to treat the situation of 1D systems with logarithmic interaction (or log gases) [SS4]. Many questions remain, first and foremost the complete minimization of  $W$ , but also more accessible ones: proving the complete existence of the thermodynamic limit for the Coulomb gas, determining the limiting statistical law of the electric field, understanding a true large deviations principle for the problem at next order, understanding whether a phase transition to a crystalline state happens at a finite temperature or only as the temperature tends to 0, etc.

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Picture of Sylvia Serfaty: courtesy and © by Olivier Boulanger



## Phase transitions in $k$ -mer systems

by ALESSANDRO GIULIANI (Recipient of the 2012 IUPAP Young Scientist Prize)



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A central issue in equilibrium statistical mechanics is to unveil the microscopic mechanisms underlying the emergence of different phases in a given particle system. Even very simple microscopic interactions among the constituents of the system can give rise to unexpectedly rich phase diagrams, whose nature is in many cases still to be understood. As a reference example, it is enough to think to classical particles in the three-dimensional space, interacting via a hard-core potential plus an attractive tail: for any “reasonable” choice of the parameters (radius of the core, radius and strength of the attractive tail) the system is believed to display a phase diagram that resembles the one of several common substances, like water or noble gases. However, to date, no satisfactory understanding of the system is available outside the high temperature/low density region, where perturbative methods allow one to get very detailed informations about the gas phase. That is, no proof of liquid-vapor, or liquid-solid phase transition is known nowadays for particle systems in the continuum interacting via a simple pair-potential like the one mentioned above (however, it is remarkable that if we allow both two- and four-body interactions with finite range then it is possible to prove the existence of a liquid-vapour phase transition [18, 24]).

An even more striking and surprising example is the one of particles interacting via purely hard core interactions, essentially the simplest interaction one could ever think of. While also in this case the low density behavior of the system is well understood, no satisfactory understanding of the high density phase is available, with the exception of the case of maximal density for hard-core spheres, which is realized by the face centered

cubic (FCC) lattice [12]. Numerical simulations suggest that at high but finite density the system of hard-core spheres displays long range crystalline order of FCC type, but no proof of this fact is available yet. If the shape of the particles is changed from spherical to elongated (cigar-like) or flattened (pancake-like) shape, yet another phase, in addition to the gas and solid ones may emerge, which has very interesting features. I refer to the so-called nematic liquid crystal, which is a phase in which the distribution of orientations of the particles' axes is anisotropic, while the distribution of the particles in space is homogeneous. In the following, I will mostly focus on this phase, whose existence is confirmed by simulations on colloidal particles and whose understanding is of fundamental importance for the physics of liquid crystals and for their widespread applications (e.g., think of the liquid crystal screens of our televisions and computers).

What is really surprising about the existence of a non-trivial phase diagram for particle systems interacting via purely hard core interactions is that the only contribution to the free energy of the system is of entropic nature: in mathematical terms, the partition function, from which we can derive thermodynamic information on the system, is equal to the total volume in phase space of the allowed configurations of non-overlapping particles, whose logarithm is simply the configurational entropy of the system. Therefore, one would naively think that since the only contribution to the free energy of the system comes from entropy, the system would just like to become as disordered as possible, at all densities. Contrary to this intuition, there is a competition between different classes of configurations (and, therefore, between different contributions to the total entropy), which may induce the system to order spontaneously.

The underlying mechanism is simplest in the context of elongated molecules, in which case it was first proposed by Lars Onsager [21]: roughly speaking, his idea was to picture each molecule as a long, thin rod, each characterized by a translational and a rotational degree of freedom, so that the total entropy can be thought of as a sum of a translational and a rotational entropy. At very low densities, so low that the average intermolecular distance is much larger than the rod's length  $\ell$ , every molecule has enough space to rotate freely around its center; therefore, the system is in an isotropic gas phase. At higher densities, each molecule is surrounded by other molecules at an average distance smaller than  $\ell$  and, therefore, it is not free to rotate in all directions. In some cases, it may be favorable for the molecules to align spontaneously: it may be convenient for the system to substantially reduce its orientational entropy, the loss being compensated by a much larger gain of translational entropy. To quantify this effect, Onsager computed an approximate equation of state by truncating at second order the virial expansion and by computing explicitly the first two Mayer coefficients. Then he expressed the equation of state as a functional of the distribution function of the rods' orientations and verified that for certain geometric shapes of the molecules there exists a non-vanishing interval of densities where the free energy of the system is lowest for an inhomogeneous distribution of the rods' orientations. Onsager's theory is the first example of an *entropy-driven* phase transition, i.e. an ordering transition induced by the competition of two entropic effects, each of which would separately like to make the system as disordered as possible. The mechanism he proposed is often referred to as *Onsager's excluded volume effect*. Since

then, there have been several attempts to prove in concrete models the correctness of this proposal, whose rigorous understanding is still an open problem in the context of three-dimensional systems with continuous translational and rotational symmetries.

The difficulties encountered in developing a mathematical theory of phase transitions in the realistic context of particles in the three-dimensional continuum forced people to analyze simpler models, which may serve as toy examples of the phenomena of interest, for which a rigorous construction of the phase diagram is more feasible. In order to understand the behavior of hard-core systems, a natural simplified class of models is that of particles randomly thrown on  $\mathbb{Z}^2$  with the condition that no particles can overlap. The only degrees of freedom one is left with are the particles' density and the particles' shape. If one is willing to understand systems of elongated molecules, like those involved in Onsager's theory mentioned above, the simplest shape one can choose is a  $1 \times k$  rectangle, also called a  $k$ -mer ( $k \geq 2$  in order not to have a "trivial" shape). The  $k$ -mer model at finite (or even maximal) density is a popular and very appealing model, studied since decades ago in the theoretical and mathematical physics communities. It is the simplest model<sup>1</sup> in which to understand the validity (or failure) of Onsager's proposal; if correct, the excluded volume effect should induce a nematic-like phase at intermediate densities, that is, a phase characterized by broken rotational symmetry and unbroken translational symmetry: the  $k$ -mers' orientations should display long range order, while their centers should be distributed essentially independently.

Most of the known results concern the case of  $k = 2$  (dimers). At maximal density, Kasteleyn [16] computed the configurational entropy of the system exactly; his method also allows one to compute a class of dimer-dimer correlations, which display algebraic decay of correlations, similarly to Ising spin systems at the critical point. At all finite densities but the maximal one, Heilmann and Lieb [13] proved that the pressure and correlation functions are analytic in the density, which means in particular that the dimer-dimer correlations decay exponentially at large distances. This means no phase transition and no nematic-like phase at intermediate densities for the dimer system. In an attempt at constructing a system of finite size anisotropic molecules exhibiting a nematic-like behavior, in 1978, O. Heilmann and E. Lieb [14] proposed a variant of the dimer system, by adding *attractive* forces between aligned dimers. By reflection positivity, they proved the existence of orientational order. Presumably, orientational order comes *without* translational order [20], as it should in a nematic phase. *However*, the attractive forces completely change the mechanism driving the ordering transition. So, is the excluded volume effect enough to induce order for  $k$ -mers, with  $k \geq 3$ ? This question remained open for more than 30 years, and after a while people raised doubts about the very existence of a nematic-like phase in  $k$ -mer systems with purely hard core interactions. To put it in Heilmann and Lieb's words [14]: *It is doubtful whether hard rods on a cubic*

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<sup>1</sup>Of course the notion of "simplest" is subjective. In this paper I focus on the description and review of results related directly to  $k$ -mer models. Another simple class of systems of anisotropic molecules with discrete orientations is related to the Widom-Rowlinson model [27], either on the lattice or on the continuum, for which some important rigorous results are due to Lebowitz-Gallavotti [17] and Ruelle [26]. For a more comprehensive history of previous results, see the introduction of [4] or [10]

*lattice without any additional interaction do indeed undergo a phase transition.*

The first clear indication that Onsager's excluded volume effect is enough to produce a nematic-like phase in  $k$ -mers systems came in 2006, due to D. Ioffe, Y. Velenik and M. Zahradnik [15], who proposed a “polydisperse” variant of the model, where rods of all possible lengths are allowed, with statistical weight depending on  $k$ . They mapped the model into the exactly solvable two-dimensional Ising model and by the exact solution they proved the existence of an isotropic-nematic transition, which is the image under the mapping of the usual paramagnetic-ferromagnetic transition in the Ising model. This result strongly suggests that the same transition should take place in the pure  $k$ -mer system, but unfortunately the method of [15] breaks down in the presence of apparently harmless changes in the relative rods' weights.

Almost simultaneously, in 2007, A. Ghosh and D. Dhar [9] numerically identified in a very clear manner a nematic phase for pure  $k$ -mers with  $k \geq 7$  at intermediate densities. Their result further confirmed, from a different perspective, the expectation arising from the work of [15].

Motivated by these results, M. Disertori and I recently reconsidered the  $k$ -mer problem on the two-dimensional square lattice [4]. We succeeded in proving the following. Let  $\rho \in (0, 1/k)$  be the density of rods.

*For  $k$  large enough, if  $k^{-2} \ll \rho \ll k^{-1}$ , the system admits two distinct infinite volume Gibbs states, characterized by long-range orientational order (either horizontal or vertical) and no translational order, selected by the boundary conditions.*

This result is the first rigorous proof of the existence of a nematic phase (i.e., a phase that breaks rotational symmetry but does not break translational symmetry) and of the validity of Onsager's volume effect in a system of anisotropic particles of finite fixed size. Of course the result is quantitatively far from being optimal: while the “large enough” in real life is expected to be “larger than 7”, as suggested by the simulations in [9], our theorem requires  $k$  to be larger than a few hundreds. Still it is what we desire from a conceptual point of view.

The proof of the result above, which can be found in all details in [4], is based on a series of standard methods in the theory of phase transitions, most notably, contours expansion (see e.g. [8]) and Pirogov-Sinai theory [23, 29]. In order to be a bit more explicit and to give a flavor of the methods involved, let me outline the main ideas involved in the proof, and its main steps.

- 1) The first step consists in coarse graining  $\Lambda$  in square tiles  $\Delta$  of side  $\ell = k/2$ : this means that we partition the original box into a collection of tiles, each of which contains in average many ( $\sim \rho k^2 \gg 1$ ) rods; in this respect the tiles can be thought of as being mesoscopic. On the other hand, each tile  $\Delta$  is so small (the side is half the rods' length) that only rods of the same orientation can have centers in  $\Delta$ , due to the hard rod condition.
- 2) Given a tile  $\Delta$ , once we prescribe the orientation of the rods with centers in it, the effective interaction between rods of the same orientation, say horizontal, is weak: in fact, the hard core repulsion just prevents two rods from occupying the same row, an event that is very rare, since the density of occupied rows is  $\sim \frac{\rho k^2}{k} \ll 1$ . It is remarkable that

the standard *cluster expansion* [8, 7, 25] allows us to quantify how close to Poissonian the distribution of centers in  $\Delta$  is, once the orientation is prescribed.

3) Each tile can thus be of three types:  $+1$  (horizontal),  $-1$  (vertical) or  $0$  (empty). In this way, we can associate with every allowed rod configuration on  $\Lambda$  a corresponding *spin configuration* on the coarser lattice of the tiles' centers. By summing over all the rod configurations corresponding to a given spin configuration, we are left with a partition sum over spins, which defines an effective spin model. This spin system has the following features: (i) the interaction between spins is short range and strongly attractive, due to the hard core; (ii) The vacuum configurations are unlikely, since the probability that  $\Delta$  is empty is  $\sim e^{-(\text{const.})\rho k^2}$ . Therefore, the typical spin configurations consist of big connected clusters of “uniformly magnetized spins”, separated by boundary layers (the contours), which contain zeros or pairs of neighboring opposite spins.

4) We are left with studying this effective contour theory. The idea is to use a Peierls argument or cluster expansion methods. However, there are a few issues that make life complicated.

(4.a) First of all, the inter-contour interaction turns out to be an exponentially decaying  $N$ -body interaction, with  $N$  arbitrary; if the nature of this  $N$ -body interaction were generic then we would really be in trouble: nobody knows how to work out a convergent cluster expansion for generic  $N$ -body interactions, even if exponentially decaying. Luckily enough, the  $N$ -body interaction we need to deal with is quasi one-dimensional, in the sense that only contours whose horizontal or vertical projections overlap interact among each other. This makes the problem treatable. The right strategy is to follow Brydges' suggestion [3]: *if at first you do not succeed, then expand and expand again!* In other words, we perform a second Mayer expansion of the multi-contour interaction and we collect together the connected components (polymers); the resulting polymers have purely hard-core interactions, and the polymers' activities satisfy similar bounds to those of the original contours.

(4.b) The activity of the contour is defined in terms of a ratio of partition functions that at the beginning of the story we do not know how to compute (yet). Moreover, the contour theory is not exactly symmetric under spin flip, due to the finite size of the rods (i.e., the activity of a given contour is not the same in the presence of  $+$  or  $-$  boundary conditions, unless the shape of the contour itself is rotation invariant). To solve these two issues, we use the methods of Pirogov-Sinai theory [23, 29]. By induction, we show that the polymers satisfy a Peierls' condition, i.e., the probability that a given contour or polymer occurs is exponentially small in the size of its geometric support. For details, see [4].

The methods developed in [4] for proving the existence of an orientationally ordered state for the two-dimensional  $k$ -mer system opens the way to facing some other problems in the same spirit. For example, it is likely that a simple extension of the methods of [4] will allow us to consider anisotropic molecules in the three-dimensional continuum at intermediate densities, provided the number of allowed orientations remains finite. By playing with the shape of the molecule, it may be possible to construct examples of transitions from, say, a nematic-like to a smectic-like phase (by smectic phase we mean a



phase where rotational symmetry is completely broken and translational is just partially unbroken - say a phase of oriented molecules with centers arranged periodically in one space direction and randomly in the other two). Another interesting question, which is non trivial already at the level of  $k$ -mers in  $\mathbb{Z}^2$  is whether by increasing density towards close packing we encounter a second phase transition to a “dense phase” or not. The conjecture, based on a simple and appealing variational computation [9], is that there should be a high-density phase where orientational order is lost, but maybe a hidden striped order could survive. The variational computation underlying this conjecture gives a bound on the entropy of the close packed system in the form:  $S_{cp}/\text{Volume} \geq C_s k^{-2} \log k$  for an explicitly known constant  $C_s$ ; the “minimal” thing to prove, in order to give some rigorous support to the conjecture about the structure of the dense phase, would be an upper bound of the same asymptotic form:  $S_{cp}/\text{Volume} \sim k^{-2} \log k$ , asymptotically as  $k \rightarrow \infty$ . Some attempts in this direction indicate that the estimate is not trivial at all. The problem may be of interest also for the community that studies random packings and glassy behavior in hard core particles; see [22] and references therein.

“Of course”, the most important and difficult open problem is to prove the existence of orientational order in a model with genuine rotational invariance. There are some examples of two or three-dimensional gases with a continuous internal degree of freedom, interacting via a tensorial *attractive* interaction, where the existence of nematic order (or quasi-order in 2D) can be proved by a combination of cluster expansion and reflection positivity [1, 2, 11, 28]. However, the understanding of Onsager’s excluded volume effect is completely open in this case. I think that this last problem is substantially more difficult than the previous two. We have a serious lack of understanding of continuous symmetry breaking phenomena, as witnessed by the state of art in the problem of existence of spontaneous magnetization in 3D classical and quantum ferromagnets: there are only very few special cases where spontaneous magnetization can be rigorously proved [6, 5], the proof being based on reflection positivity, which is very fragile under apparently harmless changes in the Hamiltonian. It is very likely that future progress on the theory of the Heisenberg ferromagnet will also help us in understanding the validity of Onsager’s excluded volume effect in continuous three-dimensional systems. It may also be of help to try to attack the issue of continuous rotational symmetry by first looking at mean-field like models, possibly in the spirit of [19].

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## Letter to the Editor: On the early history of the CLR estimate

Dear colleagues,

In the article by Barry Simon, in the July issue of the Bulletin, 2012, the story of the CLR (Cwikel-Lieb-Rozenblum) estimate, the phase space estimate for the number of bound states of the Schrödinger operator, is presented on pp. 7,8. In particular, it is written “In the mid-1970s, I spent a lot of time thinking about semiclassical estimates.” and then the story is told how Simon introduced the problem to E. Lieb, C. Fefferman, and M. Cwikel, and this resulted in the appearance of two proofs of the phase space estimate in the winter 1975/1976, by Lieb and Cwikel.

After that, in the summer of 1976, B. Simon visited the USSR, and, during the meeting with the Birman-Solomyak group, among other things, informed them about the phase space estimate. And, by B. Simon’s words,

“In further discussion, they said that a younger worker in their group, Gregory Rozenbljum (as his name was then transliterated by the AMS!), had announced a result that implied the semiclassical bound I conjectured. Given Cwikel’s work (and, of course, I also told them of Lieb’s work), they decided Gregory had better write his stuff up awfully fast. And so were born the CLR bounds.”

This is the story as seen from Princeton. However, the actual picture looks somewhat different.

G. Rozenblum discovered the bound, that later became known as CLR (in fact, a much more general inequality concerning eigenvalue estimates for the operators generated by ratios of quadratic forms) in the summer of 1971, i.e, five years before the meeting in question. The short paper with formulations and some explanations appeared in Doklady in 1972, in Russian (see [MR0295148](#)), and was translated into English in due course. Also in 1972 a detailed proof was submitted to the journal, Izvestia VUZov (see [MR0430557](#)).

Meanwhile, the estimate in question was being applied to different problems. In particular, in the large Birman-Solomyak paper [MR0364898](#), part 2, on spectral asymptotics, published in 1973, it played the crucial role.

In Birman-Solomyak lectures [MR0482138](#), published in Russian in 1974, a full exposition of Rozenblum’s proof for an important particular case (that included the polyharmonic operator) was presented.

The paper [MR0430557](#), due to the terrible slowness of the publishing process in the USSR, appeared in February 1976, but still it was essentially before the meeting B. Simon writes about.

M. Solomyak and D. Yafaev participated in the meeting mentioned by Barry Simon, but G. Rozenblum could not attend. The main topic of the discussion was the phase space

estimate. M. Birman and M. Solomyak told B. Simon that this result had been known since 1971, it is due to G. Rozenblum, his note in DAN was published in 1972, and a reprint was sent to B. Simon (among others).

Thus, by no means was the phase space estimate for Schrödinger operators new for the people present, to say nothing about them not believing in it. Moreover, M. Birman and M. Solomyak pointed out to B. Simon that their above mentioned lectures contain its full proof. B. Simon certainly became interested, and later this resulted in the publication of the English translation of the lectures (see [MR0562305](#)).

So the course of events as B. Simon presents it, that G. Rozenblum had to hurry up and write his stuff awfully fast, is not correct: the stuff had been written 5 years before and published.

Of course, both Lieb's and Cwikel's proofs are extremely beautiful and no one questions their contribution, however the correct course of events, including the correct timing, should become known to the general public.

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## Rudolf Haag – Celebrating 90 Years

Rudolf Haag, one of the founding fathers of modern mathematical physics, turned 90 on August 17th, 2012. In order to celebrate this happy event, a special day was devoted to him at the satellite meeting of ICMP12 “*Mathematical Aspects of Quantum Field Theory and Quantum Statistical Mechanics*”. It took place in Hamburg where he had been University Professor of Theoretical Physics for more than 20 years. Despite his advanced age he participated in this meeting and his profound questions and comments revealed to all participants his undiminished interest and distinctive view on central problems of quantum physics.

Within the broad community of mathematical physicists, Rudolf Haag is most widely known as founding editor of *Communications in Mathematical Physics*. He conceived this journal together with Res Jost around 1964 and led it to the forefront during his editorship until 1973. For his seminal scientific contributions he received in 1970 the Max-Planck Medal, the highest award of the German Physical Society, and he was the first recipient of the Henri Poincaré Prize of IAMP, together with Maxim Kontsevich and Arthur Wightman. He is also member of several academies (Leopoldina, Göttingen, Bavaria and Austria).



Rudolf Haag

Rudolf Haag’s many scientific accomplishments were highlighted on the occasions of his past major birthdays in special issues of prominent journals [1, 2, 3]. He has also given a very personal review of his work in [4] which is a highly interesting read. In his research he addresses primarily conceptual problems in quantum field theory and quantum statistical mechanics. These deep insights laid the foundation for the proper formulation and rigorous treatment of concrete models in quantum physics and thereby became an essential ingredient in current mathematical physics.

In his early work Rudolf Haag was concerned with the relations between particles and fields. His paper “On Quantum Field Theories” from 1955 marks a crucial breakthrough, shedding light on the underlying mathematical structure and its physical interpretation. The subsequent papers on scattering theory clarified the origin of the particle structure of quantum field theory as a consequence of locality of observables. Based on these insights he developed a new conceptual framework for quantum field theory (in collaboration with Araki, Kastler and Schroer), nowadays called “algebraic quantum field theory” or, synonymously, “local quantum physics”. As these terms indicate, the system of algebras of local observables (associated with measurements performed in bounded space-time regions) plays a decisive role in this approach. These insights initiated a fruitful and lasting exchange between quantum physics and the mathematics of operator algebras.



R. Haag discussing with H. Araki, his first Ph.D. student

Roberts) which is related to and in fact a precursor of Jones' theory of subfactors in mathematics. The latter theory in turn stimulated the operator algebraic approach to conformal field theory by Kawahigashi, Longo, Rehren and others.

Rudolf Haag received and provided inspiration in the field of mathematics. For him mathematics is primarily a tool to formulate in precise and meaningful terms his physical insights, not so much an arena for knotty reasoning. Indeed, his main interest was and still is physics. Amongst important pending problems he deplores most our lack of understanding of the physical roots of the very successful gauge theories. Since he is convinced that this conceptual problem could be solved by methods of algebraic quantum field theory he regards this gap as a missed opportunity. He also explored the foundations of supersymmetry and clarified in a frequently cited article (with Lopuszanski and Sohnius) all *a priori* possible supersymmetries of the S-matrix. Moreover, he contributed to our understanding of gravitational effects in quantum field theory, another topic of current interest. In particular, he clarified (in collaborations with Narnhofer, Stein and Fredenhagen) the local singularity structure of correlation functions resulting from physically meaningful stability conditions. These papers provided a foundation for the modern approach to quantum field theory on curved space-times. In more recent years his interest moved towards the foundations of quantum physics. There he tries to base the theory on the concept of events and their causal relations. He suspects that such a radical change of the framework of quantum physics is necessary if one wants to incorporate into the theory also quantum effects of gravity.

A detailed account of these topics can be found in the book of Rudolf Haag [5] which he wrote after his retirement. This book is by now a standard reference for the subject, and an indispensable source of insight for everybody interested in the conceptual foundations of almost all aspects of quantum physics.

At the Hamburg meeting in his honor, a number of new developments were presented which are intimately related to his work as well as to intriguing suggestions he has made. Jones and Takesaki talked on aspects of operator algebras, Fewster, Hollands and Pinamonti outlined their work on quantum field theory in curved space-times. Yngvason presented ideas and rigorous results on the concept of phenomenological entropy,

Some highlights are his work on the characterization of equilibrium states by means of the KMS condition (together with Hugenholtz and Winnink); it had a strong impact in mathematics on Modular Theory, initiated by Tomita, which was developed further by Takesaki and others. Operator algebras also played a prominent role in his work on superselection sectors and particle statistics (with Doplicher and



somewhat reminiscent of the way entropy was introduced by Rudolf Haag in his thermodynamics lectures. Lechner explained a novel, purely operator algebraic construction of a huge class of interacting quantum field theories on two dimensional space-time.

Doplicher discussed the possible quantum structure of space-time and its consequences for cosmology. Englert, Fröhlich and Sewell presented their, to some extent diverging, views on the foundations of quantum physics. There were also talks by Paycha and Brouder on mathematical aspects of renormalization, a talk by Loll on a construction of quantum space time by numerical simulations of causal lattices, and a talk by Schach-Møller on Pauli-Fierz models. A number of shorter talks on various aspects of quantum field theory and lively discussions during the talks and breaks topped off the scientific part of the meeting. Life and work of Rudolf Haag were given proper recognition in a laudation by Buchholz.

The meeting provided ample evidence of the fact that the “school of thought founded by Rudolf Haag” (as Hollands expressed it in his talk) is flourishing and very productive. It is open to new developments and in fruitful contact with other current approaches to quantum physics, in accord with the intentions and inspiring example of its Nestor.



K. Fredenhagen and D. Buchholz visiting R. Haag  
at home in Schliersee-Neuhaus



R. Haag and K. Fredenhagen at the meeting

Many of his former students and coworkers as well as colleagues who entered only more recently the field founded by Rudolf Haag attended this meeting. In the name of all of us we want to express here again our deep gratitude and affection. It is not only his scientific work which had an enormous impact on the subject and, in particular, on our own work. It is also the particular atmosphere of open-mindedness and humanity which he creates and which made this meeting (as many others) a gathering of friends.

Very best wishes from all of us

Detlev Buchholz and Klaus Fredenhagen



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## MECHANICS: CLASSICAL, STATISTICAL AND QUANTUM

### A celebration for Giovanni Gallavotti's 70th birthday

Last summer, from July 2 through 5, 2012, the conference “Mechanics: classical, statistical and quantum” in honor of the 70th birthday of Giovanni Gallavotti took place in Rome. Many colleagues and friends attended the event, which resulted in one of the most exciting scientific gatherings of the year.

Giovanni is one of the leaders in the field of mathematical physics since the end of the 60s. His first achievements are related to the microscopic description of phases in spin and particle models both at high and low temperatures, via cluster expansion methods. In particular, his results on the separation line between the two dimensional Ising model's  $+$  and  $-$  phases are of fundamental importance and are still at



the basis of the recent exciting advances on the SLE's behavior of the separation line at criticality. Among the other fundamental contributions of Giovanni in the 70s let us recall his rigorous derivation of the linear Boltzmann equation for the Lorentz gas in the Grad-Boltzmann limit, and the renormalization group construction of low dimensional ultraviolet  $\phi^4$  theories, started in collaboration with the “Roman school” (G. Benfatto, M. Cassandro, F. Nicolò, E. Olivieri, E. Presutti and E. Scacciatelli). In the 80s, Giovanni's interests concentrated on constructive field theory and KAM theory. These very successful research lines developed significantly in the following years, leading in particular to the construction of a large class of one-dimensional non-solvable interacting Fermionic theories (Luttinger liquids), on the one hand, and to an explicit diagrammatic construction of invariant surfaces (invariant tori and stable and unstable whiskers) in quasi-integrable Hamiltonian systems, on the other. Among Giovanni's achievements in the 90s, it is impossible not to mention the Gallavotti-Cohen fluctuation theorem and the chaotic hypothesis, which opened the way to a novel theory of non-equilibrium steady states, of great interest both from a theoretical and a practical/experimental point of view.

Among Giovanni's merits, there is certainly his well-known ability in being a great teacher and mentor, as proved by the several researchers in mathematical physics, who formed under his guidance. His textbooks on classical mechanics, statistical mechanics, fluid mechanics and ergodic theory, are nowadays classics in these fields and clearly express Giovanni's deep point of views both on the technical and the foundational and “philosophical” aspects of these disciplines.

The Rome conference in his honor represented an occasion for giving a wide perspective on the principal current research lines in mathematical physics, and to show the

cohesion of the discipline, notwithstanding the variety of techniques and problems investigated. The main subjects of the conference touched most of Giovanni's interests of the last 40 years, and ranged from integrable systems to partial differential equations, from quantum mechanics to many body theory, from quantum field theory, to equilibrium and non-equilibrium statistical mechanics. The list of speakers included some of the top scientists in the field, all very good friends and collaborators of Giovanni: M. Aizenman, G. Benettin, G. Benfatto, M. Berti, F. Bonetto, D. Brydges, E. G. D. Cohen, M. Disertori, P. Falco, J.-P. Francoise, J. Fröhlich, P. Garrido, K. Gawedzki, G. Jona-Lasinio, S. Kuksin, A. Kupiainen, J. Lebowitz, E. Lieb, V. Mastropietro, S. Miracle-Solé, S. Olla, G. Parisi, M. Procesi, V. Rivasseau, D. Ruelle, R. Seiringer, T. Spencer, H. Spohn, F.-L. Toninelli, J. Yngvason, and F. Zamponi.

The basic cohesion of the field has clearly emerged from several connections among the fields listed above discussed during the conference: as two illustrative examples we mention the connection between KAM theory and disordered statistical mechanics explained by T. Spencer, and the one between the theory of PDEs and non-equilibrium statistical explained by S. Kuksin.

The conference included some very interesting debates at the end of the seminars, such as a discussion on the foundations of statistical mechanics between E.G.D. Cohen, Joel Lebowitz and David Ruelle, who can be considered among the founders of modern statistical mechanics. We mention another exciting discussion about the role of topological insulators between J. Yngvason, E. Lieb and J. Fröhlich, which took place at the end of Jurg Fröhlich's talk.



The conference has been an exceptional event, both for the level of speakers and for the large diversity of subjects, quite remarkable in a period where conferences and workshops tend to become more and more specialized. The result has been an exciting *panorama* of modern mathematical physics, in a particularly productive period for the field: old problems are being solved by sophisticated new techniques, and more and more ambitious problems are being presented to the new generations as the next important goals to be reached. The presence of young brilliant speakers next to the well known *maestri* of the field demonstrates the vitality of mathematical physics.

The conference was also an occasion for informal and pleasant discussions, and all the participants were very happy to congratulate Giovanni for his birthday, looking forward to all the next exciting discoveries that Giovanni is ready to make in the next years. We are all looking forward to the next birthday's conference!

Happy birthday Giovanni, and best wishes for many more years of exciting research!

Guido Gentile

Alessandro Giuliani

Vieri Mastropietro

## The Poincaré Seminar

The **Poincaré Seminar** (*Séminaire Poincaré*) was created in 2002, and this year celebrates its 10<sup>th</sup> anniversary with its XVI<sup>th</sup> edition. During the 2012 ICMP Congress in Aalborg, Krzysztof Gawędzki, Editor-in-Chief of the *Annales Henri Poincaré*, kindly invited us to present this Seminar at the end of the *Annales Henri Poincaré* Prize Ceremony.



Poster for the VII<sup>th</sup> Poincaré Seminar, “*Einstein, 1905-2005.*”

The Poincaré Seminar offers a forum for focused and up-to-date lectures on topics of great current interest in physics. Each Seminar is devoted to a specific physical phenomenon or concept. Five lectures treat both the theoretical and experimental aspects of the chosen topic, generally with one offering some historical background. Inspired by the **Nicolas Bourbaki Seminar** in mathematics, hence nicknamed “*Bourbaphy*”, the Poincaré Seminar is held roughly twice a year at the Institut Henri Poincaré in Paris, with written contributions prepared in advance.





Poster for the XIV<sup>th</sup> Poincaré Seminar, “Chaos.”

The Seminar places particular emphasis on the pedagogical aspects of the lectures, in order that they be accessible to a wide audience of scientists and mathematicians. We distribute a soft-cover collection of the lecturers’ texts (most in English) on the day of the seminar, and put up electronic versions on our website (<http://www.bourbaphy.fr>) the following day. Beginning with the XV<sup>th</sup> Seminar, we have also made the lecturers’ slides available online. The Seminar also publishes the English texts some months later in a book in the *Progress in Mathematical Physics* series by **Birkhäuser**. (See <http://www.springer.com/series/4813?detailsPage=free>, which also offers a direct link to the Poincaré Seminar titles.<sup>2</sup>)

<sup>2</sup><http://www.springer.com/birkhauser/mathematics/mathematical+physics?SGWID=0-1724523-12-961704-0>



## Recent and Forthcoming Seminars

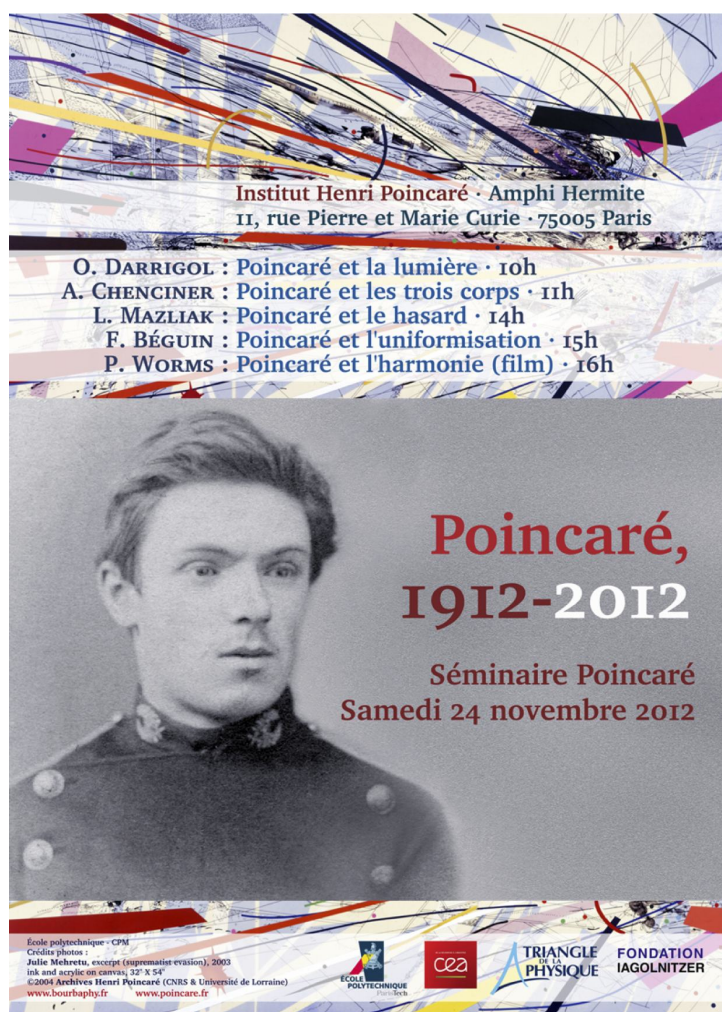
The two most recent Poincaré Seminars were devoted to **Chaos** and **Time** respectively. The latter book is in press, and the former will appear early next year.



Poster for the XV<sup>th</sup> Poincaré Seminar, “Time.”

The XVI<sup>th</sup> Poincaré Seminar, entitled ‘**Poincaré, 1912-2012**’, will be devoted to the legacy of Henri Poincaré in physics and mathematics. It will be held on November 24, 2012 at the Institut Henri Poincaré in Paris, with the following program:

- Olivier Darrigol · *Poincaré et la lumière [Poincaré’s Light]* · 10h
- Alain Chenciner · *Poincaré et les trois corps [Poincaré & Three Bodies]* · 11h
- Laurent Mazliak · *Poincaré et le hasard [Poincaré’s Odds]* · 14h
- François Béguin · *Poincaré et l’uniformisation [Poincaré’s Uniformization]* · 15h
- Philippe Worms · *Poincaré et l’harmonie [Poincaré in Harmony]* (film) · 16h.



Poster for the forthcoming XVI<sup>th</sup> Poincaré Seminar, “*Poincaré, 1912-2012.*”

This Seminar is one of numerous commemorations in France marking the centenary of Poincaré’s death. You may consult the general program of the centenary year at <http://www.poincare.fr>.

The Poincaré Seminar is currently supported by the École Polytechnique, the Direction des sciences de la matière of the Commissariat à l’énergie atomique et aux énergies alternatives (CEA), the Triangle de la Physique, and the Daniel Iagolnitzer Foundation.

**Bertrand Duplantier & Vincent Rivasseau**  
Pres & VP of the *Association Bourbaphy*

## Message from the Treasurer

by LÁSZLÓ ERDŐS (Treasurer of the IAMP)



When the previous Executive Committee (EC) took office in 2009, one of its main goals for the period 2009-2012 was to revise our membership database and to broaden the base of paying members. While IAMP boasted almost 1400 registered members, many of them were not active, especially not in their dues payment. As you see from the table below, out of 946 ordinary members, only 163 were in good standing. While the Statutes of IAMP clearly states that membership shall be terminated upon non-payment of dues beyond one year arrear, this drastic measure has never been enforced until 2010. The

EC in 2009 decided to be tough on this issue. With the endorsement of the General Assembly in Prague in August 2009, we have changed the dues structure and implemented measures to make sure that rules are obeyed. We introduced a partial amnesty that allowed members in long-term arrears to restore their status if dues back from 2008 were paid. We made the rules for lifetime membership more attractive to young members and we introduced stricter rules for the “reduced dues” (R) membership category. Finally, the dues were increased by about 35%.

Before terminating membership of delinquent members, we made many attempts to help them restore their good standing status. First we updated our database which contained several hundred obsolete contact data. With Pavel Exner and my secretary, Edith Höchst, we spent many hours of investigation through web-searches and personal connections. During the fall 2009 we then sent out several reminders in increasingly dramatic tone warning members of the serious consequences of not paying. After all these preparations, the moment of truth arrived on January 10, 2010. On this day, the membership of all non-paying members was terminated, and as a result, the number of IAMP members more than halved overnight. Reduced in number, yet heightened in spirit, and the new membership base was real! Ever since, we have consistently enforced good standing. We currently have only about 30 members who are in payment arrears, but they are permanently being reminded and in due time their membership will be terminated.

These drastic measures had several positive effects. Most importantly, almost 100 members used the partial amnesty, paid back dues and restored their membership. The number of R-members is reduced and now this category contains only first year members and members with economical hardship. Members from developed countries are excluded and members from developing countries must reconfirm their eligibility status every three years. Finally, the new lifetime membership rules proved to be attractive and we have about 55 more lifetime members than in 2009.

Membership category	June 2012	June 2009
Total registered	681	1386
Ordinary (O)	367	946
In good standing	333	163
In payment arrears	34	783
Lifetime (L)	266	210
Reduced dues (R)	48	119
Unknown	0	111
Paying members (L+O in good standing)	599	373

As a consequence of the increased dues and more discipline in payment, the total IAMP assets have increased by about 30K Euros in the last three years despite the increase of our spending on conference support. With this surge of income we have reversed the decade-long trend of depletion of our assets that was mostly due to the fact that IAMP had to support both ICMP Lisbon and ICMP Rio. (IAMP provides a limited financial guarantee for the ICMP organizer for the “worst-case” scenario.) The next table shows the development of the assets in our three main accounts.

### ASSETS AS OF JUN 30, 2012

Account	Balance	Currency	Euro equivalent
Bielefeld Checking	12,896	EUR	12,896
Bielefeld Savings	42,932	EUR	42,932
Paris Checking	5,831	EUR	5,831
Paris Savings	29,594	EUR	29,594
US Checking	4,244	USD	3,352
US Savings	39,174	USD	30,940
TOTAL Checking:			22,079
TOTAL Savings:			103,466
TOTAL (EUR):			125,545
<hr/>			
As of Jun 30, 2003:			116,000
As of Jun 30, 2006:			112,598
As of Jun 30, 2009:			96,963

Our main source of income is the membership dues. IAMP also welcomes donations. About 3-4 members donated smaller amounts on a regular basis and one member donated a very substantial amount (7,000 USD). We are very grateful to all donors and we hope more members will follow their path.



Our main expenses are conference grants. IAMP normally supports conferences in mathematical physics with 1,000-3,000 euros on a competitive basis; funding requests are evaluated by a three-member committee and the final decision is made by the EC. While the ICMP's are the most important conferences of our community, they are expected to raise sufficient money to balance their own budget. Nevertheless, on an emergency basis, IAMP is prepared to help ICMP organizers out; this was necessary in Lisbon and Rio. ICMP Prague and ICMP Aalborg needed no IAMP support thanks to sufficient funds raised by the organizers. Further expenses are administrative; the convenience of paying membership dues by credit card does not come for free. In the last six years, IAMP paid a modest amount for secretarial help; her duties were to organize and digitize IAMP archives. This task being completed, her employment is terminated.

IAMP also receives indirect support in the form of certain services at no cost thanks to the generosity of some members. Our webpage has been redesigned with the support of Manfred Salmhofer. We have a completely new database of members and online platform for dues payment. This was developed by Dietmar Kähler with the financial support of Volker Bach. The IAMP webpage is maintained by Jan Philip Solovej and hosted by the University of Copenhagen. The Bulletin is edited by dedicated IAMP members under the chief-editorship of Valentin Zagrebnov. My secretary, Edith Höchst helps with the bookkeeping of the finances. Günter Stolz, taking over the job from Larry Thomas in 2010, maintains our US bank accounts while Vincent Rivasseau takes care of our French bank accounts and the contact with the Daniel Iagolnitzer Foundation.

### FINANCIAL ACTIVITY 2009-2012

Activity	Amount (EUR)	2006-2009	2003-2006
Dues and donations	57,494	14,902	18,107
Interest	3,326	4,813	3,000
Total Bank/CC fees	- 3,269	-2,389	-1,000
Admin. support	-4,500	-4,500	N/A
Conference support	-22,137	-16,406	-17,663
ECA Prize	-3,000	N/A	N/A
ICMP Rio/Lisbon	N/A	-12,055	- 5,793
Total Gain/Loss	27,914	-15,635	-3,349



**LIST OF SUPPORTED CONFERENCES (2009-2012)**

Location/Year	Amount (EUR)
Aalborg [DK] (2009)	1,500
Cetraro CIME [IT] (2010)	1,100
Regensburg [DE] (2010)	2,000
Bressanone [IT] (2011)	2,000
Protvino [RU] (2011)	1,500
Nottingham [UK] (2011)	1,000
Dubna [RU] (2012)	1,000
Yerevan [ARM] (2012)	2,037
Barcelona [SP] (2012)	2,000
Rome [IT] (2012)	2,000
Munich [DE] (2012)	1,000
Vienna [AT] (2012)	3,000
Hamburg [DE] (2012)	2,000
Total:	22,137

In 2009 the EC, with the endorsement of the General Assembly, set the following targets related to finances for the period 2009-2012.

- i) Target: Increase the regular paying members from 163 to 250-300.  
Fact: 330
- ii) Target: Activate our associate members.  
Fact: 11 regularly paying Associate Members
- iii) Target: Maintain the investment income level.  
Fact: Decreased to about two-thirds, due to the financial crisis.
- iv) Target: Increase the net yearly income to about 8-9,000 Euros.  
Fact: cc. 17,000 (but only about 13,000 is sustainable).

Finally, I give some estimates on the future finances based upon the situation in 2012. After subtracting the running expenses and setting aside 1K Euro/year for the Early Career Award, the net income in 2012 will be around 13-14K Euro. Given our enlarged membership base and increased discipline in dues payment, this income level seems sustainable in the future years as well. Assuming that ICMP will not need support

from IAMP, we may spend the whole net income on conference support and still maintain our capital level.

### ESTIMATED YEARLY INCOME (EUR)

	2012 (Estim.)	2009
Dues of ordinary members	8910	3,260
Dues of associate members	2970	800
Capital spending of life members	2660	2,000
Investment income	1,000	1,500
Operational costs	-1,000	-2,200
Early Career Award	-1,000	-1,000
Net income (approx)	13,540	4,360

The following targets for the period 2012–2015 seem realistic.

- i) Increase the number of the ordinary members and lifetime members from the current 599 to 700.
- ii) Recruit more associate members.
- iii) Encourage regular donations
- iv)  $\Rightarrow$  Maintain the net yearly income at the estimated sustainable level 13-14K; try to increase it to about 15-16K.

On the medium term, our goal is not to increase our capital further. As a general guideline, the EC plans to spend the total net income on conferences in the next years. Furthermore, given the meagre investment income on capital, and assuming very strong applications for conference support, the EC is prepared to accept a controlled reduction of our capital at a rate of max. 5000 Eu/year.

## News from the IAMP Executive Committee

### New associate member

The IAMP welcomes the [Courant Institute for Mathematical Sciences](#) at New York University as a new associate member.

### New individual members

IAMP welcomes the following new members

1. Dr. Oskari Ajanki, Mathematisches Institut, Ludwig-Maximilians-Universität München, Germany
2. Dr. David E. Bruschi, School of Electronic and Electrical Engineering, University of Leeds, United Kingdom
3. Dr. Jeremy Clark, Mathematics Department, Michigan State University, East Lansing, MI, USA
4. Dr. Ivan Corwin, Mathematics Department, Massachusetts Institute of Technology, USA
5. Prof. Ovidiu Costin, Mathematics Department, The Ohio State University, Columbus, OH, USA
6. Dr. Wojciech De Roeck, Institut für Theoretische Physik, Universität Heidelberg, Germany
7. Prof. Bergfinnur Durhuus, Department of Mathematical Sciences, University of Copenhagen, Denmark
8. Prof. Alice Guionnet, Department of Mathematics, Massachusetts Institute of Technology, USA
9. Dr. Razvan Gurau, Perimeter Institute, Canada
10. Dr. François Huvneers, CEREMADE, Université Paris Dauphine, France
11. Dr. Anosh Joseph, Theory Division, Los Alamos National Laboratory, USA
12. Dr. Sasha Kocic, Department of Mathematics, The University of Mississippi, University, MS, USA
13. Dr. Liang Kong, Department of Mathematics and Statistics, University of New Hampshire, Durham, NH, USA
14. Prof. Ji Oon Lee, Department of Mathematical Sciences, Korea Advanced Institute of Science and Technology, South Korea

15. Tomàs Lungenstrass, Mathematics Department, Pontificia Universidad Católica de Chile, Santiago de Chile
16. Prof. Hiroyuki Osaka, Department of Mathematical Sciences, Ritsumeikan University, Japan
17. Dr. Mauro Paternostro, Department of Applied Mathematics and Theoretical Physics, Queen's University Belfast, United Kingdom
18. Dr. Morten Grud Rasmussen, Fakultät für Mathematik, Technische Universität München, Germany
19. Dr. Itaru Sasaki, Department of Mathematical Sciences, Shinshu University, Japan
20. Dr. Makiko Sasada, Department of Mathematics, Keio University, Japan
21. Dr. Yasuhiko Sato, Department of Mathematics, Kyoto University, Japan
22. Prof. Wilhelm Schlag, Department of Mathematics, The University of Chicago, USA
23. Dr. Mikko Stenlund, Courant Institute of Mathematical Sciences, New York University, USA
24. Prof. Lee-Peng Teo, Department of Applied Mathematics, University of Nottingham Malaysia Campus, Malaysia
25. Prof. Alessandro Teta, Dipartimento di Ingegneria e Scienze, dell'Informazione e Matematica, Università degli Studi dell'Aquila, Italy
26. Dr. Jérémie Unterberger, Mathématiques, Université Henri Poincaré, Vandoeuvre-lès-Nancy, France
27. Christian Webb, Department of Mathematics and Statistics, University of Helsinki, Finland
28. Prof. Makoto Yamashita, Mathematics Department, Ochanomizu University, Japan

## Recent conference announcements

### [The XVI<sup>th</sup> Poincaré Seminar](#)

November 24, 2012, Institut Henri Poincaré, Paris

organized by

[Bertrand Duplantier](#) and Vincent Rivasseau

### [Winter School on Mathematical Physics](#)

Jan 14 – Feb 1, 2013, IIMAS-UNAM, Mexico City.

organized by

[Ricardo Weder](#), Pablo Barberis, Rafael del Rio, Juan Manuel García-Islas, Luis O. Silva.

### [Quantum Fields, Gravity & Information](#)

– joint efforts and new directions in mathematical physics

April 3 – 5, 2013, School of Mathematical Sciences, The University of Nottingham

organized by

[David Edward Bruschi](#), Antony R. Lee, Nicolai Friis, Sara O. G. Tavares

### [The 8th Symposium on Quantum Theory and Symmetries](#)

August 5 – 9, 2013, El Colegio Nacional, Mexico City

organized by

[Kurt Bernardo Wolf](#), Octavio Novaro, Roelof Bijker, Octavio Castaños, Roco Jáuregui, Renato Lemus.

This conference is partially funded by the IAMP.

## Open positions

### Postdoctoral positions in Mexico

Posdoctoral Positions in Mathematical Physics. The National Autonomous University of Mexico (UNAM) offers postdoctoral positions (Becas posdoctorales DGAPA-UNAM) for one year, with the possibility of extension for a second one. Applications are accepted twice a year, usually in September (to start the following March) and June (to start the following September). People interested in applying to work in Spectral and Scattering Theory, Quantum Mechanics, Quantum Field Theory, Quantum Information, and Quantum Optics, in collaboration with researchers of the Department of Mathematical Physics of the Institute for Research in Applied Mathematics and Systems ([IIMAS](#)) and who would like more information are welcomed to contact [Dr Ricardo](#)



Weder, [weder@unam.mx](mailto:weder@unam.mx), or the Academic Secretary of IIMAS, Dr Ricardo Berlanga, [berlanga@unam.mx](mailto:berlanga@unam.mx). In order to have time to prepare the required documents please write sufficiently earlier than the application deadline.

### Two W2-professor positions in Munich

The Faculty of Mathematics, Informatics, and Statistics of Ludwig-Maximilians-Universität München invites applications for two *Professorships (W2) in Applied Mathematics* commencing as soon as possible. One is a fixed-term position until September 30, 2017, the other may become tenured, subject to an evaluation. See

<http://www.uni-muenchen.de/aktuelles/stellenangebote/profs/20120926141928.html>

The successful candidates will represent mathematical analysis, including analytic methods in stochastics or geometry, in research and teaching. He or she is expected to actively participate in the Elite Masters Program *Theoretical and Mathematical Physics* and other study programs of the Mathematics Department.

Applications comprising a curriculum vitae, documentation of academic degrees and certificates as well as a list of publications under the keyword “math” should be submitted to the Dean of the Faculty Mathematics, Informatics, and Statistics, Prof. Dr. Volker Heun, Theresienstraße 39, 80333 Munich, Germany.

The deadline for applications is November 1, 2012.

More job announcements are on the job announcement page of the IAMP

[http://www.iamp.org/page.php?page=page\\_positions](http://www.iamp.org/page.php?page=page_positions)

which gets updated whenever new ads come in.

**Manfred Salmhofer** (IAMP Secretary)

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