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Women in Mathematical Physics

International Association of Mathematical Physics

Contacts. http://www.iamp.org and e-mail: bulletin@iamp.org

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A focus on our community

by EVANS HARRELL

This month we are pleased to bring you a thematic issue on 'Women in mathematical physics." The April issue celebrates Olga Ladyzhenskaya, whose 100th birthday was on 7 March of this year, as well as several others of succeeding generations. The report of the human rights session on the position of women at the 2021 International Congress is published here along with original work analyzing data from 2021 and previous meetings. We are very grateful especially to Serena Cenatiempo and Eva Miranda for their energy and vision in bringing this thematic issue about.

Future April issues will similarly be devoted to topics of social or political interest to the membership of the IAMP. The appalling war that has begun in Europe, the barycentric continent of our community, is already pointing the way to a likely theme for April, 2023. (The recent statement by the IAMP Executive Committee about the war in Ukraine follows below.) Suggestions about this and other future themes for the *News Bulletin* are welcome at bulletin@iamp.org. Meanwhile, several scientific societies have organized resources for people who wish to provide help now, including the American Mathematical Society and the American Physical Society, at

- https://www.ams.org/displaced/ukraine-resources and
- https://aps.org/programs/international/ukraine.cfm,

respectively.

To all IAMP members: the following

Statement about the war in Ukraine

was approved by the IAMP Executive Committee on Monday, March 7, 2022.

The Executive Committee of the International Association of Mathematical Physics (IAMP) stands in solidarity with the Ukrainian people in general and Ukrainian scientists in particular and is unambiguous in its condemnation of the violence unleashed by the military offensive in their country. We also acknowledge the large number of Russian colleagues who have expressed solidarity for their Ukrainian colleagues and have spoken out against the war. From its origin, the IAMP has aimed to foster international research exchange and cooperation and is committed to continue to do so across all boundaries. We join scientists from all over the world in calling for an immediate end to the military conflict.

The Vlasov equation as the mean-field and semiclassical limit of many interacting fermions

by CHIARA SAFFIRIO

Chiara Saffirio is an assistant professor at the Department of Mathematics and Computer Science of the University of Basel, where she heads the mathematical physics research group. Her research is concerned with many-body systems, and she has contributed to the theory of the Boltzmann equation, the Hartree-Fock equation and the equations of plasmas.

At the 2021 International Congress on Mathematical Physics (ICMP) in Geneva, Saffirio was awarded the Young Scientist Prize of the International Union of Pure and Applied Physics (IUPAP) *for her important contributions towards the mathematical understanding of the dynamics of classical and quantum many-body systems, leading to rigorous derivations of effective evolution equations.*

1 Introduction

The rigorous derivation of effective macroscopic equations obtained from the underlying microscopic laws of classical and quantum dynamics is a major question in non-equilibrium statistical mechanics, whose mathematical interest goes back to Hilbert's VI problem. As many relevant physical systems are typically composed of an enormous number of constituents, a quantitative analysis from the fundamental laws of motion (such as the Newton or Schrödinger equations) is practically impossible. In most cases, however, the time evolution of the system is expected to be well approximated by emergent effective laws of motion, involving a smaller number of degrees of freedom. Depending on the scaling regime, different suitable averaging mechanisms take place at a mesoscopic scale, allowing for such a dimension reduction and giving rise to effective equations describing the many-body system in the considered regime. The derivation problem consists in understanding the range of validity of the effective descriptions from the fundamental laws of motion.

In the last decade a renewed interest in the derivation problem originated from new methods developed to obtain explicit estimates on the error in the approximation has arisen and advances have been made, for instance to better understand the Gross-Pitaevskii and Boltzmann-Grad limits. Among the different possible regimes, the mean-field one is so far the most understood and considerable progress has been made in recent years. However, both for classical particles and for particles obeying the Fermi statistics the rigorous derivation leading to the so-called Vlasov equation in presence of a singular interaction potential is mathematically very challenging. Nonetheless this class of potentials includes the Coulomb and gravitational interactions, which play a central role in applications.

In [30, 12] we could tackle the problem in the case of fermions in a mean-field and semiclassical regime for moderately singular potentials, locally in time. Very recently we extended the result globally in time in [13]. The details of this progress will be presented below.

Mathematical Setting. We consider a system of N interacting fermions in a state $\psi_N \in L^2_{\rm a}(\mathbb{R}^{3N})$, the space of square integrable functions antisymmetric w.r.t. permutations of variables. The associated Hamiltonian

$$H_{\rm trap} = \sum_{i=1}^{N}(-\Delta_{x_i} + V_{\rm ext}(x_i)) + \lambda \sum_{i<j}^{N} V(x_i - x_j) \tag{1}$$

is a self-adjoint operator acting on $L^2_{\rm a}(\mathbb{R}^{3N})$, where λ is a coupling constant, V is a real-valued two-body interaction potential, and $V_{\rm ext}$ is an external potential confining the N particles in a volume of order one.
We are interested in the mean-field regime, thus we want to balance the contributions given by the kinetic and potential energies. On the one hand, because of Pauli's principle, the kinetic energy of N particles trapped in a volume of order one is at least of the order $N^{\frac{5}{3}}$, as suggested by the Lieb-Thierring kinetic-energy inequality

$$\langle \psi_N, \sum_{i=1}^{N} -\Delta_{x_i} \psi_N \rangle \geq C \int \rho_{\psi_N}(x)^{\frac{5}{3}} dx,$$

where $\rho_{\psi_N}(x) := N \int |\psi_N(x, x_2, \dots, x_N)|^2 dx_2 \dots dx_N$. On the other hand, the potential energy is of order $\lambda\, N^2$. Hence the choice $\lambda = N^{-1/3}$ allows us to get kinetic and potential energies of the same size.
Since we are interested in the dynamics of the system, we set the external potential $V_{\rm ext}$ to zero and let the particles evolve in time. To catch effects at macroscopic time scales of order one, we scale also the time variable by a factor $N^{-1/3}$. Thus, multiplying the equation by $\hbar^2 = N^{-2/3}$, the rescaled many-body Schrödinger equation reads

$$i\hbar\partial_t \psi_{N,t} = \left[\sum_{i=1}^{N} -\hbar^2 \Delta_{x_i} + \frac{1}{N}\sum_{j\neq i}^{N} V(x_i - x_j)\right] \psi_{N,t} \tag{2}$$

with

$$\hbar = N^{-1/3} \tag{3}$$

the Planck's constant. This means that, for a large number of particles, we look at big quantum numbers, hence $\hbar$ small. In other words, because of the relation between N and $\hbar$, the fermionic mean-field limit is naturally coupled with a semiclassical regime, witnessing the scale separation between the wavelength and the length scale of the observables.
Having clarified the regime to look at, we switch to the von Neumann representation of the dynamics and define the N-particle density matrix $\boldsymbol{\rho}_N$

$$i\hbar\partial_t \boldsymbol{\rho}_N = [H_N, \boldsymbol{\rho}_N], \tag{4}$$

where $[A,B] := AB - BA$ and

$$H_N = \sum_{i=1}^{N} -\hbar^2 \Delta_{x_i} + \frac{1}{N}\sum_{i<j}^{N} V(x_i - x_j)\,.$$

We distinguish two cases: when $\mathrm{Rank}\,(\boldsymbol{\rho}_N) = 1$ we say that $\boldsymbol{\rho}_N$ represents a pure state; when $\mathrm{Rank}\,(\boldsymbol{\rho}_N) > 1$ we say that $\boldsymbol{\rho}_N$ represents a mixed state.
We define the one-particle reduced density matrix

$$\boldsymbol{\rho}_{N:1} = \mathrm{Tr}_{2\ldots N}(\boldsymbol{\rho}_N).$$

When the number of particles is sufficiently large, we expect $\boldsymbol{\rho}_{N:1}$ to approach in some topology a solution $\boldsymbol{\rho}$ to the time-dependent Hartree-Fock equation

$$i\hbar\partial_t \boldsymbol{\rho} = \left[-\hbar^2\Delta + V_{\boldsymbol{\rho}} - X_{\boldsymbol{\rho}}, \boldsymbol{\rho}\right]\,, \tag{5}$$

where $\boldsymbol{\rho} = \boldsymbol{\rho}(t)$ is a time-dependent nonnegative, bounded, trace class operator such that $\mathrm{Tr}(\boldsymbol{\rho}) = \hbar^{-3}$; $V_{\boldsymbol{\rho}}$ is the mean-field potential given by the multiplication operator by $V * \rho$; $\rho(x) = \mathrm{diag}(\boldsymbol{\rho})(x) := \hbar^3 \boldsymbol{\rho}(x,x)$ is the spatial density and $X_{\boldsymbol{\rho}}$ the exchange term given through its integral kernel $X_{\boldsymbol{\rho}}(x,y) = V(x-y)\boldsymbol{\rho}(x,y)$. Notice that we have used $\boldsymbol{\rho}(x,y)$ to denote the kernel of the operator $\boldsymbol{\rho}$.
Observe that the choice of the normalization of $\boldsymbol{\rho}$ guarantees that $\int_{\mathbb{R}^3} \rho(x)\,dx = \hbar^3\,\mathrm{Tr}(\boldsymbol{\rho}) = 1$. Thus equation (5) still depends on the number of particles N both through $\hbar$ and $\boldsymbol{\rho}$. It is therefore natural to investigate the limit $\hbar \to 0$, or $N \to \infty$. To this end, we recall the definition of the Wigner transform

$$f_{\boldsymbol{\rho}}(x,v) = \int e^{-y\cdot v/\hbar} \boldsymbol{\rho}\left(x + \frac{y}{2}, x - \frac{y}{2}\right)\,dy$$

as a function on the phase space with mass

$$\iint_{\mathbb{R}^6} f_{\boldsymbol{\rho}}(x,v)\,dx\,dv = \hbar^3\,\mathrm{Tr}(\boldsymbol{\rho}) = 1.$$

Then, looking at the Wigner transform of the solution $\boldsymbol{\rho}$ to (5) we have

$$\begin{aligned} &i\hbar\partial_t\, f_{\boldsymbol{\rho}} \\ &= \int i\hbar\partial_t \boldsymbol{\rho}\left(x+\frac{y}{2}, x-\frac{y}{2}\right)\, e^{-iv\cdot y/\hbar} dy \\ &= \int \left(-\Delta_{x+\frac{y}{2}} + \Delta_{x-\frac{y}{2}}\right)\, \boldsymbol{\rho}\left(x+\frac{y}{2}, x-\frac{y}{2}\right)\, e^{-iv\cdot y/\hbar}\,dy \\ &+ \int \left[V_{\boldsymbol{\rho}}\left(x+\frac{y}{2}\right) - V_{\boldsymbol{\rho}}\left(x-\frac{y}{2}\right)\right]\, \boldsymbol{\rho}\left(x+\frac{y}{2}, x-\frac{y}{2}\right)\, e^{iv\cdot y/\hbar}\,dy. \end{aligned}$$

Performing the change of variable $y \to \hbar y$ and using that $-\Delta_{x+\frac{\hbar y}{2}} + \Delta_{x-\frac{\hbar y}{2}} = -2\hbar\nabla_x\cdot\nabla_y$ and $\left[V_{\boldsymbol{\rho}}\left(x+\frac{\hbar y}{2}\right) - V_{\boldsymbol{\rho}}\left(x-\frac{\hbar y}{2}\right)\right] \simeq \hbar\, y\cdot\nabla V_{\boldsymbol{\rho}} + O(\hbar^2)$, dividing by $i\hbar$ and integrating in y, we heuristically get

$$\partial_t\, f_{\boldsymbol{\rho}} = -2\nabla_x f_{\boldsymbol{\rho}} + \nabla V_{\boldsymbol{\rho}}\cdot\nabla_v f_{\boldsymbol{\rho}} + O(\hbar). \tag{6}$$

Hence we expect $f_{\boldsymbol{\rho}}$ to approach in the limit of large N (or equivalently $\hbar \to 0$, because of the relation (3)) f, solution to the Vlasov equation

$$\partial_t f + 2v \cdot \nabla_x f - \nabla(V * \rho_f) \cdot \nabla_v f = 0, \tag{7}$$

where $f = f(t,x,v)$, $(x,v) \in \mathbb{R}^3 \times \mathbb{R}^3$, is the classical phase space distribution of particles, $V * \rho_f$ is the self-induced force field and ρ_f is the spatial density defined as $\rho_f(t,x) = \int f(t,x,v)\,dv$. Equation (7) describes the classical dynamics of a large number of interacting particles when collisions are not dominant and subject to many weak interactions whose collective effect can be approximated by an averaged mean-field potential. This model is extensively used in plasma physics and astrophysics.

State of the art. The first rigorous derivation of the Vlasov equation (7) from the N-body Schrödinger equation (2) in a combined mean-field and semiclassical limit was proven in [35, 44] in the case of smooth potentials, later reconsidered in [24, 11]. In order to obtain explicit bounds on the convergence rate and to relax the regularity assumptions on the interaction, the Hartree-Fock equation has been used as a bridge between the many-body Schrödinger equation and the Vlasov system. The derivation of the Hartree-Fock equation for $N = \hbar^{-3}$ large but finite has been considered in [16] for analytic potentials and for short times. It was extended in [8] to arbitrarily large fixed times and to smooth potentials, with explicit rates of convergence. The proof in [8], based on second quantization techniques reminiscent of [19, 39], holds for smooth interactions and strongly relies on assumptions on the semiclassical structure of the initial data:

$$\mathrm{Tr}\,|[x,\boldsymbol{\rho}]| \le CN\hbar, \qquad \mathrm{Tr}\,|[i\hbar\nabla,\boldsymbol{\rho}]| \le CN\hbar. \tag{8}$$

In the same spirit, partial results have been obtained for singular interactions in [40, 41].
The same problem has been studied in different regimes in [17, 5, 18, 4, 37] for the Coulomb interaction. Recently, new techniques reminiscent of the ones used in the mean-field limit for systems of classical particles have been developed in [20, 21, 23, 22]. Once the validity of the Hartree-Fock approximation is established, one can investigate its semiclassical limit $\hbar = N^{-1/3} \to 0$. The semiclassical limit towards the Vlasov equation was proven in [33] in a weak topology, including singular potentials such as the Coulomb interaction. Explicit rates in stronger topologies were then obtained in [3, 1, 7] for regular potentials, in [42, 43] for a certain class of singular potentials and in [21, 28, 29] for regular and singular interactions in a weak topology. In [32] the semiclassical limit was proven for infinite gases for local perturbations of stationary states.

Latest progress. Although much progress have been achieved in recent years, explicit rates of convergence for the most relevant physical cases in which particles interact through the Coulomb or gravitational potentials are still out of reach. An effort to tackle this problem has been recently made in [30, 12, 13], where the Vlasov equation has been derived from a system of N fermions in three dimensions, interacting through an inverse-power law potential of the form

$$V(x) = \pm\frac{1}{|x|^a} \tag{9}$$

globally in time for $a \in [0, \frac{1}{2})$, and for times of order $\hbar^{\frac{1}{2}}$ for $a \in [\frac{1}{2}, 1]$.
Before stating the result, we introduce some notation. Let $\mathcal{L}^p$ be the semiclassical analogue of Lebesgue spaces equipped with the rescaled Schatten norm

$$\|\boldsymbol{\rho}\|_{\mathcal{L}^p} = \hbar^{\frac{3}{p}} \|\boldsymbol{\rho}\|_p = \hbar^{\frac{3}{p}} \operatorname{Tr}(|\boldsymbol{\rho}|^p)^{\frac{1}{p}},$$

and $\mathcal{L}^\infty$ the space of bounded operators equipped with the operator norm $\|\boldsymbol{\rho}\|_{\mathcal{L}^\infty}$. For $\mathrm{m} := 1 + |\boldsymbol{p}|^n$, where $\boldsymbol{p} = -i\hbar\nabla$ is the momentum operator, we define the weighted semiclassical Lebesgue norms by $\|\boldsymbol{\rho}\|_{\mathcal{L}^p(\mathrm{m})} := \|\boldsymbol{\rho}\,\mathrm{m}\|_{\mathcal{L}^p}$ and the semiclassical analogue of Sobolev norms by

$$\|\boldsymbol{\rho}\|^p_{\mathcal{W}^{1,p}(\mathrm{m})} := \|\boldsymbol{\rho}\|^p_{\mathcal{L}^p(\mathrm{m})} + \textstyle\sum_{j=1}^3 \left\|\boldsymbol{\nabla}_{v_\mathrm{j}}\boldsymbol{\rho}\right\|^p_{\mathcal{L}^p(\mathrm{m})} + \left\|\boldsymbol{\nabla}_{x_\mathrm{j}}\boldsymbol{\rho}\right\|^p_{\mathcal{L}^p(\mathrm{m})} \qquad p \in [1,\infty),$$

$$\|\boldsymbol{\rho}\|_{\mathcal{W}^{1,\infty}(\mathrm{m})} := \|\boldsymbol{\rho}\|_{\mathcal{L}^\infty(\mathrm{m})} + \sup_{j=1,2,3}\left(\left\|\boldsymbol{\nabla}_{v_\mathrm{j}}\boldsymbol{\rho}\right\|_{\mathcal{L}^\infty(\mathrm{m})} + \left\|\boldsymbol{\nabla}_{x_\mathrm{j}}\boldsymbol{\rho}\right\|_{\mathcal{L}^\infty(\mathrm{m})}\right) \quad p = \infty,$$

where

$$\boldsymbol{\nabla}_x\boldsymbol{\rho} := [\nabla, \boldsymbol{\rho}] \quad \text{and} \quad \boldsymbol{\nabla}_v\boldsymbol{\rho} := \left[\frac{x}{i\hbar}, \boldsymbol{\rho}\right].$$

Given a function f on the phase space, let $\boldsymbol{\rho}_f$ be the Weyl transform, defined as the operator with integral kernel

$$\boldsymbol{\rho}_f(x,y) = \int_{\mathbb{R}^3} e^{-2i\pi(y-x)\cdot v} f\left(\frac{x+y}{2}, \hbar v\right) dv.$$

Furthermore, for $\mathfrak{h} := L^2(\mathbb{R}^3)$, let $\mathfrak{h}^{\wedge n} = \mathfrak{h} \wedge \cdots \wedge \mathfrak{h}$ be the n-fold antisymmetric tensor product of $\mathfrak{h}$ and define the fermionic Fock space over $\mathfrak{h}$ by

$$\mathcal{F} := \mathbb{C} \oplus \bigoplus_{n=1}^\infty \mathfrak{h}^{\wedge n}$$

equipped with the norm induced by the inner product on $\mathcal{F}$.
We define the number-of-particles operator

$$\mathcal{N}\psi = \left(n\psi^{(n)}\right)_{n\in\mathbb{N}}$$

where $\psi^{(n)}$ is the n-particle sector of the vector in the Fock space $\psi \in \mathcal{F}$. As done previously, we denote by $\mathcal{L}^p(\mathcal{F})$ the semiclassical Lebesgue spaces on the Fock space with norm $\|\boldsymbol{\rho}_N\|_{\mathcal{L}^p(\mathcal{F})} := \hbar^{\frac{3}{p}} \operatorname{Tr}(|\boldsymbol{\rho}_N|^p)^{\frac{1}{p}}$, so that $\|\boldsymbol{\rho}_N\|_{\mathcal{L}^1(\mathcal{F})} = 1$ and $\|\mathcal{N}\boldsymbol{\rho}_N\|_{\mathcal{L}^1(\mathcal{F})} = N$.
We now state some assumptions that will represent the main hypotheses on the initial states we consider:

(A1) *Normalization constraints.* Let $\boldsymbol{\rho}$ be a bounded operator satisfying the following normalization constraints

$$\|\boldsymbol{\rho}\|_{\mathcal{L}^\infty} = C_\infty, \qquad \operatorname{Tr}(\boldsymbol{\rho}) = \hbar^{-3}$$

for some constant $C_\infty > 0$.

(A2) *Quantum propagation of regularity, uniform in $\hbar$.* Let $\boldsymbol{\rho}$ be a nonnegative operator satisfying

$$\boldsymbol{\rho} \in \mathcal{W}^{2,2}(\mathrm{m}) \cap \mathcal{W}^{2,4}(\mathrm{m})$$
$$\sqrt{\boldsymbol{\rho}} \in \mathcal{W}^{1,2}(\mathrm{m}) \cap \mathcal{W}^{1,q}(\mathrm{m})$$

with $q \in \left[\frac{6}{1-2a}, \infty\right]$, where a is the power of the singular interaction.

(A3) *Classical propagation of moments and regularity.* Let f be a function on the phase space such that

$$(1+|x|^8+|v|^8)\nabla_x^\ell \nabla_v^m f \in L^\infty(\mathbb{R}^6) \cap L^2(\mathbb{R}^6), \qquad \ell + m \leq 9$$

We are now ready to state our result.

Theorem 1. *Let $a \in (0, \frac{1}{2})$ in (9), $n \in \mathbb{N}$ such that $n > 3$ and $\boldsymbol{\rho}$ be a solution to the Hartree-Fock equation (5) with initial datum $\boldsymbol{\rho}^{\mathrm{in}} \in \mathcal{L}^\infty(\mathrm{m})$ satisfying assumptions* (A1) *and* (A2)*. Let f be a nonnegative solution to the Vlasov equation (7) with initial datum f^{in} satisfying* (A3)*. Let $\boldsymbol{\rho}_N$ be a solution to the Liouville-von Neumann equation (4) with initial condition $\boldsymbol{\rho}_N^{\mathrm{in}}$ such that $\boldsymbol{\rho}_N^{\mathrm{in}} \in \mathcal{L}^1(\mathcal{F})$ and $\left[\mathcal{N}, \boldsymbol{\rho}_N^{\mathrm{in}}\right] = 0$. Then for every $T > 0$ there exist an operator $\boldsymbol{\rho}_{N,f}^{\mathrm{in}} \in \mathcal{L}^1(\mathcal{F})$ and a constant $C_T > 0$ such that*

$$\left\|\boldsymbol{\rho}_{N:1} - \boldsymbol{\rho}_f\right\|_{\mathcal{L}^1} \leq C_T \left(\frac{1}{N} + \hbar\right)\left(1 + \left\|(\mathcal{N}+N)^k(\boldsymbol{\rho}_N^{\mathrm{in}} + \boldsymbol{\rho}_{N,f}^{\mathrm{in}})\right\|_{\mathcal{L}^1(\mathcal{F})}\right), \tag{10}$$

for $k \geq \frac{1}{2} + \frac{3}{2}\lceil\frac{\ln N}{\ln(N\hbar^2)}\rceil$.

We observe that the leading order in the approximation is dictated by the semiclassical parameter $\hbar = N^{-\frac{1}{3}}$. This is indeed expected to be the optimal rate of convergence as suggested by the heuristic computation (6). Notice moreover that the semiclassical structure assumed in (A2) on the initial state is the (generalized) analogue of assumption (8), that was the main request on the initial state in [8].
Furthermore, if we consider a cut-off version of the Coulomb potential, for instance of the form

$$V_R(x) = \int_0^{R^{-2}} \frac{e^{-\pi|x|^2}s}{\sqrt{s}} ds \longrightarrow \frac{1}{|x|} \quad \text{as } R \to 0,$$

then we get

$$\left\|\boldsymbol{\rho}_{N:1} - \boldsymbol{\rho}_f\right\|_{\mathcal{L}^1} \leq \frac{C_T\, e^{t/\sqrt{R}}}{\sqrt{N}} + C_T\,\hbar,$$

that, due to the constraint $k \geq \frac{1}{2} + \frac{3}{2}\lceil\frac{\ln N}{\ln(N\hbar^2)}\rceil$, implies a convergence result for times $t \ll \hbar^{\frac{1}{2}}$.
Furthermore, we can get an analogue of Theorem 1 in Hilbert-Schmidt norm:

$$\left\|\boldsymbol{\rho}_{N:1} - \boldsymbol{\rho}_f\right\|_{\mathcal{L}^2} \leq C_T \left(\frac{1}{\sqrt{N}} + \hbar\right)\left(1 + \left\|(\mathcal{N}+N)^k(\boldsymbol{\rho}_N^{\mathrm{in}} + \boldsymbol{\rho}_{N,f}^{\mathrm{in}})\right\|_{\mathcal{L}^1}\right).$$

Notice that if $\left(1 + \left\|(\mathcal{N}+N)^k(\boldsymbol{\rho}_N^{\mathrm{in}} + \boldsymbol{\rho}_{N,f}^{\mathrm{in}})\right\|_{\mathcal{L}^1}\right) \leq C$, we obtain convergence in L^2 for the functions on the phase space using that $\left\|\boldsymbol{\rho}_f\right\|_{\mathcal{L}^2} = \|f\|_{L^2}$, thus proving the quantitative bound

$$\|f_{N:1} - f\|_{L^2(\mathbb{R}^6)} \leq C_T\left(\frac{1}{\sqrt{N}} + \hbar\right),$$

where $f_{N:1}$ is the Wigner transform of the one-particle reduced density matrix $\boldsymbol{\rho}_{N:1}$.

2 Fermionic Mean-Field Regime: Pure States

Step 1. Purification. The very first difficulty we encounter arises from considering mixed states instead of pure states. For a spectral set $\{\lambda_j, \psi_j\}_{j\geq 0}$, $\lambda_j \in [0,1]$ and $\psi_j \in \mathcal{F}(\mathfrak{h})$, we can express $\boldsymbol{\rho}_N$ as

$$\boldsymbol{\rho}_N = \sum_{j\geq 0} \lambda_j |\psi_j\rangle\langle\psi_j|,$$

which is clearly not a rank one projection. However, we can see it as a pure state on a larger Fock space by observing that

$$\boldsymbol{\rho}_N^{\frac{1}{2}} = \sum_{j\geq 0} \lambda_j^{\frac{1}{2}} |\psi_j\rangle\langle\psi_j| \simeq \sum_{j\geq 0} \lambda_j^{\frac{1}{2}} \psi_j \otimes \overline{\psi_j} \in \mathcal{F}\otimes\mathcal{F}$$

and by noticing that there exists an isomorphism U such that

$$\mathcal{F}(\mathfrak{h}) \otimes \mathcal{F}(\mathfrak{h}) \simeq_U \mathcal{F}(\mathfrak{h}\oplus\mathfrak{h}) =: \mathcal{G}. \tag{1}$$

With such a purification procedure, we can recast our problem for mixed states to a Cauchy problem for states that in the larger double Fock space $\mathcal{G}$ exhibit the structure of pure states (see [2, 14, 6]).

On $\mathcal{G}$ we introduce the left and right creation and annihilation operators. For every $f \in \mathfrak{h}$, let the left and right creation and annihilation operators be

$$\begin{aligned} a_l^*(f) &:= a^*(f\oplus 0), \quad & a_r^*(f) &:= a^*(0\oplus f),\\ a_l(f) &:= a(f\oplus 0), \quad & a_r(f) &:= a(0\oplus f), \end{aligned}$$

where a and a^* are the usual annihilation and creation operators on $\mathcal{F}$, satisfying the canonical anticommutation relations. Moreover, for an observable O with distributional kernel $O(x,y)$, we define the left and right second quantization of O by

$$\begin{aligned} d\Gamma_l(O) &:= d\Gamma(O\oplus 0) = \textstyle\int_{\mathbb{R}^6} O(x,y)\, a_{x,l}^*\, a_{y,l}\, dx\, dy\\ d\Gamma_r(O) &:= d\Gamma(0\oplus O) = \textstyle\int_{\mathbb{R}^6} O(x,y)\, a_{x,r}^*\, a_{y,r}\, dx\, dy \end{aligned}$$

respectively, where $a_{z,l}$, $a_{z,r}$ and $a_{z,l}^*$, $a_{z,r}^*$, are the left and right annihilation and creation operator-valued distributions at the position z. In particular the number-of-particles operator is given by

$$\mathcal{N} := \mathcal{N}_l + \mathcal{N}_r = d\Gamma_l(1) + d\Gamma_r(1).$$

In this new setting the solution of (4) with initial datum $\boldsymbol{\rho}_N^{\mathrm{in}}$ in the interaction picture

$$\boldsymbol{\rho}_N = e^{-iH_N t/\hbar}\boldsymbol{\rho}_N^{\mathrm{in}} e^{iH_N t/\hbar}$$

can be expressed as a vector $\Phi(t) \in \mathcal{G}$ such that

$$\Phi(t) := e^{-iL_N t/\hbar}\Phi^{\mathrm{in}}\ ,$$

where $L_N = U(H_N \otimes 1 - 1 \otimes H_N)U^*$, with U the isomorphism given in (1). Hence, we can write the one-particle reduced density matrix of $\boldsymbol{\rho}_N$ in terms of $\Phi(t)$ as

$$\boldsymbol{\rho}_{N:1}(x,y) = \langle \Phi(t),\, a^*_{x,l}\, a_{y,l}\, \Phi \rangle. \tag{2}$$

Step 2. Bogoliubov transformation. The advantage of second quantization in this problem consists in the use of the so-called Bogoliubov transformation, that allows us to represent a quasi-free mixed state as a rotation of the vacuum of the double Fock space $\mathcal{G}$. In simple terms, this corresponds to changing the reference frame (by a rotation) and looking at the problem from a different perspective, in which many problematic terms get automatically cancelled by the Bogoliubov rotation. In our setting, to eliminate several troublesome contributions we define the Bogoliubov transformation in terms of $\boldsymbol{\rho}$, the solution to the Hartree-Fock equation (5). More precisely, for

$$u = \sqrt{1-\boldsymbol{\rho}} \quad \text{and} \quad v = \sqrt{\boldsymbol{\rho}},$$

we can construct a unitary map $R_{\boldsymbol{\rho}} : \mathcal{G} \to \mathcal{G}$ such that

$$R^*_{\boldsymbol{\rho}}\, a_{x,l}\, R_{\boldsymbol{\rho}} = a_l(u_x) - a^*_r(\overline{v}_x), \quad R^*_{\boldsymbol{\rho}}\, a_{x,r}\, R_{\boldsymbol{\rho}} = a_r(\overline{u}_x) + a^*_l(v_x), \tag{3}$$

where we used the notation

$$u_x(y) = u(y,x), \qquad v_x(y) = v(y,x).$$

Notice that u, v are well defined because $\boldsymbol{\rho}$ is a fermionic operator, i.e. $0 \le \boldsymbol{\rho} \le 1$. Moreover, since $\mathrm{Tr}(\boldsymbol{\rho}) = N$, the p-Schatten norms of $\boldsymbol{\rho}$ and v are finite, for $p \in [1,\infty]$. However u, despite being bounded in $\mathcal{L}^\infty$, is not bounded in other Schatten norms and this makes the analysis more delicate.

The choice of R_ρ allows us to construct a quasi-free state with one-particle reduced density matrix $\boldsymbol{\rho}$ on $\mathcal{G}$. To this end, let $\Phi_{\boldsymbol{\rho}}$ be the rotation of the vacuum $\Omega_{\mathcal{G}} \in \mathcal{G}$ by the Bogoliubov transformation:

$$\Phi_{\boldsymbol{\rho}} = R_{\boldsymbol{\rho}}\Omega_{\mathcal{G}} \in \mathcal{G}. \tag{4}$$

Then, it is readily seen that the one-particle reduced density matrix associated with $\Phi_{\boldsymbol{\rho}}$ is

$$\langle \Phi_{\boldsymbol{\rho}},\, a^*_{l,y}\, a_{l,x}\, \Phi_{\boldsymbol{\rho}} \rangle = (v^*\, v)(x,y) = \boldsymbol{\rho}(x,y),$$

where we used the relations (3).

Step 3. Fluctuation dynamics. Using (4), for $\Psi_{\text{fluct}} \in \mathcal{G}$ defined as

$$\Psi_{\text{fluct}} := R_{\boldsymbol{\rho}}^{*}\Phi(t) = R_{\boldsymbol{\rho}}^{*}e^{-iL_N t/\hbar}R_{\boldsymbol{\rho}^{\text{in}}}\Omega,$$

and $\boldsymbol{\rho}_{N:1}$ as in Theorem 1, we can estimate the error in the mean-field approximation by the mean number of particles of the fluctuation dynamics around a quasi-free state, i.e.

$$\|\boldsymbol{\rho}_{N:1} - \boldsymbol{\rho}\|_{\mathcal{L}^1} \leq \frac{C}{\sqrt{N}} \left\|(\mathcal{N}+1)^{\frac{1}{2}}\Psi_{\text{fluct}}\right\|_{\mathcal{G}}^{2}. \tag{5}$$

To obtain (5), we notice that

$$\boldsymbol{\rho}_{N:1}(x,y) = \boldsymbol{\rho}(x,y) + \langle \Phi(t),\, a_{y,l}^{*}\, a_{x,l}\, \Phi(t)\rangle = \langle \Psi_{\text{fluct}},\, R_{\boldsymbol{\rho}}^{*}\, a_{y,l}^{*}\, R_{\boldsymbol{\rho}}\, R_{\boldsymbol{\rho}}^{*}\, a_{x,l}\, R_{\boldsymbol{\rho}}\, \Psi_{\text{fluct}}\rangle.$$

By (3) for any observable O, we obtain

$$\mathrm{Tr}(O(\boldsymbol{\rho}_{N:1} - \boldsymbol{\rho})) = \langle \Psi_{\text{fluct}},\, \big(d\Gamma_l(u\,O\,u) - d\Gamma_r(\overline{v}\,O\,v) - d\Gamma_{l,r}^{+}(v\,O\,u) - d\Gamma_{r,l}^{-}(v\,O\,u)\,\Psi_{\text{fluct}}\big)\rangle,$$

where $d\Gamma_{\sigma,\sigma'}^{+}(O) = \int O(x,y)\, a_{x,\sigma}^{*}\, a_{y,\sigma'}^{*}\, dx\, dy$ and $d\Gamma_{\sigma,\sigma'}^{-}(O) = \int O(x,y)\, a_{x,\sigma}\, a_{y,\sigma'}\, dx\, dy$. Using that $\|u\|_{\mathcal{L}^\infty} \leq 1$ and $\|v\|_{\mathcal{L}^\infty} \leq 1$, and that $d\Gamma(O)$ can be bounded in terms of the number-of-particles operator, we get

$$\mathrm{Tr}(O(\boldsymbol{\rho}_{N:1} - \boldsymbol{\rho})) \leq C\, \|O\|_{\mathcal{L}^\infty} \left\|(\mathcal{N}+1)^{\frac{1}{2}}\Psi_{\text{fluct}}\right\|_{\mathcal{G}}$$

which by duality yields (5). Whence, to control the mean-field approximation error, we need to bound the expectation of the number of particles in the fluctuation state Ψ_{fluct}.

Step 4. Control on the growth of $\mathcal{N}$ in the fluctuation state. We introduce the notation

$$\mathcal{U}(t,s) := R_{\boldsymbol{\rho}}^{*}\, e^{-iL_N t/\hbar}\, R_{\boldsymbol{\rho}^{\text{in}}}$$

so that Ψ_{fluct} can be written as $\Psi_{\text{fluct}} = \mathcal{U}(t,0)\,\Omega$. We refer to $\mathcal{U}(t,s)$ as the fluctuation dynamics, the unitary two-parameter semigroup with generator G_t satisfying

$$i\hbar\partial_s \mathcal{U}(t,s) = G_t\, \mathcal{U}(t,s), \qquad \mathcal{U}(s,s) = 1.$$

The generator G is given by

$$G_t = d\Gamma_l(H_{\boldsymbol{\rho}}) - d\Gamma_r(\overline{H}_{\boldsymbol{\rho}}) + D + (Q + \widetilde{Q} + \text{h.c.}),$$

where

$$H_{\boldsymbol{\rho}} = -\hbar^2\Delta + V_{\boldsymbol{\rho}} - X_{\boldsymbol{\rho}} \tag{6}$$

is the Hartree-Fock Hamiltonian, D contains terms that commute with $\mathcal{N}$, Q and $\widetilde{Q}$ contain terms that do not commute with the number operator.

In step 3 we highlighted that to give an explicit bound on the mean-field approximation error we need to bound the number of particles in Ψ_{fluct}. In other words, we consider

$$i\hbar\partial_t\, \langle \mathcal{U}(t,0)\psi,\, (\mathcal{N}+N)\,\mathcal{U}(t,0)\,\psi\rangle \tag{7}$$

for $\psi \in \mathcal{G}$ and bound it by means of Grönwall's Lemma. The result is the following

Proposition 1. *For $k_0, k > 0$ and $\psi \in \mathcal{G}$, it holds*

$$\left\|(\mathcal{N}+N)^{k_0}\mathcal{U}(t,0)\,\psi\right\|_{\mathcal{G}} \leq C\,e^{\lambda\,t}\left(\left\|(\mathcal{N}+N)^{k_0+\frac{3}{2}k}\psi\right\|_{\mathcal{G}} + \frac{\hbar^{\frac{k}{2}}t}{N^{\frac{k}{2}-k_0}}\left\|(\mathcal{N}+N)^{\frac{3}{2}k}\psi\right\|_{\mathcal{G}}\right),$$

where λ depends on $\left\|\boldsymbol{\nabla}_v\sqrt{\boldsymbol{\rho}}\,\mathrm{m}\right\|_{\mathcal{L}^{q_0}}$ and $\left\|\boldsymbol{\nabla}_v\sqrt{\boldsymbol{\rho}}\,\mathrm{m}\right\|_{\mathcal{L}^{q_1}}$, $1 \leq q_0 < q_1 < \infty$ such that $\frac{1}{2}\left(\frac{1}{q_0}+\frac{1}{q_1}\right) = 1-\frac{3}{a+1}$.

Notice that $\left\|\boldsymbol{\nabla}_v\sqrt{\boldsymbol{\rho}}\,\mathrm{m}\right\|_{\mathcal{L}^{q}}$ can be bounded uniformly in $\hbar$ (see [12, Part II] and [13]). To prove Proposition 1 we show that each term in the generator G_t that do not commute with $\mathcal{N}$ is bounded uniformly in $\hbar$. However, two difficulties arise:

(i) When $i\hbar\partial_t$ acts on $\mathcal{U}(t,s)$ in (7), we get the generator G_t containing terms where the singular interaction appears;

(ii) We need to cancel the $\hbar$ on the right-hand side of (7), and therefore to exploit the hidden commutator structure, in particular the fact that $[u,v]=0$.

To summarize, we need to control the terms in the generator that do not commute with the number-of-particle operator, namely $\widetilde{Q}$ and Q (quartic terms in the creation and annihilation operators). We first focus on

$$\widetilde{Q} = \frac{1}{2N}\int_{\mathbb{R}^3} d\Gamma^+_{l,r}(uV_xv)\,d\Gamma^+_{l,r}(u\delta_xv) - d\Gamma^+_{l,r}(v\delta_xu)\,d\Gamma^+_{l,r}(uV_xv)\,dx,$$

where $u\delta_xv$ denotes the operator with kernel $(u\delta_xv)(y,z) = u(y,x)v(x,z)$ and $V_x(y) = V(x-y)$. The term in which $\widetilde{Q}$ appears can be bounded uniformly in $\hbar$ by exploiting the commutator structure

$$uV_xv = vV_xu + u\,[V_x,v] - v\,[V_x,u]\,.$$

The contributions of Q are more difficult to handle. They are of the form

$$\frac{1}{N}\iint\limits_{\mathbb{R}^6} V(x-y)\,a^*_l(u_x)\,a^*_r(\overline{v}_x)\,a^*_l(u_y)\,a_l(u_y)\,dx\,dy.$$

In order to exploit the hidden commutator structure we further decompose this term using that $[u,v]=0$. Combining all the terms we obtain the decomposition

$$Q = (P + \widetilde{P} + \mathrm{h.c.}).$$

The terms in P are the ones responsible for the restriction to inverse power law potentials with $a<\frac{1}{2}$; the terms in $\widetilde{P}$ do not present cancellations and we will deal with them in Step 5 below. The terms in P are of the form

$$\mathfrak{P} = \frac{1}{N}\int_{\mathbb{R}^3} a^*_l(u_x)\,a^*_r(\overline{v}_x)\,d\Gamma_l(V_x)\,dx.$$

By the Cauchy-Schwarz inequality, for every $\psi_1,\ \psi_2 \in \mathcal{G}$ we get

$$\begin{aligned}|\langle\psi_1,\mathfrak{P}\psi_2\rangle| &\leq \frac{1}{N}\left(\int_{\mathbb{R}^3}\|a_l(u_x)\,\psi_1\|^2_{\mathcal{G}}\right)^{\frac{1}{2}}\left(\int_{\mathbb{R}^3}\|a^*_r(\overline{v}_x)\,d\Gamma_l(V_x)\,\psi_2\,\|^2_{\mathcal{G}}\right)^{\frac{1}{2}}\\ &\leq \frac{1}{N}\left\|\mathcal{N}_l^{\frac{1}{2}}\psi_1\right\|_{\mathcal{G}}\left(\int_{\mathbb{R}^3}\rho(x)\ \|d\Gamma_l(V_x)\,\psi_2\|^2_{\mathcal{G}}\,dx\right)^{\frac{1}{2}}.\end{aligned}$$

where in the last inequality we used that $\|v_x\|_{L^2} = N\rho(x)$. Recall that $V_x(y) = \frac{1}{|x-y|^a}$. Thus, by Hardy-Littlewood-Sobolev inequality

$$\int_{\mathbb{R}^3} \rho(x) \, \|d\Gamma_l(V_x)\, \psi_2\|_{\mathcal{G}}^2 \, dx \le C \iint_{\mathbb{R}^6} \frac{\rho(x)\, g(y)}{|x-y|^{2a}} dx\, dy \le C \, \|\rho\|_{L^{3/(3-2a)}} \, \|g\|_{L^1} \, ,$$

where $g(y) = \left\| \psi_2^{(n,m)}(y, x_1, \ldots, x_{n-1}, y_1, \ldots, y_m) \right\|^2_{L^2(dx_1 \ldots dx_{n-1}\, dy_1 \ldots dy_m)}$, where $\psi_2^{(n,m)}$ is the (n,m)-sector of ψ_2 in $\mathcal{G}$. Notice that the last estimate entails the restriction $a < \frac{1}{2}$.

Step 5. Auxiliary fluctuation dynamics. We are left with the terms in $\widetilde{P}$ that do not commute with $\mathcal{N}$ nor present cancellations due to the commutator structure. To deal with them, we modify the generator of the fluctuation dynamics $\mathcal{U}(t,s)$ using a perturbative argument. More precisely, we split the generator G_t into two parts

$$G_t = \widetilde{G}_t + B_t,$$

where B_t is small for N large, and $\widetilde{G}_t$ defines a new auxiliary fluctuation dynamics $\widetilde{\mathcal{U}}(t,s)$ as the solution of the Cauchy problem

$$i\hbar\partial_t \widetilde{\mathcal{U}}(t,s) = \widetilde{G}_t \widetilde{\mathcal{U}}(t,s)\,, \qquad \widetilde{\mathcal{U}}(s,s) = 1, \tag{8}$$

whose well-posedness has been shown in [12, Appendix A].
The terms in $\widetilde{P}$ are therefore absorbed into the new generator $\widetilde{G}_t$ and the smallness of B_t allows us to use $\widetilde{G}_t$ instead of G_t, paying the price of an additional small error term. This enables us to prove Proposition 1, which together with the estimate in (5) concludes the proof of the convergence rate for the mean-field approximation from the many-body dynamics (4) to the Hartree-Fock equation (5).

3 Semiclassical Limit

Once the mean-field approximation of the many-body evolution by the solution to the Hartree-Fock equation is obtained, it is legitimate to investigate the limit $\hbar \to 0$ in order to get the Vlasov equation. To this end, we consider the Weyl transform of the Vlasov equation (7)

$$i\hbar\partial_t \boldsymbol{\rho}_f = \left[-\hbar^2\Delta, \boldsymbol{\rho}_f\right] + A_{\boldsymbol{\rho}_f}, \tag{1}$$

where $\boldsymbol{\rho}_f$ is the Weyl transform of the solution f to the Vlasov equation and $A_{\boldsymbol{\rho}_f}$ denotes the operator with integral kernel

$$A_{\boldsymbol{\rho}_f}(x,y) = \nabla\left(V * \rho_f\right)\left(\frac{x+y}{2}\right) \cdot (x-y)\, \boldsymbol{\rho}_f(x,y).$$

We are now in the position of comparing $\boldsymbol{\rho}_f$ with $\boldsymbol{\rho}$, solution to the Hartree-Fock equation (5). In the same spirit of the Bogoliubov transformation in the mean-field context (see Step 2 in Section 2), we define the unitary transformation $\mathcal{U}(t,s)$ as the two-parameter semigroup, solution to the Cauchy problem

$$i\hbar\partial_t \mathcal{U}(t,s) = H_{\boldsymbol{\rho}}(t)\, \mathcal{U}(t,s), \qquad \mathcal{U}(s,s) = 1,$$

where $H_{\boldsymbol{\rho}}(t)$ is the time-dependent Hartree-Fock Hamiltionian defined in (6). The semigroup $\mathcal{U}(t,s)$ plays a similar role to the one of the Bogoliubov transformation, namely it changes the reference frame entailing some cancellations. More precisely, by conjugating the difference $(\boldsymbol{\rho}-\boldsymbol{\rho}_f)$ with respect to $\mathcal{U}(t,s)$, the contributions given by the kinetic part of (6) and the right-hand side of (1) disappear:

$$\begin{aligned} i\hbar\partial_t\,\mathcal{U}^*(t,s)(\boldsymbol{\rho}-\boldsymbol{\rho}_f)\mathcal{U}(t,s) =&\,\mathcal{U}^*(t,s)\left[V*(\rho-\rho_f),\boldsymbol{\rho}_f\right]\mathcal{U}(t,s) \\ &+\mathcal{U}^*(t,s)B(\boldsymbol{\rho}_f)\mathcal{U}(t,s)+\mathcal{U}^*(t,s)\left[X_{\boldsymbol{\rho}},(\boldsymbol{\rho}-\boldsymbol{\rho}_f)\right]\mathcal{U}(t,s) \end{aligned}$$

with $B(\boldsymbol{\rho}_f)$ the operator with integral kernel

$$B(\boldsymbol{\rho}_f)(x,y)=\left[(V*\rho_f)(x)-(V*\rho_f)(y)-\nabla(V*\rho_f)\left(\frac{x+y}{2}\right)\cdot(x-y)\right]\boldsymbol{\rho}_f(x,y).$$

By Duhamel's formula and taking the trace norm, we get

$$\begin{aligned} \left\|\boldsymbol{\rho}-\boldsymbol{\rho}_f\right\|_{\mathcal{L}^1}\leq&\left\|\boldsymbol{\rho}^{\text{in}}-\boldsymbol{\rho}_f^{\text{in}}\right\|_{\mathcal{L}^1}+\frac{1}{\hbar}\int_0^t\left\|\left[V*(\rho-\rho_f),\boldsymbol{\rho}_f\right]\right\|_{\mathcal{L}^1}ds \\ &+\frac{1}{\hbar}\int_0^t\left\|B(\boldsymbol{\rho}_f)\right\|_{\mathcal{L}^1}ds+\frac{1}{\hbar}\int_0^t\left\|\left[X_{\boldsymbol{\rho}},(\boldsymbol{\rho}-\boldsymbol{\rho}_f)\right]\right\|_{\mathcal{L}^1}ds, \end{aligned} \tag{2}$$

where we used that $\mathcal{U}(t,s)$ is a unitary operator. We now estimate each term on the right-hand side of (2).

Error terms. The term $B(\boldsymbol{\rho}_f)$ as well as the exchange term $X_{\boldsymbol{\rho}}$ turn out to be sub-leading in the cases we are interested in, namely $a\in(0,1]$. It has been proven in [30, Proposition 4.4] that

$$\left\|B(\boldsymbol{\rho}_f)\right\|_{\mathcal{L}^1}\leq C\hbar^2\,\|\rho_f\|_{L^1\cap H^m}\left\|\nabla_v^2\right\|_{H^{2n}_{2n}(\mathbb{R}^6)}, \tag{3}$$

where $m=(n+a-1)$ and $n>\frac{3}{2}$, and $H^{2n}_{2n}(\mathbb{R}^6)$ denotes the Hilbert space $W^{2n,2}(\mathbb{R}^6)$ weighted with $(1+|x|^2+|v|^2)^n$. Taking into account the factor $\hbar^{-1}$ in the second line of (2), we conclude that the term containing $B(\boldsymbol{\rho}_f)$ gives a contribution of order $\hbar$. As for the term containing the exchange operator, we rely on [30, Proposition 5.1], that proves the following bound:

$$\left\|\left[X_{\boldsymbol{\rho}},(\boldsymbol{\rho}-\boldsymbol{\rho}_f)\right]\right\|_{\mathcal{L}^1}\leq c\hbar^{3-a}\,\left\||\boldsymbol{p}|^{\frac{a}{2}}\boldsymbol{\rho}\right\|_{\mathcal{L}^2}\left(\|\boldsymbol{\rho}\|_{\mathcal{L}^1}+\left\|\boldsymbol{\rho}_f\right\|_{\mathcal{L}^1}\right). \tag{4}$$

Taking into account the factor $\hbar^{-1}$ in the second line of (2) we conclude that the term containing $X_{\boldsymbol{\rho}}$ gives a contribution of order at most $\hbar$ because, if $a\in(0,1]$, $\hbar^{-1}\hbar^{3-a}\leq\hbar$ for $\hbar\ll1$. Therefore, for the class of interaction potentials we are considering, the exchange term does not change the order of the rate of convergence given by the term $B(\boldsymbol{\rho}_f)$.

Leading-order term. The main contribution comes from the commutator term $\left[V*(\rho-\rho_f),\boldsymbol{\rho}_f\right]$. Indeed, writing explicitly the convolution we obtain

$$\left\|\left[V*(\rho-\rho_f),\boldsymbol{\rho}_f\right]\right\|_{\mathcal{L}^1}\leq\int|\rho(x)-\rho_f(x)|\,\left\|\left[V(x-\cdot),\boldsymbol{\rho}_f\right]\right\|_{\mathcal{L}^1}dx. \tag{5}$$

To cancel the factor $\hbar^{-1}$ in front of the time integral in the second term of the first line of the right-hand side of (2) we seek some smallness arising from the commutator structure. More precisely, the following estimate holds true.

Proposition 2 (Theorem 4.1 in [30]). *Let* $\mathfrak{b} = \frac{3}{a+1}$ *and* $\mathfrak{b}'$ *be the conjugated Hölder exponent of* $\mathfrak{b}$. *Then for* $\varepsilon > 0$ *and* $\tilde{\varepsilon} \in \left(0, \frac{\varepsilon}{2\mathfrak{b}'}\right)$, *there exists* $C > 0$ *such that*

$$\left\| \left[V(x-\cdot), \boldsymbol{\rho}_f\right] \right\|_{\mathcal{L}^1} \leq C\hbar \left\| \mathrm{diag}(|\boldsymbol{\nabla}_v \boldsymbol{\rho}_f|) \right\|_{L^{\mathfrak{b}'-\varepsilon}}^{\frac{1}{2}+\tilde{\varepsilon}} \left\| \mathrm{diag}(|\boldsymbol{\nabla}_v \boldsymbol{\rho}_f|) \right\|_{L^{\mathfrak{b}'+\varepsilon}}^{\frac{1}{2}-\tilde{\varepsilon}} .$$

Notice that Proposition 2 provides a uniform bound in the x variable on the trace norm of the commutator $\left[V(x-\cdot), \boldsymbol{\rho}_f\right]$, hence the integral in x on the right-hand side of (5) is bounded by the L^1-norm of the difference of the spatial densities ρ and ρ_f. By duality and using that $\rho = \mathrm{diag}(\boldsymbol{\rho})$, $\rho_f = \mathrm{diag}(\boldsymbol{\rho}_f)$, we obtain the bound

$$\|\rho - \rho_f\|_{L^1} = \sup_{O \in L^\infty,\ \|O\|_{L^\infty} \leq 1} \left| \int_{\mathbb{R}^3} O(x)\ (\rho(x) - \rho_f(x))\ dx \right| \leq \left\| \boldsymbol{\rho} - \boldsymbol{\rho}_f \right\|_{\mathcal{L}^1} ,$$

that allows as to close the the Grönwall-type inequality. More precisely we get

$$\left\| \boldsymbol{\rho} - \boldsymbol{\rho}_f \right\|_{\mathcal{L}^1} \leq \left(\left\| \boldsymbol{\rho}^{\mathrm{in}} - \boldsymbol{\rho}_f^{\mathrm{in}} \right\|_{\mathcal{L}^1} + C_0(t)\hbar + C_1(t)\hbar^{s-1} \right) e^{\lambda(t)},$$

with $C_1(t)$, $C_2(t)$ and $\lambda(t)$ functions depending only on weighted Sobolev norms of the solution to the Vlasov equation, for which the regularity theory is well-established (see for instance [34, 38] and [30, Appendix A]).

4 Conclusions

Despite recent progress, the analysis on time intervals of order one of the most interesting case of particles interacting via the Coulomb potential remains a major open problem, as does the companion problem of deriving the Vlasov equation with Coulomb interaction from the dynamics of many classical particles. In the context of classical mechanics, the derivation problem can be formulated as follows. We consider an N-particle configuration on the phase space $(x_1, v_1, x_2, v_2, \ldots, x_N, v_N) \in \mathbb{R}^{6N}$. Its evolution in time is given by the Newton equations

$$\frac{d\,x_i}{dt}(t) = v_i(t), \qquad \frac{d\,v_i}{dt}(t) = -\frac{1}{N}\sum_{j=1}^{N} \nabla V(x_i(t) - x_j(t)), \qquad i = 1, \ldots, N, \tag{1}$$

where $V : \mathbb{R}^3 \to \mathbb{R}$ is a two-body interaction potential. The problem of justifying the Vlasov equation (7) starting from the dynamics of N particles obeying Newton's laws has been proved for smooth potentials in the pioneering works [36, 10, 15] (see also [45]). The class of potentials was then extended to locally Hölder continuous interactions in [26, 27]. In [9] the convergence towards the Vlasov equation is proven for potentials with a vanishing cut-off (as $N \to \infty$) converging to singular interactions, including the Coulomb potential. A further improvement has been achieved in [31], where the size of the cut-off is comparable to the mean inter-particle distance. Moreover, in [25] a class of potentials slightly singular at zero has been treated. Thus the derivation of the Vlasov equation in the cases of Coulomb and gravitational interactions, which are the relevant models for applications to plasma physics and astrophysics, is still an open problem.

We also remark that, although the derivation procedure offers a trail towards the quantitative analysis of the many-body problem, the price to pay in this approach is that usually the resulting evolution equation is nonlinear, and in many interesting cases even proving the well-posedness of the effective dynamics is a very challenging mathematical problem. On the one hand, this claims for a deep mathematical study of the effective evolution equations, that is in many cases extremely challenging. On the other hand, the emergent nonlinearity is often responsible for spectacular collective phenomena, such as turbulence for the evolution of fluids or Landau damping phenomena. The rigorous understanding of these phenomena represents an outstanding challenge in mathematical physics, that uses and at the same time motivates the development of new mathematical methods, for instance in the study of nonlinear PDEs.

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O. A. Ladyzhenskaya and the problem of the global unique solvability of the Navier-Stokes equations

by GREGORY A. SEREGIN and TIMOFEY N. SHILKIN

One of the most important models in mathematical hydrodynamics is the following Navier–Stokes system of equations in $\mathbb{R}^3 \times (0, +\infty)$:

$$\begin{cases} \frac{\partial u}{\partial t} - \nu\,\Delta u + (u \cdot \nabla)u + \nabla p \;=\; 0; \\ \qquad \operatorname{div} u \;=\; 0, \end{cases} \tag{2}$$

which describes the flow of a viscous incompressible fluid. The variables in system (2) are the vector field u and the scalar function p; they play the role of the fluid velocities field and pressure, respectively. One of the central questions regarding this system is whether the given model provides a deterministic description of fluid dynamics, in other in words, whether the definition of the initial data

$$u|_{t=0} \;=\; u_0 \tag{3}$$

uniquely determines the solution of system (2) for all t.

To answer these and similar questions in the 20th century, the theory of partial differential equations began to develop new approaches based on the methods of functional analysis. Among the scientists who contributed to the development of new ideas, one can name N.M. Gunther, S.L. Sobolev, J. Leray, R. Courant, K. Friedrichs and many others. In particular, using the concept of weak derivatives introduced by S.L. Sobolev, J. Leray [14] proved the global existence of weak solutions, later called Leray–Hopf solutions, and established the existence of strong solutions on a finite time interval. Today, the uniqueness of the Leray–Hopf solutions is an open problem. On the other hand, the strong solutions are unique in the class of Leray–Hopf solutions, but their global existence is unknown. This was the situation in the theory of the Navier–Stokes equations by the time when O. A. Ladyzhenskaya joined the research in this area.

In 1957, the work "On the existence and uniqueness of the solution of the nonstationary problem for a viscous, incompressible fluid" was published; see [3]. It can be said that already in this work Olga Aleksandrovna's position was determined, which she adhered to in her studies of the Navier–Stokes equations throughout her life. Olga Aleksandrovna always considered the primary question of the uniqueness of solutions (and not of their regularity) for nonstationary problems, and considered the formulation of a problem to be "correct" if the question is to find functional classes in which it would be possible to prove simultaneously both the global existence of solutions and their uniqueness. While for the Navier–Stokes equations the uniqueness problem for solutions is closely related to the question of their regularity (for the Navier–Stokes system, the following "weak-strong" uniqueness theorem holds: the existence of a smooth solution implies that given any weak solution, the Leray–Hopf solution must coincide with it), Olga Aleksandrovna admits that solutions of equations describing the dynamics of viscous fluids in principle can form singularities over time, but despite this, mathematically should be described by models that give a deterministic description of such flows.

In [3], different variants of functional classes are presented, in which the theorem of uniqueness of solutions to initial-boundary value problems for equations (2) is valid; the existence of solutions in these classes on a finite time interval is proved, with a lower bound for the lengths of the corresponding intervals. Olga Aleksandrovna adhered to the philosophy of this work all her life—to look for functional classes in which the global unique solvability holds. She believed that the class of Leray–Hopf solutions is unacceptably large and there is no uniqueness in it. This problem is still open, but work [2] partly confirms the conjecture.

In 1958, O. A. Ladyzhenskaya [4, 5] proved the global unique solvability of the initial-boundary value problem for the following two-dimensional Navier–Stokes equations in $Q_T := \Omega \times (0, T)$:

$$\begin{cases} \frac{\partial u}{\partial t} - \nu\,\Delta u + (u \cdot \nabla)u + \nabla p \;=\; 0; \\ \qquad \operatorname{div} u \;=\; 0; \\ \qquad u|_{t=0} \;=\; u_0, \quad u|_{\partial\Omega\times(0,T)} \;=\; 0, \end{cases}$$

where Ω is an arbitrary domain in $\mathbb{R}^2$. This generalizes the results of J. Leray [15] for the two-dimensional Cauchy problem. In contrast to J. Leray, she uses her concept of choosing the correct functional class. Her proof is based on the following interpolation inequality:

$$\|u\|^4_{L_4(\mathbb{R}^2)} \;\leq\; 2\,\|u\|^2_{L_2(\mathbb{R}^2)}\|\nabla u\|^2_{L_2(\mathbb{R}^2)},$$

which is currently called the Ladyzhenskaya inequality.

Later, Olga Aleksandrovna continued to search for cases such that the global unique solvability of the Navier–Stokes equations holds. In 1961, the first edition of her book "The Dynamics of a Viscous Incompressible Fluid" was published (see [6]). This book, translated into English in 1963, became the basic textbook on the mathematical theory of the Navier–Stokes equations for many generations of mathematicians around the world for many years. In 1968, paper [9] was published, in which the global unique solvability of the Cauchy problem for the Navier–Stokes equations for axisymmetric initial data without the angular velocity component was proved (similar results were obtained in [18], also published in 1968). In [8], O.A. proves that the uniqueness theorem for the three-dimensional Navier–Stokes equations holds in the class of weak solutions with the following norm:

$$\int\limits_0^T \|u(\cdot,t)\|^l_{L_s(\Omega)}\,dt < +\infty,$$

$$\frac{3}{s} + \frac{2}{l} \leq 1, \quad s \in (3, +\infty], \quad l \in [2, +\infty]. \tag{4}$$

Observe that class (4) was earlier considered in [16] and [17]. In the same work, O.A. also proves the smoothness of the solutions satisfying condition (4). For the axisymmetric Navier–Stokes equations without the angular velocity component, O.A. [10] constructs an example of non-uniqueness of weak solutions with a finite energy norm. In the constructed counterexample, an initial-boundary value problem is considered in the axisymmetric domain Q_T defined in cylindrical coordinates (r, φ, z) by

$$Q_T := \{\, t \in (0,T),\ r, z \in (a\sqrt{t}, b\sqrt{t})\,\},\ 0 < a < b.$$

Non-unique solutions constructed by O.A. belong to the Leray–Hopf class and, moreover, satisfy condition (4) for all $s > 3$ and $l \geq 2$ such that

$$\frac{3}{s} + \frac{2}{l} > 1.$$

O.A. indicates that the constructed example of non-uniqueness gives reason to believe that in the Cauchy problem for the Navier–Stokes equations, the class of Leray–Hopf solutions is possibly too large for uniqueness, and in the energy class, the initial-boundary value problem for the Navier–Stokes equations may be incorrect. This was a very bold hypothesis at that time.

On the other hand, when studying the three-dimensional Navier–Stokes equations, O.A. also considered it illegal to deliberately narrow the functional class of "physically correct" solutions to the class of infinitely smooth functions. She always emphasizes (see, for example, [11]) that the primary question concerning the Navier–Stokes equations is that of global unique solvability, that is, the question of finding a functional class in which one can establish both the global existence of solutions and their uniqueness. She believed that the formulation of the "Sixth problem of the millennium" proposed by C. Fefferman (see [1]) and replacing the problem of a deterministic description of fluid dynamics by the study of the global existence of smooth solutions, to some extent transferred the problem from the philosophical plane into the category of purely sporting achievements.

O.A. presented her views on the "Sixth problem of the millennium" in [11], as well as in her talk at a seminar on May 3, 2001 at Princeton University.

In this regard, in parallel with the study of the Navier–Stokes equations, Olga Aleksandrovna was also looking for other nonlinear hydrodynamic models that, on the one hand, would allow the existence of nonsmooth solutions. On the other hand, it was expected that global unique solvability would hold in the energy class. Such models were announced in a report [7] at the Mathematical Congress in Moscow in 1966 and are currently called "Ladyzhenskaya models." Later it was found that this class includes many models well known in fluid mechanics and turbulence theory, in particular, generalized Newtonian fluids and the Smagorinsky model.

The proof of the global unique solvability of the "modified Navier–Stokes equations" (as O.A. called them) was published in [12, 13]. Later, in the 90s of the 20th century, these equations became a favorite topic of O.A. and she devoted numerous works to their study.

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Olga Ladyzhenskaya (1922-2004)

by DARYA E. APUSHKINSKAYA and ALEXANDER I. NAZAROV

Olga Alexandrovna Ladyzhenskaya was born on March 7, 1922, in the tiny town of Kologriv in Kostroma region, in 500 kilometers northeast of Moscow. The youngest of three daughters, Olga showed mathematical skills very early. Her father, a school mathematics teacher and a former nobleman, was executed in 1937, during the Great Purge. For this reason, Olga was considered as "the daughter of an enemy of the people," and, despite the excellently passed entrance exams, could not enter the Leningrad University. From 1939 to 1941 she studied at the Leningrad Pedagogical Institute named after Pokrovsky.

After the beginning of the Great Patriotic War, Olga returned to Kologriv, where she taught mathematics at her former school. In 1943, Ladyzhenskaya was allowed to enroll at Moscow University as a second year student. In 1947, she received a diploma with honors (her supervisor was I.G. Petrovsky[1]). Olga's graduation work was devoted to the well-posedness of the Cauchy problem for 2b-parabolic equations.

After marriage with Leningrad mathematician Andrey Kiselev,[2] she moved to Leningrad and entered PhD program at Leningrad University with S.L. Sobolev[3] as advisor. After two years she got her PhD for the work "Solution of the Cauchy problem for hyperbolic systems by the method of finite differences." Her Habilitation thesis "The Mixed Problem for a Hyperbolic Equation," where she justified the Fourier method for general second order hyperbolic equations in the multidimensional case, was ready in 1951. However, she could not defend it (again for political reasons) before Stalin's death in 1953.

Since 1950, Olga worked at Leningrad University, in 1955 she became Full Professor. From 1954 until the end of her life she was a Fellow of Leningrad Department of Steklov Institute (LOMI[4]). In 1961 Ladyzhenskaya organized the Laboratory of Mathematical Physics in LOMI; she headed this Laboratory up to 1998.

Olga Alexandrovna wrote and co-authored 7 monographs and textbooks, as well as more than 250 research papers. Her mathematical achievements were honored by the Leningrad University Prize (twice, in 1954 and 1961), the Chebyshev Prize of the USSR Academy of Sciences (1966), the State Prize of the USSR (1969), the Kovalevskaya Prize of the Russian Academy (1992), and the Ioffe Prize of the St. Petersburg City Administration (2002). In 1981 she became a Corresponding Member, and in 1990 Full Member of the USSR Academy of Sciences. In 2002 she was awarded the highest award of the Russian Academy, the Great Gold Lomonosov Medal. She was elected as foreign member to the oldest German academy Leopoldina (1985), the Accademia Nazionale dei Lincei (1989), and the American Academy

[1]Ivan G. Petrovsky (1901-1973) was a prominent Soviet mathematician known for his works in PDE's, ODE's, and algebraic geometry. Full Member of the USSR Academy of Sciences, Rector of Moscow University (1951-1973).

[2]divorced in 1956

[3]Sergei L. Sobolev (1908-1989) was an was an outstanding Soviet mathematician working in mathematical analysis and PDEs. Full Member of the USSR Academy of Sciences. He introduced the notions (Sobolev spaces, distributions, etc.) that are now fundamental for several areas of mathematics. Nowadays, Institute of Mathematics in Novosibirsk is named after Sobolev.

[4]Since 1991, St. Petersburg Department of Steklov Institute (POMI).

of Arts and Sciences (2001). She was heralded with the Doctoris Honoris Causa degree by the University of Bonn (2002).

The scientific legacy of Olga Ladyzhenskaya includes a huge variety of topics in PDE's and in mathematical physics. Among the most important ones were problems from the spectral theory for differential operators, justification of the Fourier method for hyperbolic equations, the convergence of finite difference methods, the calculus of variations, the theory of quasilinear equations, mathematical fluid dynamics, and the theory of attractors for PDE's. The results she achieved were spectacular and many of them remain on the top of those areas.

In the 1960s, Ladyzhenskaya co-authored with her pupil Nina Uraltseva a series of papers on the regularity of solutions to quasilinear elliptic and parabolic equations. In particular, these works completed the solution of the famous 19th and 20th Hilbert problems.

It should be emphasized that Olga Alexandrovna was not even so much interested in solving individual mathematical problems as in setting new problems. She was a true mathematical strategist. It is thanks to Ladyzhenskaya that such concepts as generalised statements of boundary value problems and generalized solutions have become mainstream.

The main mathematical love of Olga Alexandrovna was a rigorous theory of fluid dynamics. Her very influential book *The Mathematical Theory of Viscous Incompressible Flow*, published in 1961, was translated into many languages and has become a classic in the field. The global well-posedness for the Navier-Stokes equations in dimension 2 proved in 1950's is one of her great results. Jointly with Andrey Kiselev, she proved a similar result in dimension 3 as well but only on a finite interval of time. It is worthwile to notice that the global well-posedness for the 3D problem remains open to this day. Moreover, the issue of existence and uniqueness of physically reasonable solutions to the 3D-NSEs has been chosen as one of the seven Millennium "million dollar" prize problems by the Clay Mathematical Institute.

The charm of personality and the ability to distinguish the talented students allowed Ladyzhenskaya to grow a number of brilliant mathematicians whose names make up the glory of the St. Petersburg school of PDE's and Mathematical Physics. Among them are Ludvig Faddeev, Nina Uraltseva, Vsevolod Solonnikov and many other prominent scientists.

While a student at Moscow University, Olga not only took part in scientific seminars but also organised a student seminar on PDE's and convinced I.G. Petrovsky to chair this seminar. After her arrival in Leningrad in 1947, she (together with V.I. Smirnov[5]) organized a weekly seminar on Mathematical Physics. This seminar is now named after V.I. Smirnov and is about to celebrate its 75th anniversary.

Since the revival of the St Petersburg Mathematical Society (SPbMO) in 1959, Ladyzhenskaya has been one of its most active members. For more than 40 years she served as a Board Member and as Vice-President. From 1990 to 1998 she was the President of SPbMO. In 1998 she was elected an Honorary Member of SPbMO. In 2014, the Society established a scholarship named after Ladyzhenskaya.

Olga Alexandrovna had the rare courage of opinion. She reacted keenly to any injustice.

[5]Vladimir I. Smirnov (1887-1974) was a prominent Russian and Soviet mathematician known for his works in the theory of functions of a complex variable, mathematical physics, and the history of mathematics. Full Member of the USSR Academy of Sciences. He was an outstanding figure of mathematical education and phenomenal organizer of science. Smirnov is also widely known among students for his five-volume series (in seven books) *A Course in Higher Mathematics*.

She advocated time and again for students who had problems with admission to graduate school by political reasons.

O.A. Ladyzhenskaya was an extraordinary person, who was interested in life in all aspects. She was an enthusiastic traveller, had the wonderful skill of a storyteller, and was well versed in literature, arts, and music. Among her friends were famous poets, writers, musicians and painters, in particular, Anna Akhmatova, Iosif Brodsky, Alexander Solzhenitsyn, and Boris Tishchenko. Ladyzhenskaya was mentioned by Solzhenitsyn in the list of 257 'witnesses of the Gulag Archipelago', while Akhmatova devoted to her the poem In Vyborg.

Finally, it is impossible to keep quiet about the fact that Olga was a very beautiful woman. In the congratulatory address for her 60th birthday A.D. Alexandrov[6] wrote: 'Such a combination of beauty and talent in one person seems unreal, if not for Olga Alexandrovna.'

Leningrad seminar on Mathematical Physics, 1968. First row, left to right: N.N. Uraltseva, O.A. Ladyzhenskaya, V.I. Smirnov. (photo courtesy of N.N. Uraltseva)

[6]Alexander D. Alexandrov (1912-1999) was an outstanding Soviet and Russian mathematician, one the greatest geometers of XX century, also known for his seminal works in PDE's. Full Member of the USSR Academy of Sciences, Rector of Leningrad University (1952-1964).

This portrait of Ladyzhenskaya made 1959 stood on A.D. Aleksandrov’s desk.

Interview with Eva Miranda about Olga Ladyzhenskaya's work on Navier-Stokes and other problems in Mathematical Physics

**Mathematics provided me with an inner safe haven,
something that gave me a sense of freedom.**

Eva Miranda is a full professor in Mathematics-ICREA Academia at the Universidad Politécnica de Cataluña (UPC)-IMTech and a member of the Centre de Recerca Matemàtica (CRM). She is the only mathematician awarded with two ICREA Academia Prizes: one in 2016 and one in 2021. In 2017 she was distinguished with a Chaire d'Excellence from the Mathematical Sciences Foundation of Paris, becoming the first Spaniard and only the second woman in the world to occupy such a position. She has been an associate researcher at the Paris Observatory and she is a collaborating researcher with the ICMAT. In this article, we interview her about her research in geometry, dynamical systems and connections to mathematical physics with special emphasis on the work of Olga Ladyzhenskaya.

This *News Bulletin* is dedicated to the 100 birthday anniversary of Olga Ladyzhenskaya. Do you feel connected to her path somehow?

Yes, among other things Olga Ladyzhenskaya gave a proof of the existence and uniqueness of solutions of the Navier-Stokes equations in dimension two. The motion of incompressible fluids follows a system of partial differential equations called the Navier-Stokes equations for viscous fluids; when viscosity is zero they are called the Euler equations. The movement of water of the sea follows these equations. Understanding whether a blow-up – that is, the existence of certain regions of space, called singularities, where the energy of the fluid concentrates and becomes infinite – can exist for these equations is fundamental, as it clarifies whether the models are precise enough. The existence of blow-up in three dimensions would tell us that the equations are not precise enough to describe certain physical systems. Addressing Navier Stokes in dimension three is still a big challenge, as the 2-dimensional proof of Olga Ladzhenskaya did not work in three dimensions. Indeed, the Navier-Stokes conjecture about smoothness of solutions is one of the open problems in the Clay foundation list and it is not solved yet. My research took me quite in an unpredictable way to a question related to the problem of Navier-Stokes in three dimensions, so this is certainly our first clear connection.

Could you tell us something about these results?

We worked on a construction of solutions for fluid equations, which are capable of simulating any Turing machine. The flows we build codify a basic operating system that is able to execute any computational algorithm; we could say we have invented a 'water computer'. One of the remarkable consequences of our result is that it enables us to determine that certain hydrodynamic phenomena are undecidable. In other words, there exists no algorithm to decide whether a fluid particle will pass through a certain region of space in finite time. This even occurs in some fluids that, a priori, may appear to be simpler because they correspond to stationary equations (i.e., not time-dependent). We have shown that complexity of various types arise in fluids, from the dynamical, computational and logical points of view.

How is this question related to the Navier-Stokes problem and Ladyzhenskaya's result in three dimensions?

Ladyzhenskaya had proved that no initial two-dimensional fluid configuration generates singularities over time, back in 1958. She proved the existence and uniqueness of long-time solutions. A key point in her proof in dimension two was the use of an inequality – now known as Ladyzhenskaya's inequality. There is a similar inequality in the three dimensional case but the exponents are slightly different. This difference seems to be one of the main obstacles for proving existence and uniqueness of solutions of the Navier-Stokes equations in three dimensions. However, does regularity hold? Ladyzhenskaya also made important contributions to understanding three-dimensional Navier-Stokes solutions, but she was unable to crack the puzzle.

Terence Tao, professor at the University of California Los Angeles and a Fields Medal winner in 2006, has proposed a new way to attack the problem. Tao's bid is on proving the blow-up of the solutions. His approach to tackle the conjecture is related to these fluid computers that we constructed. Tao believes that it is possible to generate a blow-up in finite time, by making use of constructions that are sufficiently complex, by simulating a Turing machine. This would imply that the Navier-Stokes equations do not always admit smooth or regular solutions, and that they are subject to sudden jumps.

One of the remarkable aspects of this approach is the combination of different mathematical viewpoints: geometrical techniques, Turing machines, fluid mechanics, etc.

Yes, this is a very interesting topic in which topology interacts with Turing machines. This is an explosive combination. In 2018, Daniel Peralta came to Barcelona to give a course on dynamical systems at a school and we began collaborating together. Together with Daniel and my student Robert Cardona we extended his idea of switching from differential fields to differential forms for b-manifolds, which are used to model manifolds with boundaries. This is something I had been working on intensely since 2010, opening a new branch in Poisson geometry on b-symplectic manifolds. This correspondence 'between Beltrami fields and Reeb fields' allows you to translate questions about fluids to geometry. It is a mirror, reflecting questions on fluid dynamics as questions about geometry. We managed to obtain an initial result by starting from a fluid in a manifold with a boundary, like a recipient, and next to the boundary we interpreted it as a b-contact structure. This is the precise moment when we began intermingling our two worlds. Later, while travelling on a TGV I was reading Tao's proposal in his blog, and thought that maybe the ideas we had developed could be applied there. In the Beltrami- Reeb mirror, Tao's question about the universality of the Euler equations became a problem of geometry: Can any vector field be embedded as a Reeb field in a higher dimension? We could not resist the temptation to try to address the problem with our geometric methods, and we managed to do it.

How did you arrive at the result?

First, we obtained a construction of universality. In this paper, which is still awaiting acceptance, we include a full Turing construction in a sphere of dimension 17. It seemed a beautiful construction to me, but after speaking about it at a seminar in Zurich in November, 2019, I realized that to capture the attention of analysts we ought to get a construction in three dimensions. However, the techniques we had used heretofore did not work in low dimensions, so in December, 2019, we set to work with different techniques. We had the construction in our heads, but we had to make the part corresponding to the Turing machines fit in with it, and that took quite a long time.

What progress do you think Tao's proposal for solving the Navier-Stokes problem will make?

Different teams have been working on several approaches to solve the mystery of the Navier-Stokes conjecture. It would be fantastic to see Tao disproving the Navier-Stokes conjecture with a counter-example. He's certainly been mulling it over with very innovative ideas, at least since 2017. I do not think he will use our machine, as, in our result, the metric is not fixed. It is one more variable of the problem, which provides a lot of freedom for making all these constructions. Tao suggests that by imposing certain restrictions on the metric and obtaining a condition of invariance, it would be possible to arrive at a construction of this type. It would be fantastic that this approach was successful but we still do not know how to make it work. In any event, it is a barely explored land, and understanding the rich connection to computational mathematics will certainly have other applications.

You have still been moving forward with these ideas?

Yes. The conclusions we considered in the previous paper were about Beltrami fields; that is, a stationary case, which does not depend on time, and now we have managed to extend the Turing-machine construction to the general Euler case (time-dependent case). Everything has a price; now the dimension is increasing rapidly. What's more, we have used a result that Francisco Torres de Lizaur obtained just after, related to our paper in the PNAS. We happened to be working on that construction when Francisco was finishing his article, and that was exactly the data we needed.

We also managed to find a 3D Turing complete Euler flow in Euclidean space. The construction and way to proceed are very different. In the new construction, the metric is fixed and the construction is physical; however, analytical techniques replace geometrical ones. These solutions do not have finite energy, and the construction uses the fact that 3D space is not bounded. Yet, we found "good bounded approximations" on a flat torus.

You and Ladyzhenskaya are both women in mathematics, how did you think the situation has changed since her time?

It was not easy at all to be a woman in mathematics in the times of Ladyzhenskaya. It was probably a great challenge. Nowadays the challenge is smaller but the situation is still not ideal for women. Not enough has been done to improve the situation. We are aware that the problem exists, and we organize sessions devoted to addressing of the issue, as well as round-table discussions and talks, but so far, no solutions to remedy it are implemented. It is not enough to talk about it. We ought to be able to gain more ground and get more women to take up careers in research, otherwise we are wasting a great deal of talent. Diversity in science is vital and talented women should be included to achieve advances, and to do that it is necessary to make some bold decisions.

What might be some possible solutions?

I think that if we give more visibility to women in science we will make a lot more progress. For example, by proposing more women as plenary speakers at congresses or more women as candidates for prizes, we would give them the role of main character in science. Unconsciously, we are prone to nominate men for prizes. Science has to be more diverse and inclusive and we need to add more women to the prize's radar, promoting women more to the forefront in science. Right now, the position of women in science is still under question. We sometimes have to make twice the effort to get the same recognition as our male colleagues. If a woman's name comes up for consideration for a prize or as a plenary speaker, voices are often raised against it. We have to stop questioning women in science and put them forward for talks and prizes, without backing down and without fear of making a mistake. Everything would be easier if this was normalized.

You were the first woman mathematician to be distinguished with an ICREA Academia award and only the second to attain a Mathematical Sciences Foundation of Paris chair of excellence.

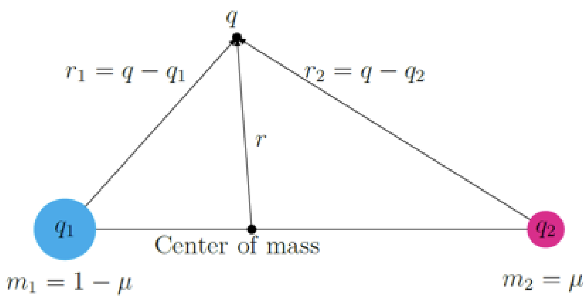
That is correct, although since then Teresa Martínez Seara and Núria Fagella have also received an ICREA Academia prize. I am pleased not be the only woman in this category. As regards the FSMP chair of excellence, this is very surprising, and only Hélène Esnault and I have received one. Hélene received it in 2011. I am glad to report that I am the only mathematician in Spain who got the ICREA Academia twice and I am a woman!

Congratulations! That is a big recognition in your career.

This came to me in an excellent moment when I have many research paths open and an intensive research program. The ICREA Academia 2016 allowed me to focus on research and attain influential results in the theory. I am looking forward to have five more years of intensification of research.

What else, other than fluids, are you working on at the moment?

One of the projects I have been working on for some time is the extension of the theory of symplectic topology to Poisson manifolds. The motivation for this can be explained by celestial mechanics, particularly with the restricted three-body problem. The restricted three-body problem is a system consisting, for example, of a satellite, the Earth and the Moon. An attraction exists between them, but as we assume that the satellite has negligible mass, so, in a way, what counts is the attraction between the Moon and the Earth. The motion of a satellite follows equations that we know, but if a collision occurs, or the satellite escapes and goes to infinity, the behavior of the satellite ceases to correspond to the Hamiltonian equations associated with the standard symplectic structure, and singularities appear. The structure now becomes symplectic structure with singularities, which is called b-symplectic. My interest lies in understanding these escape orbits. Now I am wondering when such orbits can appear. Can we prove that they always exist? This is interesting from the point of view of contact geometry, because it is related to the Weinstein conjecture on the existence of singular periodic orbits, which is a generalization of the standard Weinstein conjecture.



The three-body constraint problem

What applications do these ideas have?

It is interesting to study these escape orbits because we do not want them to occur, but if they do, we want to be able to control them. Classically, perturbation theory, also known as the KAM theory, is applied to the problem of the motion of the satellite. The singular models that I have worked on and a refinement of the KAM theory in this context could yield an improvement of the techniques in astrodynamics. There are many questions concerning three-body motion used in practice. For instance, the identification of the so-called Lagrange points, which are used to optimize fuel consumption; if you navigate close to these points you use less fuel as these points are mathematically an equilibrium point, the critical point of the potential. We all heard about L^2 recently when the James Webb Space Telescope has arrived there. The Lagrange point L^2 has been known as a good spot for space-based observatories precisely because at those points, the gravitational pull of two large bodies equals the centripetal force and the object remains there.

What results have you obtained from this?

With Daniel Peralta and with Cédric Oms, a former student of mine, we demonstrate that by using the Beltrami-Reeb mirror there exist a minimum number of escape (or singular) orbits, dictated by the form of the set of the line to infinity. The topology of the critical set determines the minimum number of orbits that may appear. We could not apply this result to the three-body problem because there are certain initial hypotheses that are not fulfilled, but we are trying to improve it. Demonstrating it in general is one of the questions I am working on right now. What's more, with Cédric I obtained a new family of periodic orbits for the three-body problem that was unknown until now. I am very proud of this result.

This work has in some way converged towards the work on fluids.

Yes, they were two topics that apparently were unrelated, but it turns out that they are and the nexuses are diverse. On the one hand, as I was explaining before, any problem involving

Beltrami field orbits can be seen as a problem of orbits for Reeb fields by means of the Beltrami-Reeb mirror. In particular, the results known as the Weinstein conjecture – whenever known to hold – can be applied to prove the existence of periodic orbits for Beltrami fields. The mirror works in both directions. So we have been able to apply a result by Karen Uhlenbeck – Abel Prize winner – on the genericity of the Laplacian eigenfunctions, in order to demonstrate the existence of orbits with singular points on the other side of the mirror. In particular, the escape orbits present this behavior. What is curious is that the escape orbits can be seen as singular orbits, sometimes periodic. This brings us back to the terrain of the Weinstein conjecture.

You're also working on Floer theory.

Yes, I was interested in questions about periodic orbits, which is a question about dynamical systems, and that led me to the Floer problems, which concern symplectic and contact topology. Andreas Floer developed an algebraic complex for studying the existence of periodic orbits of Hamiltonian fields. He was looking for a characterization of the existence of orbits, that was the driving force behind the Floer theory. He believed that this theory was going to depend on the Hamiltonian, but he ended up by demonstrating that was not the case. Even though, at first, that must have been very frustrating, it was precisely what eventually enabled him to prove the Arnold conjecture, which deals with the fixed points of a diffeomorphism. Now I am trying to translate what Floer was doing in order to identify certain types of periodic orbits in which are marked points at which the field vanishes. The motivation comes from celestial mechanics, precisely the problem I explained before, because some of these singular orbits correspond to escape orbits.

In the reflection of the Rakotzbrücke Devil's Bridge in Germany we find an artistic representation of the singular orbits studied by Miranda. The singular points on the (semi)circle are the intersection with the water.

Apart from that, you're also interested in quantization.

Yes, that is a problem with more classical flavor of mathematical physics. We are trying to combine geometrical models with models of quantum physics. There are open conjectures that pose the question whether quantization commutes with reduction. This makes sense as symmetries are ubiquitous in physical systems and this conjecture aligns with the intuition that the symmetries leave footprints at the quantum level. This conjecture is more or less well-understood when the classical system is modelled on a symplectic manifold. I have been working on extending the models to more general situations, particularly to Poisson manifolds, which respond to the existence of equilibrium points in physical problems, or to parametric versions of classical systems, where the parameter may be singular. Obtaining models that are compatible with physics is complicated, but not impossible. This theory is very rich as it connects to other facets of mathematics such as Lie theory.

In addition to your research work you also supervise doctoral theses. How important is this task?

I am a happy thesis advisor. Six of my students have already defended their PhD theses and three more will do it so soon. Collaboration with my students is often very long lasting and I learn a lot from them. I feel rewarded when I see them progress in their careers and acquire more experience. Last year I had seven PhD students simultaneously, which perhaps was a bit too much. Four of them defended their theses during 2020-2021, and there are three more still to fly the nest, plus another who is about to start.

Eva Miranda with two of her former PhD students

When did you realize that you wanted to devote yourself to mathematics?

I liked literature very much as well as mathematics; they have something in common, and that is creativity. There is also a liberating aspect that they share: they both offer an escape from 'reality'. Mathematics provided me with an inner safe haven, something that gave me a sense of freedom. I enjoyed writing as well as solving mathematical problems. When I had to make a decision, I opted for mathematics, although I often wonder what would have happened if I had chosen the other path.

How did you get interested in research?

In my third year, our differential geometry professor set us a problem in foliation theory, which studies how it is possible to slice a space into pieces in a differentiable way. It fascinated me because it seemed something beautiful and artistic, and I told him I wanted to know more about the subject. He lent me a book on foliations that I found very exciting (Geometric Theory of Foliations by Camacho and Lins Neto), and I was able to tackle some more advanced questions during my career. When I completed my degree, I chose this differential geometry professor, Carlos Currás-Bosch, as the director of my PhD, so finally I was able to realize my dream and do a thesis on lagrangian foliations, the objects that had fascinated me in my third year. Lagrangian foliations are a natural object to consider from a physics perspective as many physical systems can be modelled on a symplectic manifold. Symplectic manifolds generalize the position-momenta duality in a cotangent bundle and both position and momenta naturally define lagrangian foliations. The lagrangian ingredient added a connection to physics that has always been present in my research.

What did you do after finishing your Ph.D. thesis?

I went to France as a postdoctoral researcher and stayed there for almost three years, nearly two of them on a Marie Curie contract, and then I came back with a postdoctoral contract to Barcelona. Coming back was very hard and full of uncertainty. The reintegration path for postdocs is a hard path. In Spain, our university staff is getting very old and it is vital to let new talent in which will contribute fresh energy and new ideas to the system.

Interviewer: ÁGATA A. TIMÓN GARCÍA-LONGORIA

Eva with some of her PhD students in her office in March 2020 : From left to right Anastasia Matveeva, Cédric Oms, Arnau Planas, Eva Miranda, Joaquim Brugués, and Pau Mir

Numbers speak louder than words: A first approach to an evidence-based discussion about underrepresented groups in mathematical physics

by SERENA CENATIEMPO and SIMONE RADEMACHER

Introduction

Diverse communities have been shown to be more creative, productive and richer [1, 2]. In the past years, the discussion of diversity has become of crucial interest not only for companies but also in academia. With this article we continue the discussion of the International Association of Mathematical Physics (IAMP) on diversity (see [3, 4, 5] for recent actions), by providing the first attempt at an evidence based discussion about underrepresented groups in the community of mathematical physics [7]. The focus here is to analyze registration data of the last four International Congresses of Mathematical Physics (ICMPs) from 2012 to 2021 [6]. From these data conclusions can be drawn with respect to two dimensions of diversity: participant nationalities and their gender. The results of this analysis are discussed below.

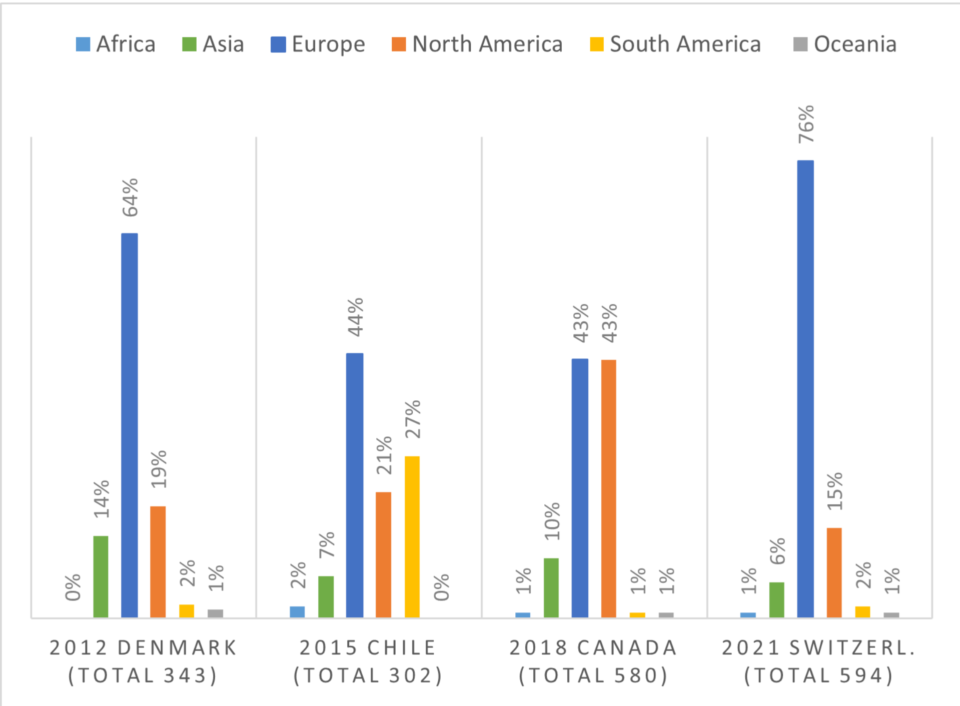


Figure 1: Percentage of participants from the different continents compared to the total number of participants.

[7]The importance of building any discussion about diversity on empirical evidences has been largely investigated in the literature, see for example [9]

Nationalities

This section focuses on diversity with regards to nationality. By nationality we refer to the continent a participant's institution is located. The choice to discuss participation on the scale of continents was made in part because the total number of participants from each country is often small and, in addition, the representation of a single country at each of the four congresses varied considerably. Each continent's participation at the last four ICMPs is discussed first, and then its representation among invited speakers [7]. For the latter, we compare our findings with recent data for the speakers at the International Congress in Mathematics (ICM) [8].

Participants

The chart in Fig. 1 shows the number of registered participants from the different continents, compared to the overall number of participants (indicated on the horizontal axes). More than 60% of the participants were European for the two ICMP congresses held in Europe. The majority was again European (around 40%) at the other congresses; however, the percentage of the participants from the hosting continent was higher compared to the other congresses (21% South American participants in 2015, compared to less 2% otherwise, and 43% North American participants in 2018, compared to approximately 20% otherwise). As for Asia, the overall participation has been around 9% at all four ICMPs, with peaks in 2012 (14%) and 2018 (10%). Participation from Africa and Oceania at all of the four congresses was less than 2%. The data from 2021 were clearly affected by the pandemic. Travel restrictions and overall uncertainties regarding the evolution of the pandemic made this congress difficult for participants to attend and organizers to plan. Nevertheless, thanks to the hybrid format and the very flexible registration options, an international presence was still possible and the number of participants was as high as in Canada 2018. Overall, there has been an increasing trend of ICMP participants over the years. Even when the conference was in South America in 2015, the number of participants was comparable to 2012, despite the fact that the majority of participants had to travel long distances.

For less represented continents, Fig. 1 shows that the location of the conference is important. This is particularly true for students – at all four ICMPs the majority of student participants came from the hosting continent (see Fig. 9 below). This suggests that rotating the location of ICMP will allow for participation from a diverse community of students. On the other hand, travel grants for graduate and PhD students could also be helpful, especially for promoting participation of students from other continents. Also, it is possible that a hybrid concept for future ICMPs might not only increase diversity in the dimension of nationality, but also in the dimension of disability [10], and be as successful as the 2021 congress. We remark that hybrid congresses might also help reduce climate costs.

Speakers

Each of the analyzed ICMP congresses was structured similarly, with around 16 plenary talks and 70 invited talks given in several thematic sessions. Fig. 2 compares the nationalities' representation among the plenary and invited speakers with the nationalities' representation among professors and doctors attending the congresses (i.e. postdocs, assistant professors, researchers

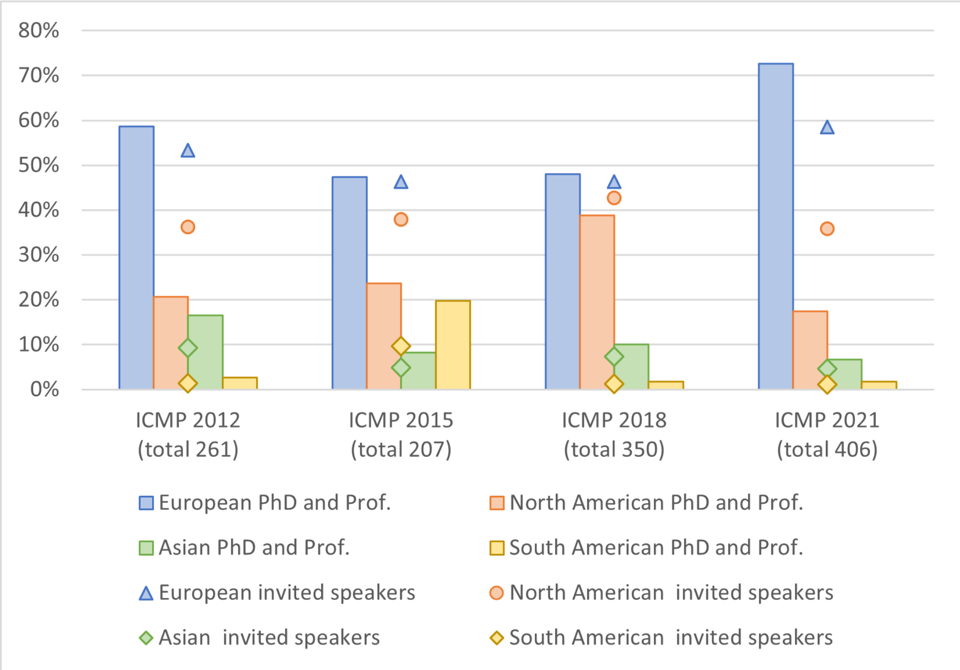


Figure 2: For each ICMP the bars represent the percentage of professors attending from Europe, North America, Asia, and South America (with respect to the total number of professors and doctors attending ICMP indicated on the horizontal axes), and the dots the percentages of plenary and invited speakers from the same continents.

with permanent positions, professors and directors of research). Recall that nationality is referred to as the location of a researcher's affiliated institution.

At all four congresses, most of the talks were given by speakers affiliated with European institutions (around 50%) and North American institutions (around 40%). A few talks were given by researchers affiliated with an Asian institution (less than 10%). The same percentage of talks (10%) was given by South American researchers in 2015, when the conference was located in South America. Furthermore one talk in 2015 was given by a researcher affiliated with an African institution, and two talks in 2018 from researchers based in Oceania. It is interesting to compare the outcomes in Fig. 2 with data from the International Congress in Mathematics (ICM). Here, one sees a similar increase in the percentage of invited speakers coming from the host continent and, specifically, the host country [8, Section 5.1].

Gender

As the ICMP registration data [6] have so far considered gender as a binary question, the following discussion on underrepresented groups in the context of gender will focus on the representation of women. The percentage of female participants at the different congresses (partitioned by career stage) is first compared with similar data for the whole field of mathematics [11, 12, 13, 14]. We then analyze the representation of female participants among speakers [7], and compare this with data from the International Congresses in Mathematics (ICM) [8].

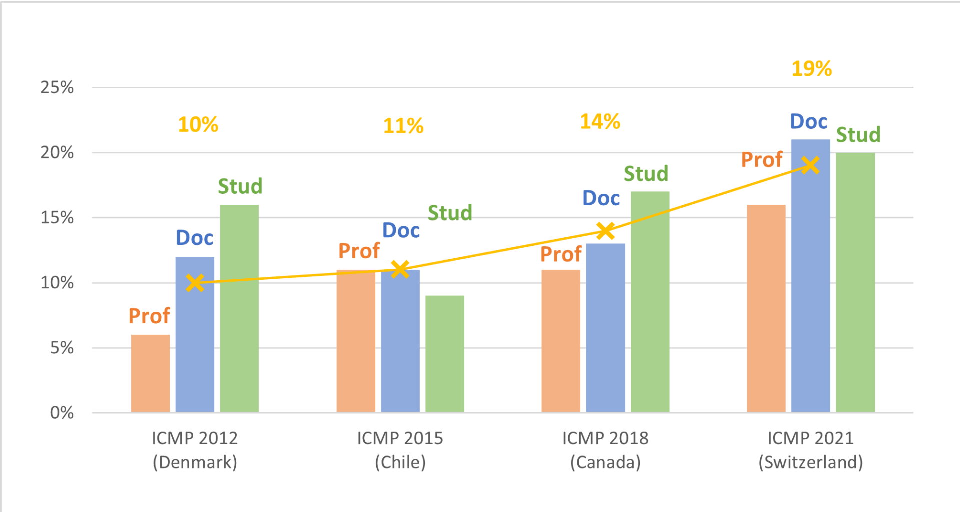


Figure 3: Percentage of female participants to the last four ICMP congresses (yellow line and numbers), together with the percentages of women among professors, doctors, and students attending the same congresses.

Participants

The percentage of female participants at the four ICMPs increased monotonically from 10% in 2012 to 19% in 2021 (see the yellow line in Fig. 3). With the data available, it was possible to split the participants into three groups based on their academic status: i) students, including bachelor, master and graduate students; ii) doctors, including early stage postdocs, tenure-track or non tenure-track assistant professors, and researchers with permanent positions; iii) professors and directors of research. The percentage of women participants in each of these different categories for the last four ICMPs is plotted in Fig. 3. The aim here is to compare the data from the ICMPs with data for the whole field of mathematics. Since the majority of students participating came from the ICMP's hosting continent (see also Fig. 9 below), it is appropriate to compare this with data of the hosting continent's mathematics community. However, as we only found comparable data up to the year 2018 for North America [14, 15, 16] and Europe [11, 12, 13], we focus our discussion on the 2012 (Denmark) and 2018 (Canada) ICMPs, see Fig. 4 and Fig. 5 below. As for South America, we are only aware of data regarding Brazil [17], where the percentage of women PhD students, postdoc, and members of the Brazilian Academy of Sciences (ABC) in 2014 were 24%, 13% and 5%, respectively.

In 2012 and 2018, the representation of female professors attending ICMP (6% in 2012 and resp. 11% in 2018) was less than female doctors (12% resp. 13%) and students (16% resp. 17 %). This phenomenon is famously known as *the leaky pipeline* and is commonly seen in academia. It pictures how female researchers drop out along the academic career path. As the statistics in this article only refer to participation at ICMP conferences over a small window of time (2012-2021), it is not possible to make any assertion about the leaky pipeline phenomenon. For this, one would need to compare recent data with appropriate points in the past. However, we still find it interesting to compare our registration data with that of the American and European Mathematical Societies.

Fig. 4 compares the 2018 ICMP registration data (bars) with data for mathematics in the US

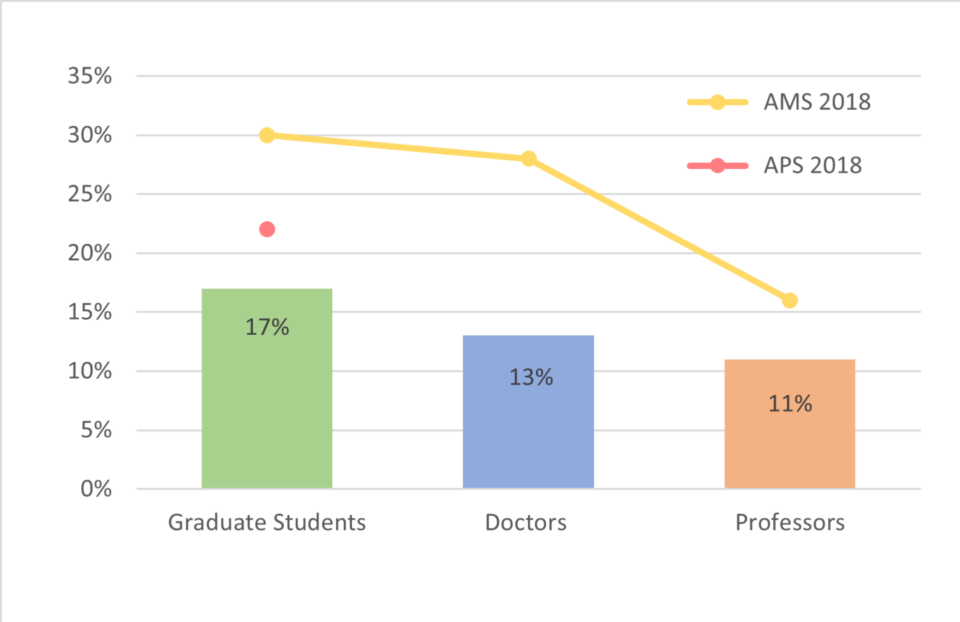


Figure 4: Percentage of women among professors, doctors, and students attending the ICMP 2018 (bars), together with percentage of women among professors, doctors, students in the field of mathematics in North America 2018 [14, Table GS2 and Table FF4](yellow line) and percentage of women among PhD students in physics in North America [15] (red dot).

[14] (yellow line). The division into categories based on academic standing for both data sets is similar, except for the fact that in [14] tenure-track positions are counted as professors, while the ICMP registration data does not differentiate between these and non-tenure positions. The ICMP 2018 data are also compared with the proportion of female physics students [15] and researchers [16]. The percentage of female physics PhD students (22%) is also plotted in Fig. 4 for comparison. As for the percentage of female researchers, we could not directly compare with IAMP data due to the different career categorization in [16]; however the percentages of women faculty in North America (including instructors and assistant professors) and full professors were 19% and 12%, respectively, in 2018.

For all three categories, the percentage of female participants at ICMP 2018 was lower than the total percentage of women working in mathematics in the US during 2018. Fig. 4 shows that the representation of women in mathematics was the same among students and doctors, but women were represented less among professors. This differs from the ICMP 2018 data where the representation of women decreases from students to doctors to professors. However, the latter decrease is less than the decrease from doctors to professors in the US. We note that the percentage of female students at ICMP 2018 is only slightly lower than the same percentage among physics students in North America in 2018 [15]. On the other hand, the percentage of women PhD recipients strongly depends on the field of mathematics considered. An AMS survey from 2017 [18, Table A.1] showed, for example, that women comprised 20% (resp. 18% and 15%) of the PhD recipients in probability (resp. analysis and geometry) in the United States between 2016-2017.

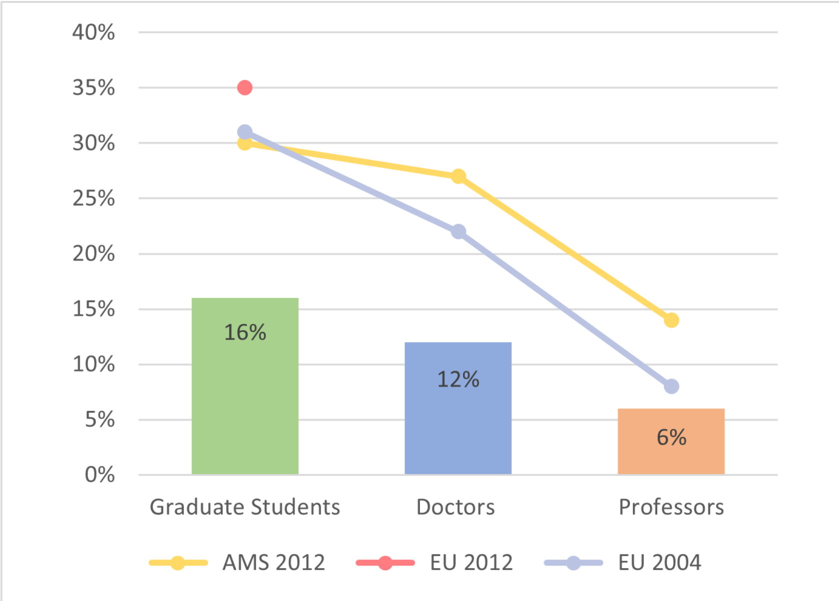


Figure 5: Percentage of women among professors, doctors, and students attending the ICMP 2012 (bars), compared with the percentage of women among professors, doctors, students in the field of mathematics in US in 2012 [14, Table GS2 and Table FF4] (yellow line), Europe in 2004 [11], [12, Table 2.3] (light blue line) and Europe in 2012 [13, Table 2.4] (red dot)

Fig. 5 compares the ICMP 2012's registration data (bars) with data for the whole field of mathematics in US in 2012 [14] (yellow line), Europe in 2004 [11, 12] (light blue line) and Europe in 2012 [13] (red dot). Our aim was to compare the registration data with the representation of women in mathematics in Europe, the hosting continent of ICMP 2012. However, the only available data for European doctors and professors were from 2005, and so North American data from 2012 are also included for comparison. The relationship between the 2012 registration data and the 2012 for North American data is similar to the one discussed for the 2018 ICMP. However, compared with Fig. 4, there is a positive trend of increased representation of women, especially among professors, for both data sets. Focusing now on the European data, Fig. 5 shows that declining representation of women from one academic level to the next is again seen in the 2012 ICMP registration data as well as for the whole field of mathematics in Europe in 2005. However, we expect that the representation of women in mathematics in Europe in 2012 was higher than what is shown in the 2005 data, as is suggested from the percentage of women European math PhD students in 2012. Still, the percentage of female participants at ICMP 2012 was less than the 2005 European data in all three academic categories. Finally, the percentage of female European physics PhD students was 35% — the same as female European math PhD students.

As mentioned before, we did not find appropriate data to compare with the IAMP conferences held in 2015 and 2021. However, the data from Fig. 3 show that the percentage of female students and doctors for 2015 was lower than the previous ICMP. As student participants mostly come from the hosting continent (see Table 9), it does not seem adequate to compare ICMPs held on different continents. Nonetheless, we remark that the representation of women among

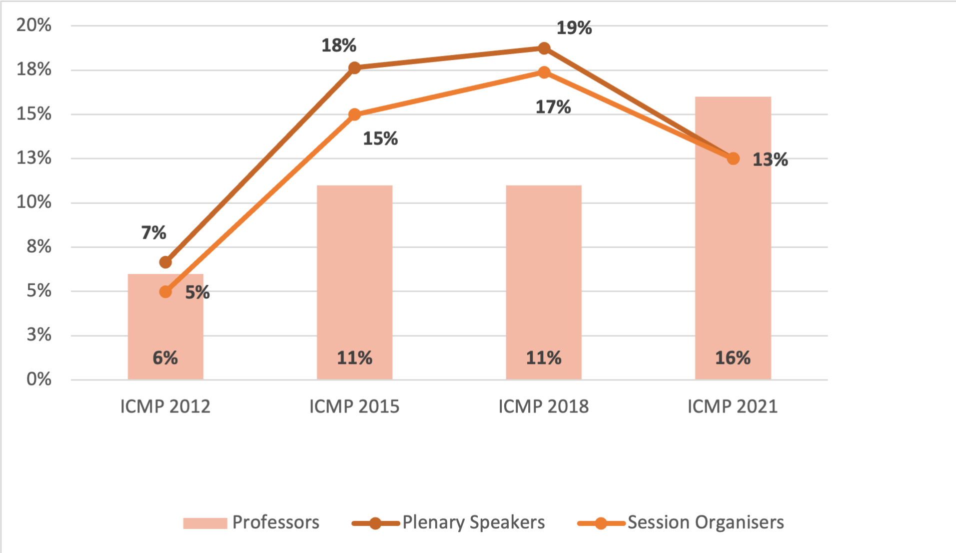


Figure 6: For each ICMP, the graph show the percentage of female plenary speakers and session organizers (with respect to the total number of invited speakers and session organizers respectively), together with the percentage of women among professors attending the congress.

European students at the ICMP 2015 was only 10%, much lower than in the previous ICMP held in Denmark, where it reached 21%.

As for ICMP 2021, the registration data were clearly influenced by the pandemic and so it is difficult to draw conclusions. Still, there was a noticeable increase of female doctor representation, possibly reflecting the increased female student representation from the ICMP 2018. Interestingly, the percentage of female students in 2021 was less than doctors.

Speakers

Recall that each ICMP congress is structured similarly. The International Scientific Committee invites around 16 speakers for plenary talks and 20-24 professors to organize and chair the 10-12 (parallel) thematic sessions. The organizers of each session invite around 6-8 established professors and doctors to give talks and choose between 6-12 contributed talks, for which most of the applicants are doctors and PhD students.

Since the representation of women speakers among the plenary and invited talks is often a debated point, we decided to compare: i) the percentage of women professors attending a given ICMP with that of female plenary speakers and session organizers (see Fig. 6); ii) the percentage of female professors and doctors attending a given ICMP, with that of female invited speakers (see Fig. 7); iii) the percentage of women students and doctor at a given ICMP, with that of female contributed talks (see Fig. 8). For ICMP 21, the proportion of women applicants for contributed talks is also known. From Fig. 6, one sees that female professors attending the congress were mostly well represented both as speakers and session organizers. Still, due to the low numbers involved, an over (resp. under) representation by a few percentiles might be caused by low total numbers of both female attendees and female speakers. Also of note is that the percentage of female speakers from the 2018 ICMP matches the percentage of female North

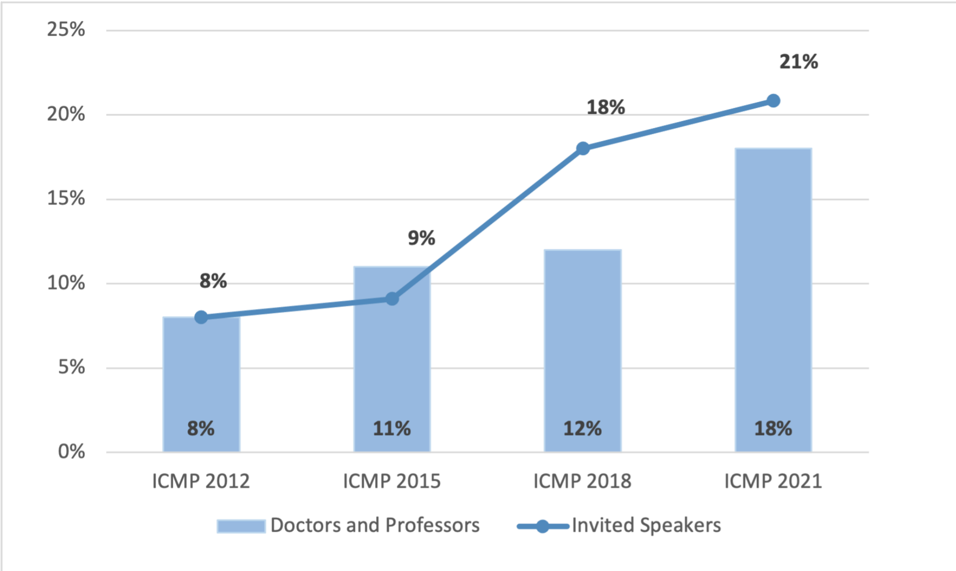


Figure 7: Percentage of female invited speakers compared to the total number of invited speakers, together with the percentage of women among professors and doctors at the ICMPs congresses.

American professors. This could be a reflection of the fact that half of all plenary speakers from this congress were from North America. On the other hand, the over representation seen at ICMP 2015 and 2018 might also be interpreted as intentionally highlighting the contributions of women mathematicians. This would have been in the spirit of suggestions made, e.g., by the European Women in Mathematics (EWM)'s conveners [19].

Similarly, female professors and doctors were well represented among invited speakers at all four ICMPs, see Fig. 7. Of note is that the percentage of female plenary and invited speakers at ICMP 2018 (about 18%) was higher than those at ICM 2018 (around 15%) [8, Figure 3].

For the contributed talks, the percentage of talks given by female researchers also increased with the percentage of female doctors and PhD students attending the conference. We expect these data to be less sensitive as the total number of contributed talks given is remarkably higher than the number of plenary or invited talks. Still, the application process works separately for each session, and there are visible differences in the participation of women in the different thematic sessions in all four ICMPs. Also for the ICMs, the representation of women among speakers differs significantly between different mathematical topics [8, Figure 5].

Interestingly, we also had the application data for the congress in 2021. This showed that while women comprised 21% of student and doctor participants, only 15% of the applicants for contributed talks were women. Moreover, the number of applications submitted by female researchers varied considerably between the twelve sessions, with a standard deviation of 13%. Despite this, the percentage of contributed talks given by women was 19%, well representing the population.

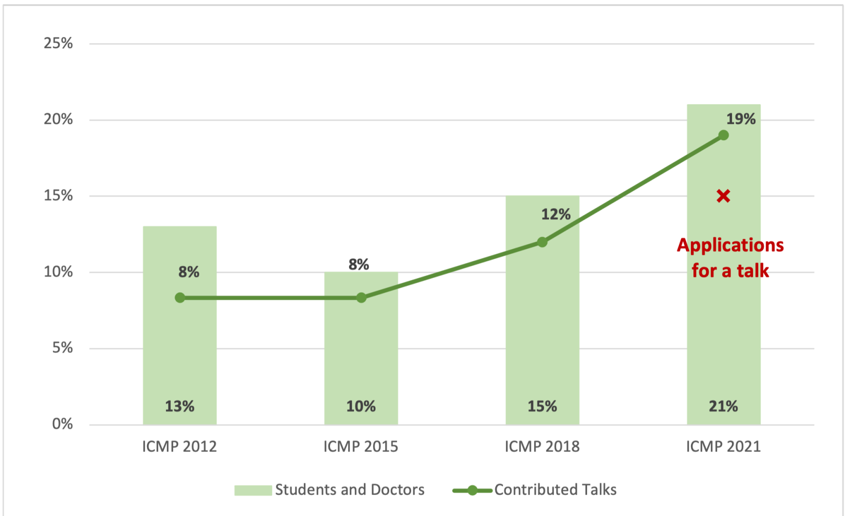


Figure 8: Percentage of contributed talks given by female speakers compared to the total number of contributed talks. Together with the percentage of women among doctors and students attending. Red cross marks the percentage of female applicants compared to the total number of applications for contributed talks.

Conclusion

To summarize, the registration data from the last four ICMPs show that the mathematical-physics community is making progress towards becoming more diverse. The representation of women researchers at the congresses has increased, both among participants and speakers, although it is below the average of the whole field of mathematics. We hope that continuing to highlight women's contributions to mathematical physics, as is suggested by the EWM conveners [19], will help to support this positive trend. However, the data suggest that, in particular, female students and doctors might still be facing difficulties as: i) the representation of women among students at the ICMP is lower in comparison with the whole field of mathematics; ii) in 2021 fewer women applied for contributed talks; iii) data from ICMP 2015 show that there might be a decrease in the participation of women from continents different from the one hosting the conference. We imagine that a support network built, for example, through mentoring programs, or an initiative for women in mathematical physics, would be a great help. This would also possibly intercept or mitigate obstacles and biases junior women researchers may experience. Moreover, diversity in the context of gender is much more than simply representation by women. We hope to broaden our community's awareness for gender minorities and their representation in the mathematical-physics community in the future.

The previous discussion also shows that there is still space for improvement in our community with respect to international diversity among participants and speakers. In particular, the data show the importance of rotating the location of the IAMP conference. In addition, we would hope that a hybrid concept for future conferences might contribute to a more international participation and allow to for increased representation in other dimensions of diversity, including for members with disabilities [10], family obligations, etc. — all at reasonable climatic costs.

Numbers of Students attending

	2012 Denmark (total 82)	2015 Chile (total 95)	2018 Canada (total 230)	2021 Switzer. (total 188)
Africa	0%	4%	1%	0%
Asia	5%	3%	10%	4%
Europe	80%	37%	37%	84%
North America	12%	14%	50%	10%
South America	1%	42%	1%	2%
Oceania	1%	0%	1%	1%

Numbers of Doctors attending

	2012 Denmark (total 111)	2015 Chile (total 106)	2018 Canada (total 154)	2021 Switzer. (total 176)
Africa	0%	2%	1%	1%
Asia	19%	9%	12%	2%
Europe	51%	37%	55%	84%
North America	25%	22%	29%	11%
South America	3%	30%	2%	2%
Oceania	2%	0%	1%	1%

Numbers of Professors attending

	2012 Denmark (total 150)	2015 Chile (total 101)	2018 Canada (total 196)	2021 Switzer. (total 230)
Africa	0%	0%	0%	1%
Asia	15%	7%	9%	10%
Europe	64%	58%	43%	64%
North America	17%	26%	46%	22%
South America	3%	9%	2%	2%
Oceania	1%	0%	1%	1%

Figure 9: Percentages of participants among students, doctors, and professors from the different continents compared to the total number of participants. Antarctica does not appear in this chart, as there were no participants at any of the four congresses. We highlighted in red (resp. in yellow) the continents with highest (resp. second highest) participation.

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Voices of women in mathematical physics: A series of five interviews

organized by
GIULIA BASTI, CHIARA BOCCATO, LEA BOSSMANN, SERENA CENATIEMPO, EMANUELA GIACOMELLI, GIOVANNA MARCELLI, SIMONE RADEMACHER

Women are an integral part of the mathematical physics community. This interview series shines a spotlight on five female members of IAMP. We selected five researchers at different career stages and working in various areas of mathematical physics, in institutions located in diverse parts of the world. **Maria Esteban** is a senior researcher in nonlinear PDEs based in France; **Eman Hamza**, an associate professor in Egypt, works on quantum many-body physics; associate professor **Makiko Sasada** from Japan explores the hydrodynamic limit; **Hanne van den Bosch** is an assistant professor in Chile investigating the Dirac operator; and **Amanda Young** from the US is currently a postdoc in Germany researching quantum lattice models. Starting from the same set of questions for each researcher, the interviews display a variety of perspectives on research, careers and the situation of women in mathematical physics. They are sorted alphabetically.

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Maria Esteban

Maria Esteban studied mathematics at the University of Bilbao. After her PhD in 1981 (University Pierre et Marie Curie, Paris), she obtained a permanent position at CNRS. She is now a senior researcher at CEREMADE, University Paris-Dauphine. Among the many distinctions she has received in her career, she was an invited speaker at ICM 2018 and was awarded the Blaise Pascal medal of the European Academy of Sciences (2021). She has served the scientific community in various roles, in particular as president of the International Council for Industrial and Applied Mathematics (ICIAM) and as chair of the Applied Mathematics Committee of the European Mathematical Society (EMS).

Maria, let us start by describing your research.

In the last few years, I have mainly been working in two directions. On one hand on functional inequalities, mostly in collaboration with Jean Dolbeault and Michael Loss: trying to obtain the best constants, understanding whether there are extremal functions for the inequalities, and also the qualitative properties of the extremals, like for instance, symmetry. More recently, mainly with Jean Dolbeault, Ari Laptev and Michael Loss, I have studied inequalities which are related to the presence of external magnetic fields, a much more difficult topic where many problems remain open. The other direction in which I have worked for many years, mainly with Eric Seré and Mathieu Lewin, are problems involving the Dirac operator in relativistic quantum mechanics. In addition, I have worked on many different topics over the years, for example on problems related to the interaction of solids and fluids or on the Skyrme problem in Quantum Chromodynamics.

Have you ever had interactions with researchers from other fields?

Especially in the area of relativistic quantum mechanics, we have had many interactions with physicists and chemists, and it has always been quite motivating for me to see that our work was also interesting to scientists in other fields. For example, after having proposed an algorithm for computing the eigenvalues of Dirac operators, which is based on a very abstract result but is quite easy to implement, I was invited to the annual meeting of the German chemists to present the algorithm, which was an interesting situation being the only mathematician at the conference. In this direction, we also organised probably one of the first interdisciplinary conferences in Oberwolfach, about 20 years ago.

How did your career in mathematical physics start?

I studied math in Bilbao, and during the whole five years of university studies I just attended one small physics class. After my PhD in Paris, while I was working on my habilitation my advisor gave me a variational problem that was related to mathematical physics, and then some time later, someone showed me an article by a chemist about the spectrum of Dirac operators from the chemistry point of view. We got a lot of inspiration from that article. My advisor gave his students the taste for working on mathematical problems that have a meaning, that have a relation with the real world. It was probably him who passed that taste onto me.

When you started your position as a researcher at CNRS, how did you divide your time between research, academic responsibilities and grant applications?

At the beginning of my career, I spent a number of years doing only research. I have taught a little bit, but not much, because my position is research only. I didn't have to apply for grants at the time, since in France the Ministry or the CNRS gave money to our departments, for all of us, and it was only much later that the system of applying for individual grants began. At that time people could concentrate more on research, especially young people. I am very sad when I see now that very young researchers have to spend a lot of time on grant applications and less time on research. The system has changed a lot over my career, and I am not sure it's for the better.

On the other hand we saw a long list of service activities in your CV. How did it start, and how was this experience?

At some point in my career, I decided to start taking responsibilities in my department and in the Society of Applied Mathematics in France, the SMAI. Later I worked in different European committees and networks and little by little I invested more time working at the international level. I like meeting new people and certainly working in French and international committees increased my chances to do so. I also very much enjoy talking to scientists in other fields, and learning what is going on in other areas of science. But I think that the main reason why I devoted so much time to administrative and management tasks is my sense of duty: there are many things to be done in our profession, not only research and teaching and we have to run the community all together. My belief is that we should all share this work load, take part in this kind of activities. In recent years, more than half of my time has gone into that.

Have you had any "leaky pipeline" experiences in your career?

If you refer to gender gaps: I never felt that some activity or position was not possible for me just because I was a woman. I think that I have been very lucky, also because I was educated in a family where my sisters and I never heard about specific limits for women. I think that it helped a lot because that gave us some kind of self confidence. I have many women friends in math and in other sciences, and I see that some of them could aim for positions to which they do not apply because they think they do not have the appropriate level. Women sometimes impose very high levels of criticism on themselves, much more than men. Also, I do not have children. Maybe when you have children there is a time when working becomes more difficult, because you have less time.

You participated in the creation of the Standing Committee for Gender Equality in Science. What is this about?

Some years ago the International Science Council (ISC), then called ICSU, launched a project named "Gender Gap in Science: How to Measure it, How to Reduce it". There are two mathematical unions in ISC, namely the International Mathematical Union (IMU) and ICIAM. Since I was the president of the ICIAM at that time, it was natural for me to participate. I found it very interesting. In particular there was a survey studying many aspects of women's careers, in all sciences, and on all continents, whose results are published in a book available online[8]. When the project concluded, nine unions participating in that project, including ICIAM and IMU, decided to create a permanent committee to coordinate programs and activities related to acting on the gender gap in science[9], at the global level. Currently, as a sequel of the survey project, I am working with a group of women mathematicians to compare the gender gap in math and applied math with other sciences. The article will be ready in the next months. We are also working on a comparison between the situation in Africa and the rest of the world which is already yielding interesting results.

Overall, what is your impression of the situation of women in mathematics in Europe?

The situation depends a lot on the country we are speaking about. One should not only measure the number of women in academia, but also the balance among junior and senior professors. In Italy and Spain there are more women researchers than in France, but in my opinion, the balance between junior and senior professors is better in France. It also depends

[8] https://zenodo.org/record/3882609#.Yf7kyerMLEY
[9] https://ems.press/content/serial-article-files/12092

on the fields. Globally, in applied math there are more women than in pure math. I think that this happens because the applied math community works in a more collaborative way, and women like to be part of networks.

I would say that the situation in Europe is not satisfactory at all. How to improve it is not clear. I think that efforts are necessary at all levels, starting at the family and elementary school levels, and ending at the university or when women decide to go into academic positions. Nowadays, many universities are looking for women to fill their open positions, since if you do not have enough women in the department this looks bad, and often they do not find enough or adequate candidates. The problem is that we are not training enough women, that has to change. When looking at where the proportion of women versus men diminished during the cycle of study, we see that we lose women at all passages between one level and the next.

I am not totally pessimistic. Nowadays, I see more young women who attend conferences and are active in the scientific community. Also the attitude of society is changing, and the attitude of colleagues is also changing. More attention is being given to this problem. And the fact that you pay attention to something already acts on it. But the situation is still bad, and in mathematical physics, still worse than in other fields of mathematics.

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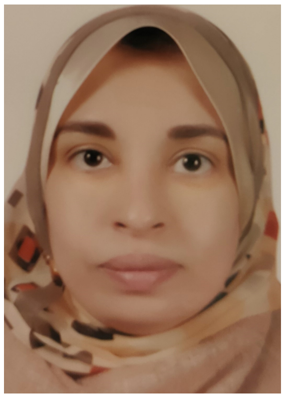
Eman Hamza



Eman Hamza studied physics at Cairo University in Egypt. She got her master's degree in mathematics at the University of Alabama at Birmingham in the U.S., where she also obtained her PhD on random unitary operators under the supervision of Günter Stolz. After spending a period as a postdoc at Michigan State University, she obtained her current position as an associate professor at Cairo University. She was a research fellow at the Erwin Schrödinger International Institute and a Fulbright scholar at the University of California, Davis. She is currently a visiting professor at LMU Munich.

Eman, could you briefly describe the area of your research?

Throughout my career I have been working on mathematical problems related to quantum mechanics. My PhD thesis was about the effect of disorder on certain quantum systems, a topic that I have continued working on during my post-doctorate career. I have also worked on quantum spin systems as well as the BCS theory of superconductivity. A long term project of mine concerns the mathematical understanding of many body problems originating from condensed matter physics.

What do you like most about being a researcher in mathematical physics?

Physics and mathematics are two of the oldest branches of science, and I really like that we mix them to rigorously study physical systems. I think it is amazing that there are still many things to explore. Nowadays you get the feeling that everything is computerized and that might lead one to think that it is only a matter of calculating, but ideas are what drive calculations. I think it is great that we can still be surprised by new aspects of old problems.

How did you discover your passion for mathematical physics?

As a child, we had a huge library at home and I sort of drifted into reading science books. I knew early on that I wanted to study physics. This is why I chose to get my bachelor degree in physics. At some point I realized that I wanted to do mathematical physics: when I started to look for a project for my master's thesis, I came across the paper by Elliott H. Lieb, Robert Seiringer, and Jakob Yngvason, *Bosons in a trap: A rigorous derivation of the Gross-Pitaevskii energy functional* and I realized that this is the kind of research that I wanted to do. I mean, I wanted to prove things rigorously.

What would be your advice to a younger colleague, a PhD student or a postdoc, who wants to pursue an academic career in mathematical physics?

The main advice I would give to people who start this path is that they should try to work on a topic they enjoy, because it can be tough sometimes. Moreover, they should be patient, because it might take longer than initially planned to get the work done. In general, I think that if someone has the passion and the commitment to start an academic career, then with hard work and a bit of patience, the journey can be enjoyable.

How would you describe the situation of women in mathematical physics in your group or your department?

I always had the impression that there is a quite good representation of women in mathematical physics compared to other branches of mathematics. The reason could be that the mathematical physics community is a rather close-knit group. We definitely have a number of remarkable female mathematicians who give a brilliant impression of our field.

In Cairo University, as in other Egyptian public universities, there is no bias when it comes to faculty positions. This encourages female students to study science, because they know that if they prove themselves, they are as likely to get positions as their male colleagues.

Do you think there is a stage in which it is more difficult to progress?

In Egypt probably the most difficult stage is the transition from graduate to post-graduate studies. Not a lot of people take this step and I think for women in particular taking this step is not always easy. Since this is usually the time when one considers starting a family, which incurs certain responsibilities that, traditionally, fall disproportionally on women more than men. This might be one of the reasons why fewer female students progress from undergraduate to graduate studies or that it takes them a bit longer to go through the PhD or master's phase. After that the proportion is more or less stable.

And do you think something can be done to fix this?

In my university we are already trying to address this problem. We are trying to distinguish between two stages, the master's and the PhD. We try to encourage our students to complete their master's by creating conditions so that they have other career options outside academia after their studies, in industry for example. We also try to convey the fact that the master's doesn't have to be completed in one or two years. This is particularly important for female students that might be starting a family: they can take some time off and then come back, conclude the master's and continue for a PhD if this is what they want. I think this somehow takes away the stigma on women to choose between either continuing doing research or having a family.

In Egypt, particularly in the public sector and universities, women have the option to take time off for family reasons. This is great, but I think it also helps that, when they come back, they don't feel that they are lagging behind their male colleagues. This is the sort of support that we, as a community, should offer them.

We would like to change topic and go back to your career path. Have you been supported by a mentor during your career?

I have been extremely lucky to have some fantastic mentors throughout my career. Beginning with my PhD advisor, Günter Stolz, who is and remains my teacher, advisor and mentor. I was also very fortunate to work with Christian Hainzl, Alain Joye and Bruno Nachtergaele who have been mentors and advisors. Each of them helped me develop both personally and scientifically and continue to do so. I am very grateful for their support and advice. It is thanks to these mentors that I feel a sense of community in our field.

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Makiko Sasada

Makiko Sasada studied mathematics at the University of Tokyo, where she received her PhD in mathematics in 2011. After being assistant professor at Keio University in 2011–2015, she is now an associate professor at the University of Tokyo in 2015. In addition to research and teaching, Makiko organizes a colloquium series aimed at a general mathematical audience in Japan and runs the program *Suri-Joshi* promoting interest in mathematics to girls.

Makiko, could you briefly describe your research?

I am mainly working on the hydrodynamic limit. In this area, we try to find a mathematically rigorous way to connect the microscopic and the macroscopic world: for stochastic systems of interacting particles with a large number of degrees of freedom, we rigorously derive macroscopic PDEs, such as the heat equation or the Burgers equation. I have for example worked on interacting random walks on $\mathbb{Z}^d$, and recently also on a generalization to more general lattices, which is not so trivial. Another topic I am interested in is integrable systems. Recently, I have been working in the direction of generalized hydrodynamics, i.e., hydrodynamics for integrable systems, and derived a generalized hydrodynamic equation.

What do you like most about being a researcher and mathematical physics?

In physics, we often use intuition or some heuristic ideas to understand some really existing phenomena. The really interesting point in mathematical physics is to start from some very intuitive or heuristic discussion, and to finally get something rigorous—the fog will disappear, and some clearer view appears eventually.

You studied mathematics. How did you end up doing mathematical physics?

I've never learned physics in university or in graduate school except in the liberal arts courses. I was interested in probability theory itself as a mathematical topic. It was really surprising and interesting to me that one can formulate randomness in this rigorous way. Randomness is something intuitive that all people understand in some fashion. Still, it is very difficult to define or to discuss it rigorously, but we can do it. When it comes to physics, especially to connecting the microscopic and the macroscopic world, it is also very difficult to discuss problems rigorously—but we can do it, using probability theory. This seemed very nice to me, so I started to work in this direction.

You organize the Catch-all Mathematical Colloquium of Japan[10]. What is this about?

One of the organizers suggested to start some colloquium for a general math audience that would be at a friendly time for people living in Asia. Since it's a very special occasion to gather mathematicians from different areas, I suggested that we could also talk about other topics, such as diversity or an inclusive research environment. We just started in October, and so far it's really impressive. Each time, the first part is a mathematical talk for a general math audience. After that, the speaker will talk about their journey as a mathematician. We ask them to choose any focus of their talk—sometimes it's about women in math, or about being a foreigner in Japan, or how to find collaborators in different areas —we try not to make the topic too narrow.

The main idea is that, especially in Japan, mathematicians often talk only about math and not much about other topics. In Europe, I feel from my experience that mathematicians also talk much about other topics, such as hobbies, families, movies, and so on. In Japan, we are not really used to talking about any non-mathematical topics, in particular how to engage people from different backgrounds, which is not so good for creating an inclusive research environment. Around 30–40 people usually attend the talks, and we divide them into groups of four or five to discuss. After the colloquium, we have tea time (of course online) and continue to discuss the topic of the day. I feel it's going well so far; we find people who also want to discuss diversity and inclusion in the math community.

You also run a website called "Suri-joshi"[11]. Could you tell us a bit about it?

The main idea is to show how nice mathematics is to a broad audience. The project is divided in four categories: the connection between math and other fields like economy, art or sport; the jobs you can do after studying math; the math content itself; and the life of women in math. We publish new articles on these four subjects once or twice a month. We also organize workshops for girls and their mothers, where they attend exactly the same workshop in different

[10] `sites.google.com/view/catch-allmathematicscolloquium`

[11] `http://www.suri-joshi.jp/`, English translation: `http://www.suri-joshi.jp/walk/english/`

rooms. The reactions are very different and very interesting. Usually the mothers have a more strict way of thinking, while the children have very free ideas. Here is an example: in one of our workshops, we consider a die with four faces, two people roll their die and the biggest number wins. The task is to compose your own die and put numbers on each face that you can choose freely, but the sum of the numbers should be 10. We let them discuss about their strategy. Question such as “Can we use zeroes?” or “Can we use negative numbers?” come more often and faster from the children. The message we want to convey with this game is that we can change rules. Math is not just the game of finding a solution given a rule, but it is also the activity of making rules. This is not so often underlined in schools.

We read your article where you collected data about the situation of women in math in Japan[12]. For example, you mention that only around 9% of PhD students in math are female. Is the situation better for other subjects? Can you briefly summarize your insights?

At the PhD level, the proportion of women is around 30% counting all subjects, in science around 20%, and only 9% in math. In the math community of Japan, the topic of gender equality is not considered so important, compared to the US or Europe. There are still many workshops that only have male speakers and are organized almost exclusively by men. In the last one or two years, though, the topic has given much attention not only in the mathematics community but throughout Japan, probably because of the pressure from abroad. This led the government to put some pressure on the universities to think about gender issues more seriously, and somehow resulted in the attempt of the math departments to hire more women. But of course, it’s not so easy because we don’t have many female PhD students. I worry that we just focus on the ratio of women, while we should talk more about why we want women in math and what kind of research environment we want.

Sometimes young men feel that they lose their chance to get positions because of the affirmative action of universities trying to hire more women. But this impression is not true, and instead many female students and researchers leave or just give up their career because of family reasons, sexual harassment, isolation in the community, or social bias even before they start to compete for positions. A strong gender bias actually exists in Japan, and talking about DEI is still challenging. One of the reasons for this might be the fact that the ratio of Japanese is very large in Japan’s population composition. Because of this, gender is the most clear attribute of people, and we tend to use gender to categorize people from the very beginning, with children in schools, where boys and girls are often approached differently.

The gender bias also imposes constraints on men’s careers. For example, there is still the idea that if you are a woman, people will understand that your research productivity might drop for a while after having a child. Men instead are expected to have the same productivity. It is a very strange idea but it still certainly exists.

Do you think there is a particular moment in the career of a mathematician that is more difficult to overcome for women?

I think undergraduate and graduate school years might be very tough. It is a period when you are not yet fully accomplished as a researcher, when you are uncertain about your career,

[12] See http://www.math.keio.ac.jp/~bannai/Report_MathGender_en.pdf

and also a time of intense competition. Moreover, students are often too young to build good friendships between men and women, and so it is easy for them to encounter harassment and gender bias. Having a few reliable friends or faculty members in their research community would really help young female students. Moreover, in Japan, some parents and teachers recommend their daughters or students to study subjects (medicine for example) which can lead to a job that is more popular and considered more solid than being a researcher. So just deciding to study math can already be hard for some female students (it can also happen to male students, but less frequently). Before getting a permanent position, the work-family balance is also a big challenge. Things will get really better once researchers in your generation get permanent positions, though then you will become quite busy.

As a last question: What do you wish to change in 20 years from now?

I really hope many more female students study mathematics in graduate school, and that this will lead to an environment where minorities in the math community, not just in terms of gender, feel safe, welcomed, and supported in their studies. Actually, the age diversity in Japan at university or graduate school is also very narrow. I would like to see more opportunities for people to start studying or doing research after having children, or to start mathematics after pursuing other careers. In fact, at the Catch-all Colloquium, I realized that there are already great mathematicians in Japan who have taken diverse paths. Twenty years from now, I hope that the diversity of researchers will be richer and also more visible to everyone. I believe that this will lead to an inclusive mathematical community in which everyone is able to show their abilities, and that the mathematics that emerges from such a community will be richer as well.

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Hanne van den Bosch

Hanne Van Den Bosch studied mathematics and physics at the Université Catholique de Louvain, in Belgium. She then moved to Chile, where she obtained a PhD in physics in 2017 at the Pontificia Universidad Católica de Chile, in Santiago, under the supervision of Rafael Benguria. In the same year she won the Pontificia Universidad Católica de Chile's prize for best PhD thesis in the areas of Chemistry, Physics and Mathematics. After her PhD, she worked as a postdoctoral researcher at the Center for Mathematical Modeling (CMM) (an affilaite of the Universidad de Chile and CNRS), where she obtained a permanent position as assistant professor in 2019.

Hanne, could you briefly describe your research area?

I work on several lines of research in mathematical physics. One research line is related to Dirac operators in different models, for example two-dimensional models related to physical systems like graphene; we aim at understanding the domains of self-adjointness of these operators and the interplay between the boundary conditions and the spectrum of the resulting operators. Another research line is about non-linear Dirac operators; one long-term goal here is to shed some light on solitary waves, by relying on our understanding and ideas about the corresponding linear operators. Then, a very unrelated one, which is completely new for me, deals with the rigorous study of models describing matter subject to gravitational forces.

What do you like most in being a researcher in mathematical physics?

The main thing I like about research is that you can always learn something new. In particular, in mathematical physics, you can understand many different physical systems studying a few classes of similar equations or methods. I also enjoy how this job gives a chance to interact with several people from all over the world, and I like teaching as well. Overall, I like being part of this kind of world.

How did you discover your passion for mathematical physics?

I discovered this passion very gradually. When I was studying, I did not plan on this type of career. During my master of science in physics, I was not very enthusiastic about the many courses focused on high energy physics and renormalization and so I did not imagine myself doing research in something like that. But then by a coincidence, during my Erasmus exchange at Universitá di Bologna in Italy, I discovered different types of physics, which were and are really interesting for me, like statistical physics and mathematical physics. Later, I came to Chile for personal reasons and I started my PhD in physics. Then I have continued to enjoy doing research, so step by step I have become more aware of my passion for this field.

Have you been supported by a mentor during your career?

Yes, definitely. During my career I had the chance to meet and collaborate with many people, some of them were like mentors for me. In particular, Jean Bricmont, who was my supervisor for my master thesis in Belgium and Rafael Benguria, who was my PhD supervisor in Chile. I think that they made a big difference for me, by supporting me and showing that it was possible to continue in academia.

Do you have any other responsibilities in academia? How much time do you invest in them?

Yes, I do. I am part of the committee of the Center for Mathematical Modeling, which is involved in a really big project including several people and many universities. Being part of this committee means dealing with human resources, distributing travel money, deciding which postdocs to hire, which applied research to develop and so on. It’s an administrative work, very far from my background, but I think it’s very rewarding, since you really feel like part of the university and maybe it can have a concrete and relevant impact.

What would be your advice to a younger colleague, who wants to pursue an academic career in mathematical physics?

First of all, if you come from math then you should really try to understand physics as well and vice-versa, because I think it is very important to comprehend both sides of the problem you are trying to solve. Moreover, on the one hand you should not stress too much since in this job I guess we are questioning ourselves all the time but on the other hand you cannot just relax and wait for good things to happen. I think it helps to discuss with many people. Indeed, from my personal experience you can really learn just by talking with other people in universities. What I appreciate also about the mathematical physics community is that most people, in my experience, are really open to sharing their ideas or insights and are happy to collaborate.

How do you see the situation of women in mathematical physics in your country?

When I was hired as a professor at Universidad de Chile & UMI-CNRS, I was the third woman with a permanent position in the department of Mathematical Engineering and Center for Mathematical Modeling with over 30 academics in total. After two years there are already five of us in the department. This means that something is changing. When I talk to female undergraduates I often have the impression that they think that this is not a job for women, or maybe that they are not good enough. The reason could be that, here in Chile, from the cultural point of view there are still some quite strong biases. I am from Belgium and there I have never experienced this so strongly. However, at the university I did not experience the same biases. My supervisors and also other people I met have been very supportive and aware of the situation of women working in academia. When I was in Belgium, at the department of mathematics and physics where I studied, there was not a single female professor, while, here in Chile the number of female scientists is growing quite fast.

To summarize, in general at the university there is a deep awareness about gender related problems, but there are still strong biases in the society.

In your opinion what is the most difficult career stage for women in mathematics?

I think it is between finishing a PhD and getting a permanent position. I guess that at each step you have always the same question in mind – *"should I try to find another position?"*. Answering this question is really hard because you are at an age when you might want some stability in your life, but at the same time you worked really hard to get the PhD and you also want to go on with your research.

What do you think can be done to fix the leaky pipeline?

Although I never personally had bad experiences about the gender gap in academia, I think this is an important aspect which should be faced. First of all I find it very useful to communicate with other women working in academia, at any stage of their career, to see that some problems are not individual but are shared by many women. Talking about these problems can be useful for everybody in the community. What I have in mind is like a group in which there are no specific rules: you could have a mentor but at the same time you can help other people. Moreover, nowadays there are several initiatives to close the gender gap, for example positions opened specifically for women. On one side I know that this is a good idea to increase soon the number of women working in academia. However, on the other side, the people getting these positions could be looked upon as the ones who got the position just because they are women. I do not know yet how to solve that.

What do you think will have changed twenty years in the future? What do you expect and what do you wish?

I hope it becomes more and more normal to have women in academia, so that we do not need to talk about gender gap problems anymore. I am quite optimistic for the future, I have seen a lot of changes recently. There are more women getting permanent positions than few years ago. At my university, at the undergraduate level there is still a big gap, while among PhD students the number of women is increasing. This makes me think and hope that, at some point, the gender gap will really be closed.

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Amanda Young

Amanda Young completed her undergraduate and graduate studies in mathematics at the University of California, Davis. She received her PhD in 2016 under the supervision of Bruno Nachtergaele. Afterwards, she worked as a postdoc with Robert Sims in the Mathematics Department at the University of Arizona and is currently a postdoc in the group of Simone Warzel at the Munich Center for Quantum Science and Technology (MCQST), and the Technical University of Munich.

Amanda, could you briefly describe your research area?

I study spectral properties of quantum lattice models and, in particular, quantum spin systems. The motivation for my research comes from the classification of quantum phases of matter. A (gapped) quantum phase is a collection of models that can be connected along a smooth path of gapped Hamiltonians, and so two natural questions are if a model is gapped above the ground state, and whether that gap remains open in the presence of perturbations. My research has focused on rigorously determining gaps for models conjectured to be typical of specific phases as well as investigating spectral gap stability.

What do you actually like most in being a researcher in mathematical physics?

What I enjoy from a mathematical point of view is that you can use many different kinds of mathematics to analyze these kinds of physical models. This adds a lot of creativity to the work. It is exciting when using techniques from a different area of math greatly simplifies a difficult problem, or when someone comes up with a clever new proof that gives additional insight for understanding a physical question. This is a really fun way of "solving puzzles."

I also really like being a part of this community. As a PhD student, I saw that an academic career gives you the opportunity to work with different people on a variety of projects. Not only was I excited about this, but I felt it was important to get to know the larger community, which is why I wanted to do postdocs in the US and Europe. Both experiences have been wonderful.

Have you been supported by a mentor during your career?

I've had several great mentors. Obviously, the main one is Bruno Nachtergaele. I actually started working with him as an undergraduate during a summer research program that eventually lead to completing my bachelor's thesis with him. He is a great researcher and mentor, so I was excited to continue working with him for my PhD. Bruno introduced me to mathematical physics, and has always given me invaluable guidance throughout my academic career. I can not thank him enough. Along the way there have been others mentors as well. Sven Bachmann was a postdoc at Davis during my PhD, and he regularly checked in to make sure that I was doing well – something he still does today. My first postdoc was with Robert Sims at the University of Arizona. We met at least once a week to work on projects and discuss different topics of interest. He is very encouraging, willing to listen, and gives me kind, honest advice. Now I am working with Simone Warzel, who has helped me improve over the last couple years and come into my own as a researcher. All of these people have been instrumental in my career.

What advice would you give to a younger colleague or a PhD student who wants to pursue an academic career in mathematical physics?

Since we are a cross disciplinary community at the intersection of math and physics, I think having a background in both areas is very useful, even if this requires some additional time. This is something I would have done more of as a student if I could go back. Since the academic career path is really long, it is also helpful to ask yourself how can you start making smaller steps right now, so when you get to the next big transition it is not such a big jump. This can apply to many aspects of the academic career: diversifying your research program, improving your teaching, balancing your responsibilities as a group leader, etc.

Going back to mentors, one thing I think young researchers should consider is that at some point there is a transition from being a mentee to a mentor, and it is worth thinking about how to approach this new role. I had the opportunity to participate in the *Diversity in Leadership Program* through MCQST, which focuses on topics like how to be a good and helpful mentor, skills for being a research group leader, and aspects related to supporting women and underrepresented groups. I really enjoyed the program; it helped me reflect on what it means to be a professor, beyond research and teaching, as well as identify the values I want to instill in my research group and environment.

In your opinion, what is an important attitude to have as researcher in mathematical physics?

I think a good attitude to have is being open to different ideas and perspectives, especially in how one approaches both the research and their career. In research, and in particular mathematics, there is always a right answer, but this does not mean that there is only one solution – there could be different paths. Analogously in the career, what is right for me as a researcher is not necessarily what is right for someone else. I think being open to how other people pursue their careers is helpful, especially when considering how to support underrepresented groups.

How do you see the situation for women in mathematical physics in the US?

The math physics community in the US is more spatially spread out than in European countries, so meetings tend to be smaller and it is hard to get a good perspective for the population as a whole. However, there have been many occasions where I was the only woman in the room during a conference or seminar. Speaking with other junior female researchers from a range of scientific disciplines (both in the US and Europe), it is not uncommon for us to have interactions where we question if gender played a role. These are never really malicious and typically consisted of someone saying or doing something without thinking that was inappropriate or made us feel uncomfortable. I think this is what can be rather challenging about this issue. However, going back to the math physics community, I was pleasantly surprised when I looked around at the participants from ICMP 2021. It seemed like the proportion of junior female researchers was higher than at past congresses. While more can still be done to conscientiously address the items contributing to under representation, this felt very promising.

What do you think can be done to fix the leaky pipeline?

I am not an expert in this area, and I do not claim to have all of the answers myself. However, there are several things I think everyone in our community can do to be supportive. We all value research, and taking some time to look up peer-reviewed articles on this topic and being aware of the suggestions they make will almost certainly have an impact. One finding from an interesting article [13] was that informative, evidence based programs showed promise for addressing this issue. These take time to organize and researchers are very busy, but engaging, e.g., diversity, inclusion and equity (DEI) offices to coordinate events could alleviate this burden.

We can also reflect on how we might accidentally contribute to the leaky pipeline. This is a kind and well-intentioned community, but it is worth understanding how an environment we are comfortable in might be uncomfortable for someone else. Often, it is small, unconscious instances that are disproportionately directed at a particular group that make them feel frustrated and unwelcome. Even though this is not intentional, for the people they are directed at it is significant. It is often discussed that one way these situations arise is from individual biases. These are not inherently bad – everyone has biases – but we can be self-aware of what they are and how they might inadvertently contribute to this issue. These situations will still happen, though, so being a supportive advocate that is willing to listen and acknowledge the poor behavior (rather than excuse or justify it) is particularly helpful to make sure individuals feel heard and not isolated.

What do you expect will have and what do you wish to have changed 20 years in the future?

People seem to be much more conscientious now, especially in the younger generations. This is really promising and lays a good foundation for increasing representation in the future. The recent focus has been on promoting women in science. Given that they make up half the population but far less of the research community, I can understand how this was a natural starting point. However, they are not the only underrepresented group. What I hope for in the future is that we also make an effort to engage these other communities. I think it is promising as 20 years ago DEI initiatives were not as commonly discussed, investigated, and developed as the are today. We definitely seem to be moving in a direction of being more conscientious and aware of how we interact with one another, which I think is just in general good for humanity.

[13] https://www.jneurosci.org/content/39/37/7228

SEET: Support Education, Empower Together

Physics is the same everywhere. But the education system is not.

– Faheema*, physicist from Sudan

We, the team of SEET (Support Education, Empower Together), support female refugees on their way (back) to higher education in Switzerland.

Our mentees

Faheema is a physicist from Sudan. After her master degree at one of the country's largest universities, she joined a research team at CERN in Geneva for two months. After returning to Sudan, she started working at a research institution. But then she had to flee from her home country and leave everything behind - her family, friends, supportive network, even her university certificates.

Having arrived in Switzerland, she realized that it is much more than a language barrier preventing her from working as a physicist again - the job for which she is qualified and loves. *Physics is the same everywhere, but the people and the education system is not. I realized that the educational system here in Switzerland is different than in my home country and that I needed help to understand the Swiss one*, Faheema summarizing her situation.

Bahira*, a chemist from Syria, faced similar problems. The university she studied at was destroyed by ISIS, and she could not provide the appropriate certificates to be allowed to continue her studies in Chemistry in Switzerland. *Still, I don't want to stay at home, I want to use my full potential and follow my passion to work as a chemist again.*

Over the past few years, we the team of SEET were very happy to set up a support structure for these women on their way (back) to higher education and the jobs to which they are qualified.

SEET *made me see again a light at the end of the tunnel by not having to work as an unqualified.* – Fatima*, journalist from Pakistan and mentee since 2021 (currently completing her certificates).

Our mission

Education is the key for sustainable integration, future prospects, individual fulfilment, and independence. Education is part of the declaration of the universal human rights. (Higher) education should be accessible for every human, independent of their origin. This is what we stand for: SEET provides a study support program for female refugees in Switzerland.

> *Having the prospect to work in my proper job, finally feels like being treated as a human being.* – Basima*, English teacher from Syria and mentee since 2021 (currently taking language courses).

Female refugees especially face numerous difficulties; not only at a cultural level but also at a financial and administrative one. SEET builds a bridge between our mentees and the Swiss universities to help them realize their full potentials. We are convinced that this is the most promising path for successful and sustainable integration.

Our program

SEET's program is based on a three-fold supportive system: individual mentoring, workshops and network events, and situation based financial support.

The goal of the individual mentoring is to overcome administrative and cultural obstacles by matching every mentee with a mentor with a similar academic background. The mentoring is supported in tandem by both the SEET team providing solution-oriented support, and professional coaches.

Our mentees had to leave their supportive network behind when fleeing from their home country. With workshops and networking events, mentees can build a new supportive network. In addition to social events with other mentees, mentors, alumni and the SEET team, we organize post-migration and intercultural workshops, trainings on IT skills, application processes, working psychology and more, all lead by professional coaches.

> SEET *helps me develop my skills. For example, writing a CV or absolving an interview is completely different here in Switzerland – this is really complicated for me.* – Faheema, physicist from Sudan and mentee since 2019 (now completing language classes).

To overcome situational financial gaps, we provide monetary support for our mentees not only for study-related costs, but also for language courses, translation of certificates, and train tickets to classes. Furthermore, we are happy to facilitate institutional partnerships and support fundraising campaigns to solve larger financial issues.

Our history

SEET's roots lie in the program BACK ON TRACK of the SAO Association.[14] Through this program, we started in 2018 as a pilot project of three mentees. After a couple years of experience, SEET branched off as an independent organization in 2020. Our work has paid off with our mentees successfully finding their way (back) to universities. The SEET program continues to grow: in the most recent yearly call for mentees and mentors, 15 matches were made. SEET also won the Young Caritas Award in 2020 and we are very happy to be supported by several foundations and institutions such as *Evangelische Stiftung Zürich, ETH Student Project House, and AWK Group AG.*

Our team

While SEET began in 2018 with only five members, our team has grown to 20 (PhD) students, postdocs, and young professionals who all want to share their great luck being born in a place with easy access to higher education.

> SEET *understands my situation better than others because the team and the mentors all have academic background themselves.* Bahira, chemist from Syria and mentee since 2019 (now taking language classes).

All SEET members work on a voluntary basis to stand up for the human right of education independent of one's origin. For them, the best reward is theirs mentees' success and motivation.

[14] SAO (Save - Assist - Outreach) is an association running day centers for female refugees in Athens and Lesvos that supports displaced women on their way to a new, independent, and fulfilling life.

Get involved

After two years in our program, Faheema completed an internship at a physics lab in Zurich. Currently, she is working on her qualification as a PhD student by taking language classes in German and English financed by a fundraiser organized by SEET. After one year in the program, Bahira got accepted to (re)study Chemistry in Switzerland.

> *I have a goal for my life, but I didn't know how to reach it here in Switzerland.* SEET *made me much closer.* – Zahida*, dentist from Palestine and mentee since 2019 (now studying dentistry in Zurich).

Help us supporting them: we are looking forward to new mentors, workshops offered, language tandem partners, but also for new financial supporters and members of SEET.

Contact us through seet@seet.ch or reach out to me directly at simone.rademacher@ist.ac.at.

You can also visit us on our webpage, on Facebook, Instagram and LinkedIn .

We look forward to hearing from you!

SIMONE RADEMACHER

*The names of our mentees are changed.

About the Author: Simone Rademacher has been team member of SEET, resp., BACK ON TRACK since 2018. She was originally part of the finance team and currently is a member of the program team and mentor for a physicist. She is also a postdoc in mathematical physics at IST Austria.

Equal Opportunities & Promotion of Women in Science: Summary of the ICMP 2021 Human Rights Session

For NCCR SwissMAP by Mayra Lirot, Shaula Fiorelli, Elise Raphael

The following is an extract from the panel discussion of the Human Rights Session of the XX International Congress on Mathematical Physics (ICMP), that took place in Geneva in August 2021. The main goal of the discussion was to engage in a dialogue about equal opportunities and gender balance issues in science as well as in academia. The video recording of the complete session is available online: https://youtu.be/0-ATFiDtIPo.

Participating panelists

AF - Prof. Anna Fontcuberta (EPF Lausanne): Physicist and materials scientist, Anna is also a member of the Swiss National Foundation Programmes Division of the Research Council and Co-Chair of Equal Opportunities in NCCR QSIT.

JF - Prof. em. Jürg Fröhlich (ETH Zürich): Mathematician and theoretical physicist, Jürg has also been a member of various national and international scientific associations and societies and has participated in executive committees of such societies. He is also a member of NCCR SwissMAP.

DM - Prof. Deborah Madsen (University of Geneva): Professor of American Literature and Culture, Deborah has years of experience as a mentor within the University of Geneva's Mentorat Relève programme.

EM - Prof. Eva Miranda (UPC – CRM – Observatoire de Paris): Mathematician, specializing in dynamical systems especially in symplectic geometry, Eva was a member of the organizing committee of the "Women in Geometry and Topology Workshop" in Barcelona in 2019.

Moderator

Tania Chytil (RTS): Journalist and producer at the Swiss Radio and Television, Tania is a specialist in issues related to science and other complex topics.

Contributors

PC - Participant's contributions & comments.

Context of discussion

The discussion centered around three different selected measures to promote women in science at different stages of their career. The current environment for each of these measures was presented through three videos at the start of each discussion.

Measure 1 - Outreach

Key takeaways from the video

Presented by: Tatiana Samrowski (Junior Euler Society [JES] – University of Zurich); Ivana & Viera (JES teachers & ex-participants); Anastasia (JES participant).

Obstacles

- Girls thinking that they are not good enough and that boys are more talented and gifted in STEM subjects.
- Being confronted with stereotypes from a very early age.

Advice

- Girls-only activities can help to draw them in and prove to girls that they are not alone in their passion.
- Provide women role models girls can identify with.
- Not only motivate girls to be more confident in male-dominated fields, but also encourage boys to be more confident in female-dominated areas.

Key discussion points raised by the panelists:

Do we need to have girls-only activities? Why is it not possible to have an event for boys and girls and just encourage girls to participate?

AF – Having an island in which girls can excel and be with their friends may help. It's not the only way but certainly for some girls, it is a way to evolve.

EM – As societies judge through stereotypes, to balance things up, we need to put more weight on the other side of the scale, by doing things that may sound a bit artificial. Also, since the crisis, there has been a step back and women are losing confidence. Girls-only activities is a good strategy to advance and improve the situation.

JF – Let me give an example about what could be done. France had an École normale supérieure de jeunes filles only for girls, the experience was that more women started subjects like mathematics and physics than now that they are mixed. So, perhaps it would make sense to reintroduce some special schools for girls.

For well over 10 years, various initiatives, including TV programmes and outreach activities, have been working to promote women in science and combat stereotypes by providing women role models. Why has this not been enough?

DM - This is a battle against the entire culture. The academy is embedded within a very particular society. I say a society but of course, there are multiple societies, each national system is embedded within a particular culture and within microcultures. We also have the global entertainment culture, to take one example of a very powerful purveyor of gender stereotypes. So, the battle is against very powerful opposing forces, and changing the imagery by presenting young women with senior female role models to counteract stereotypes is a very important dimension of outreach.

How was the battle for you as Professors?

EM - I did have to battle, I did have to try hard. I did have to prove that I was good enough. I believe that many brilliant mathematicians and people in mathematical physics who are women, have to strive and prove much harder. We know that gender bias is unconscious, and we know it is real. There are studies that prove this: in the 2012 Yale University 'John and Jennifer study,' two same CVs were presented; only the names were different, and John's CV was noted higher, even though both CVs were the same.

AF - For me it was not a battle I was very lucky. Very early on I had people who gave me the space and believed in me, and this gave me a base from which I could fly. Even though later on in life I could find some people that were less nice with me, I had this strong belief in myself.

JF - To have a career in academia, at the beginning of your career, you should have some well-established well-known people who believe in you and support you. If that is missing, the career can be incredibly difficult. I was lucky enough as I had some people who believed in me. If we have good young people, we should support and of course promote them, independently of whether they are men or women.

Are outreach initiatives for school children, such as UZH's JES effective? Why is there so much importance attributed to being with like-minded individuals?

AF - Programmes such as JES, are important in planting seeds at the age when children are trying to find their identities, to find people like them. It is kind of eye-opening for them and they feel reinforced in their passion. Otherwise, maybe the passion would diminish. So, it's very important for both, boys and girls.

Measure 2 - Mentoring

Key takeaways from the video

Presented by: Prof. Anton Alekseev (University of Geneva & NCCR SwissMAP Deputy Director & Mentor); Arina Voorhaar (University of Geneva, NCCR SwissMAP PhD & Mentee).

Obstacles

- Often people with more serious problems are silent and are not necessarily seeking help.
- The idea of approaching well-known mathematicians can be very intimidating. People who succeed do not really talk about their own failures which makes them appear more inaccessible.

Advice

- Provide informal settings for discussions between potential mentors and mentees.
- Increase visibility of mentorship experience success stories in order to encourage others to participate, to speak openly about life's obstacles and to avoid the feeling of isolation.
- Breaking boundaries between the fields and between the institutions can lead to greater openness.

Key discussion points raised by the panelists:

Did you have a mentor?

AF - I think we all have mentors. In the book by Sheryl Sandberg 'Lean in', there is a chapter on mentoring where it describes a mentor as someone you could meet whilst having a coffee, or during a conference, and gives you good advice that serves you all your life, even though you only meet this person once.

JF - I don't think I had a mentor when I was a student, I think the social life is in many ways much more important than mentoring. You need a social framework that gives you some kind of security.

PC: It's very important for students to have a social life to get support. Even scientific exchanges can happen during social life. However, it can be more difficult for women to go and say to men, let's go to have a beer together. Maybe in that aspect, organizing and giving opportunities for students in the department to meet could be very important.

How are mentoring programmes organized?

DM - For the past 17 years my university has run two mentoring programmes. One for doctoral students and early career researchers, and the other is a complement to a grant, where young researchers can apply to have a certain percentage of their work replaced by someone else. A condition for this grant, which is intended to be devoted to professional development, is that they work with a mentor. This is significant because the university is putting its money where its mouth is. The schemes have been very successful. They both require that the mentor and the mentee be in different faculties ideally, but in different departments, which means that this is a flat horizontal, non-hierarchical relationship. Further, in this relationship of equality, the mentor can be the cheerleader of the mentee. This is important, especially for people who are not necessarily well informed about how the University works or in terms of how the discipline functions. Ideally, the supervisor would be providing this kind of professionalization but unfortunately that doesn't always happen.

JF - I think cheerleading is not the right notion. PhD studies are not always cheerful, there are periods when you are stuck in your problem and your advisor doesn't really understand the problem either and then you have to fight. What is more important is to express faith in your students and that they will overcome problems and accomplish something.

EM - You could think of a PhD like a mountain you start going up the path and this path maybe doesn't get you to climb up to where you thought initially but then you can descend, and you can find another path, and this is where the cheerleading effect is important. This is an important aspect which advisors are maybe not able to bring because maybe they're more technical. There is a human side of the advising process for sure.

AF - About whether or not people should be forced to join a mentoring programme, I am a member of the female mentoring programme in Suisse Romande and I have never been contacted by anybody. I am not sure of the reasons and how efficient this kind of network is. Maybe people don't dare as it can be intimidating to see a professor, but the professor was a student as well, we all go through the same path.

Have you ever been a mentor? How is it different from being an advisor?

AF - Hopefully I'm a mentor of all my PhD students. I think this is one of the roles that we have as professors, to have the door open and have students come and ask questions that have to do with the discipline or something else. Also, the mentor also learns from the mentee, so it's not just one directional.

EM - My former PhD students still come back to me with all kinds of problems, and not only mathematics, so I feel that I am almost a sort of 'guru' mentor.

JF - In a good relationship between the student and the advisor, you don't need too many further mentors. Unfortunately, fairly frequently there's something wrong in the relationship between the PhD student and the advisor. It is important that if you have a problem with your advisor, you know where to go to discuss the problem and to get some help.

DM - The advantage of having a mentor who is not your supervisor, who is not in the same department, research group, or even perhaps in the same faculty, is that it can be quite difficult to explain to a supervisor, or someone in your hierarchy, that you have an emotional problem, and that a blockage is impacting your work. That can be complicated because then you could appear to be a little bit incompetent, whereas the pressure is to be highly productive 24/7.

How about people who need help and support but are too shy to ask?

DM -This is where an external system really has value. Especially one which will dangle the carrot of a few hours or a percentage of work time that is being replaced, that would encourage people to apply for a mentoring relationship. However, people cannot be forced into a mentoring relationship because it's a personal relationship. I would also say that someone who was so shy and so introverted, that they are unable take a step to resolve their issues in the interest of their professional development, would need more assistance than a mentor can give. A mentor is not a therapist.

JF - We live in a society where the admission of weakness and of problems does not pass well. If for example you have a depression, you have to hide it and not admit it, as it is considered to be bad and people might not take you seriously anymore afterwards. It's just the way our society functions but we should try hard to change this.

PC: The mentoring programme of the doctoral programme in my institution is compulsory. Every PhD student must choose a mentor and have at least one discussion per year. I've had several mentees, and thanks to the programme I was able to detect a serious problem and I was able to help the person. So, if the system is not in place, we might lose the opportunity to detect these problems, and the students might not dare to come and a talk.

Measure 3 - Women dedicated events & travelling with young children

Key takeaways from the video

Presented by: Prof. Eva Miranda (Universitat Politècnica de Catalunya & member of CRM & affiliated to the Observatoire de Paris); Dr. Serena Cenatiempo (Grans Sasso Science Institute – Italy).

Obstacles

- Men might sometimes feel intimidated by women dedicated events.
- Building a family can coincide with the most delicate phase of a career. People often think that becoming a mother is just a matter of few months of maternity leave.

Advice

- Hybrid events should be promoted. The protagonist could be women as plenary speakers and contributed talks could be ungendered.
- Childcare funds are usually aimed at increasing women's participation to research events. However, the same type of support should be offered to male researchers facing similar issues. Childcare is a family commitment, not a women business.

Key discussion points raised by the panelists:

Why have women dedicated events?

EM - To boost contemporary female role models and to create new networking opportunities. The pandemic has left aside not only women, but also junior participants. I think now we should really promote further women activities to reintegrate them again back in the group.

Why are there not more women plenary speakers in general?

EM - When I've been in committees, I have observed that often, female and male members of the committee tend to question women's applications more than men's. If I were a PhD student now, it would be hard for me to picture myself in academia, because when I go to conferences I don't see women plenary speakers. Therefore, as a student I may not even think that it is possible to be one of them. I think that to have more women speaking at conferences is also a way to give confidence to the younger generations of women as it generates a 'call effect.'

PC: I've served on many committees, scientific committees to organize conferences, hiring committees for professor positions and all the committees I've served on, have only judged by scientific criteria it was completely gender independent. What is more, all funding agencies all over the world demand that conferences have female speakers and better yet, more than one. My childhood was in the 70s, and if I compare the development from then, it has been dramatic, as far as the gender equality goes. If you see old movies from the 70s or 80s, there are jokes being made that would be considered completely unacceptable nowadays. So, what I want to say is that there has been development and developments in societies are not fast.

EM - As a woman, I'm not able to say that I never had unconscious gender bias. The bias can be very subtle, I'm not saying that people are not giving positions to women because they are women, I'm just saying that there is some unconscious bias sometimes.

Concluding remarks:

AF - We come from decades of men dedicated events and men dedicated positions, we need to transition to something more equal. In my field there are many competent women and even there, it is and has been long dominated by men. They get to have the headlines. So, it's nice sometimes, when there is a conference that is not publicized as women only and where most invited plenary speakers are women. It feels good, even for me, who has a position, it feels so good because maybe we lack confidence as very often we're put down.

JF - I think we live in a period where we like to debate problems forever, but we never solve them. If we see problems for example, about gender inequality, let's just all try to do something about it and solve the problem. At least to the extent we can. That would be my pledge: let's act and not just debate and complain.

DM - I think that the big difference between when we were all young researchers and where we find ourselves now, is precisely that action is being taken. That there are these programmes in place, to create a critical mass of well qualified experienced women, who are in the position to accept invitations to be plenary speakers at conferences. I think action is the big difference between then and now.

Announcements

2022 Call for IUPAP conference sponsorship applications

Every year, the International Union of Pure and Applied Physics (IUPAP) supports conferences in all fields of Physics that are recommended by its different topical commissions. Applications concerning conferences in our field will be reviewed by the commission for Mathematical Physics, C18.

The deadline to apply for IUPAP conference sponsorship is set to 1 June 2022 for events taking place in 2023.

The interested organisers should review the rules and guidelines which are accessible on the IUPAP weblink (https://iupap.org/conferences/conference-policies/) and apply online.

The commission for Mathematical Physics will then make recommendations to the IUPAP Executive Council, which determines which conferences will be sponsored.

Alain Joye, IUPAP C18 Chair

Videos available from ICMP 2021

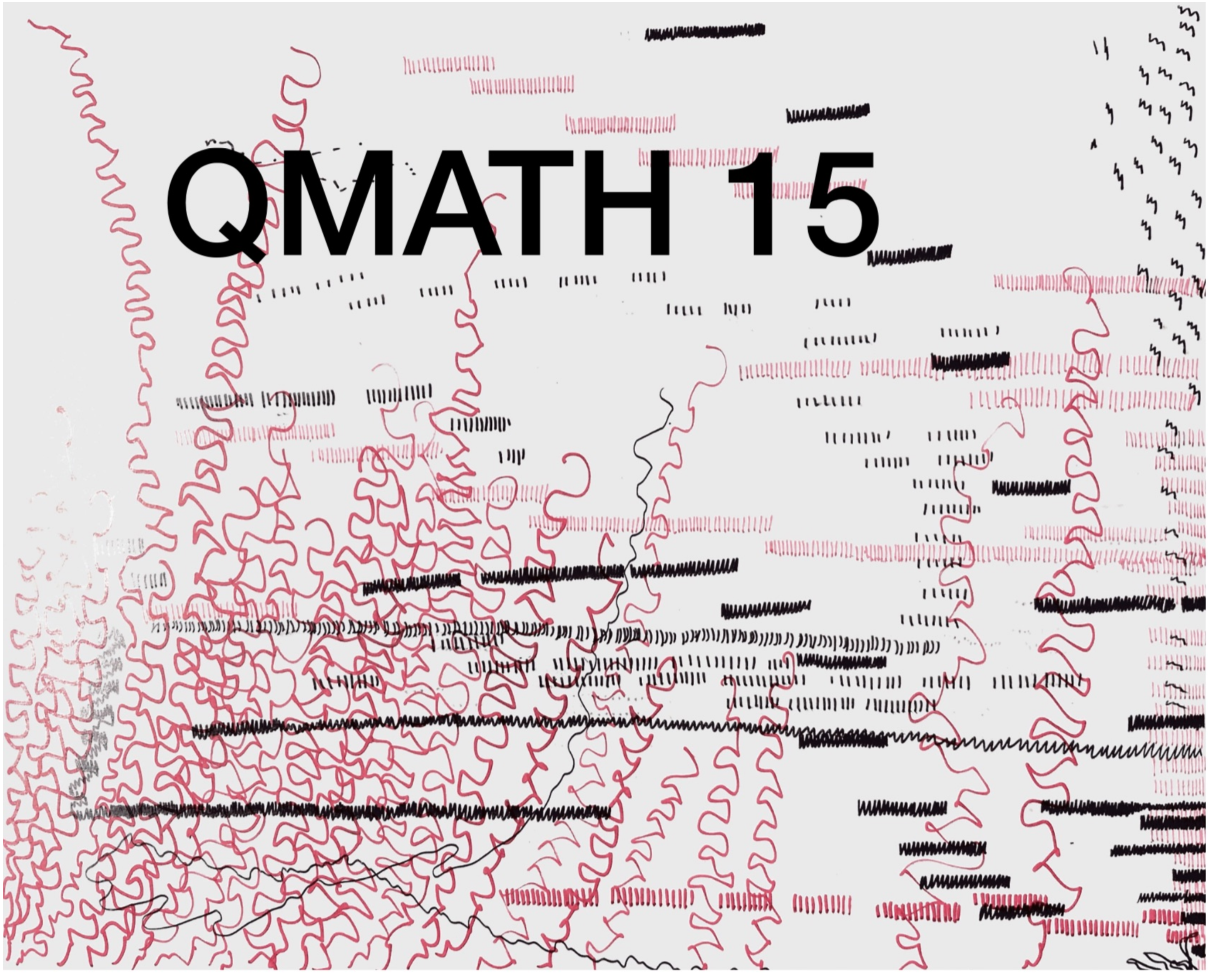
We would like to inform you that the video recordings of the XXth International Congress on Mathematical Physics (ICMP) and Young Researchers Symposium (YRS) 2021 are now available online, alongside a selection of photographs of the event.

On behalf of the Local Organizing Committee, we would also like to take this opportunity to thank you for attending the event last August in Geneva. Whether online or onsite, we hope you enjoyed the congress.

Link: ICMP videos & photographs
Link: YRS videos & photographs

SwissMAP was the main sponsor and organizer of the ICMP & YRS 2021. If you are interested in organizing a conference in Switzerland, please visit the SwissMAP Research Station website.

Benjamin Schlein and Anton Alexeev



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Sessions

- **Spectral Theory**
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- **Many-Body Quantum Physics**
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- **Disordered Systems and Random Matrices**
  Paul Bourgade, Yan Fyodorov
- **Quantum Information**
  Aram Harrow, Zhengwei Liu
- **Semiclassical Approximation and Dynamical Systems**
  Semyon Dyatlov, Nalini Anantharaman
- **New Topics in Mathematical Physics**
  Joel Moore, Vieri Mastropietro

Plenary Speakers

- **Gregory Berkolaiko**
- **Nilanjana Datta**
- **Matthew Hastings**
- **Jens Marklof**
- **Tomaž Prosen**
- **Tatyana Shcherbina**
- **Benjamin Schlein**
- **Andreas Winter**

Local organizing committee: Martin Fraas, Bruno Nachtergaele, Maciej Zworski

Advisory board: Fernando Brandao, Pavel Exner, Arthur Jaffe, Jon Keating, Michael Loss, Yoshiko Ogata, Simone Warzel

Art form the "Postcards from Utopia" collection by Yael Degany

Nominations for the 2023 Dannie Heineman Prize for Mathematical Physics

This prize recognizes outstanding publications in the field of mathematical physics. The prize consists of $10,000 and a certificate citing the contributions made by the recipient plus travel expenses to attend the meeting at which the prize is bestowed. It will be presented annually.

Establishment & Support

This prize was established in 1959 by the Heineman Foundation for Research, Educational, Charitable, and Scientific Purposes, Inc., and is administered jointly by the American Physical Society and the American Institute of Physics. Biographical information on Dannie Heineman: https://www.aps.org/programs/honors/prizes/heineman-bio.cfm.

Rules & Eligibility

This prize is awarded solely for valuable published contributions made in the field of mathematical physics with no restrictions placed on a candidate's citizenship or country of residence. "Publication" is defined as either a single paper, a series of papers, a book, or any other communication which can be considered a publication. The prize may be awarded to more than one person on a shared basis when all recipients have contributed to the same accomplishments. Nominations will be considered for three review cycles provided the nominator re-certifies the nomination before the next deadline.

Nomination & Selection Process

Deadline: Wednesday, June 1, 2022

The nomination package must include:

A letter of not more than 5,000 characters evaluating the qualifications of the nominee(s) In addition, the nomination should include:

- A biographical sketch.
- A list of the most important publications.
- At least two, but not more than four, seconding letters.
- Up to five reprints or preprints.

To start a new or update a continuing nomination, please see the Prize & Award Nomination Guidelines at https://www.aps.org/programs/honors/nomination.cfm.

Further information about the Dannie Heineman Prize and related links are at https://www.aps.org/programs/honors/prizes/heineman.cfm.

Time's Arrow

Scientific anniversaries

1822. Rudolf Clausius was born on 7 March in Köslin, Prussia.

1922. Olga Aleksandrovna Ladyzhenskaya was born on 7 March in Kologriv, Russian SFSR. (See related articles in this issue.)

Awards and Honors

Antti Kupiainen, Rémy Rhodes, and Vincent Vagas have been awarded the 2022 George Pólya Prize in Mathematics by the Society for Industrial and Applied Mathematics, for "a rigorous justification of the DOZZ formula for three-point structure constants in Liouville Conformal Field Theory."

Joshua Zak has won the 2022 Israel prize, for "the development of mathematical tools such as the 'Zak Transform' and the 'Zak Phase' for the study of quantum phenomena in crystalline solids. These tools allow for the prediction of materials with unique properties to build electronic devices."

Readers are encouraged to send items for "Time's Arrow" to bulletin@iamp.org.

News from the IAMP Executive Committee

New individual members

IAMP welcomes the following new members

1. PROFESSOR BIVUDUTTA MISHRA, BITS-Pilani, Hyderabad Campus, India
2. DR. GIOVANNA MARCELLI, SISSA, Trieste, Italy
3. PROFESSOR AMOL AGGARWAL, Columbia University, New York City, USA.

Recent conference announcements

INdAM Quantum Meetings at the Politecnico di Milano

March-May 2022, Politecnico di Milano, Italy.

Baylor Analysis Fest: "From Operator Theory to Orthogonal Polynomials, Combinatorics, and Number Theory

May 23-27, 2022, Baylor University, Waco, TX, USA

Sixth Great Lakes Mathematical Physics Meeting 2022

June 9 -12 2022, Michigan State University, East Lansing MI, USA.

Mathematical Challenges in Quantum Mechanics

Third School and Workshop, June 13-18, 2022 Como (Italy).

St. Petersburg Conference in Spectral Theory and Mathematical Physics

June 22-26, 2022, Euler International Mathematical Institute, St. Petersburg.

Operator Theory Analysis and Mathematical Physics - OTAMP2022

June 27-30, 2022, Dept. of Mathematics, Stockholm Univ. and ZOOM Please register before June 5

New Directions in Disordered Systems

June 27 - July 1, 2022, Cergy, France

Probability and Mathematical Physics

June 28th - July 7th, 2022, Helsinki, Finland.

O.A. Ladyzhenskaya Centennial Conference on PDE's

July 16-23, 2022, St. Petersburg Department of Steklov Mathematical Institute

34th International Colloquium on Group Theoretical Methods in Physics

July 18 - 22, 2022, Université de Strasbourg, France.

4th ZiF Summer School Randomness in Physics and Mathematics: From Integrable Probability to Disordered Systems,

August 1-13, 2022, Centre for Interdisciplinary Research (ZiF), Bielefeld University.

Quantum information in Many-Body Physics: A Mathematical Invitation

August 29 - September 2, 2022, IAMP-EMS Summer School in Mathematical Physics, Munich, Germany

Solid Math 2022, Trieste, Italy. Mathematical and Numerical Methods for Solid-State Physics

September 6-9, 2022, Trieste, Italy.

Qmath15

September 12 - 16, 2022, University of California, Davis CA, USA.

Asymptotic Analysis and Spectral Theory (Aspect'22)

September 26 - 30, 2022, Oldenburg, Germany.

A Tale of Mathematics and Physics: A Tribute to Krzysztof Gawędzki

November 7 - 10, 2022, École Normal Superieure, Lyon, France.

For an updated list of academic job announcements in mathematical physics and related fields visit

`http://www.iamp.org/page.php?page=page_positions`

Michael Loss (IAMP Secretary)

CHIARA SAFFIRIO
Departement Mathematik & Informatik
Universität Basel
Spiegelgasse 1
4051 Basel, Switzerland
chiara.saffirio@unibas.ch

EVA MIRANDA
Departament de Matemàtiques
Universitat Politècnica de Catalunya
Avinguda Doctor Marañon 44-50 Despatx 333
Tercera Planta 08028, Barcelona, Spain
eva.miranda@upc.edu

ALEXANDER I. NAZAROV
St Petersburg Department of
Steklov Mathematical Institute, RAS
St. Petersburg, Russia
al.il.nazarov@gmail.com

GREGORY A. SEREGIN
Mathematical Institute
University of Oxford
Oxford, OX2 6GG, United Kingdom
and St Petersburg Department of
Steklov Mathematical Institute, Russia
seregin@maths.ox.ac.uk

ÁGATA TIMÓN GARÇA-LONGORIA
The Institute of Mathematical Sciences
(ICMAT)
C/ Nicolás Cabrera, 13-15
28049 Madrid, Spain
agata.timon@icmat.es

SERENA CENATIEMPO
Mathematics Division
Gran Sasso Science Institute
Viale Crispi n.7
67100 L'Aquila, Italy
serena.cenatiempo@gssi.it

MAYRA LIROT
Communication Officer
NCCR SwissMAP
Université de Genève
mayra.lirot@unige.ch

SIMONE RADEMACHER
IST Austria
Am Campus 1
3400 Klosterneuburg, Austria
simone.rademacher@ist.ac.at

MICHAEL LOSS
School of Mathematics
Georgia Institute of Technology
Atlanta, GA 30332-0160, USA
secretary@iamp.org

EVANS HARRELL
School of Mathematics
Georgia Institute of Technology
Atlanta, GA 30332-0160, USA
bulletin@iamp.org