Poincaré Prize 2015

Alexei Borodin

I am very pleased and honored to give the laudatio for Alexei Borodin on his winning the Poincaré Prize for 2015.

Alexei was born and went to school in Donetsk in the Ukraine. In 1992 he was accepted into the famous "mech-mat" Department at Moscow State University, graduating in 1997. At Moscow State he began working with Grigori Olshanski in a marvelous collaboration that continues to this day. He received his PhD under Alexander Kirillov from the University of Pennsylvania in 2001. He was a professor at Caltech from 2003 to 2010, and since then he has been at MIT.

I met Borodin about 15 years ago when he was a student at UPenn, and I have been following his career closely since then. His mathematical interests lie in the circle of ideas which connect the representation theory of "big" groups, combinatorics, integrable interacting particle systems and random matrix theory.

When I met Borodin I was struck immediately by the freshness of his approach, combined with a professional maturity well beyond his years. He seemed to know and understand everything and I had to keep reminding myself that I was speaking to a young PhD student and not a seasoned colleague of many years standing. Borodin's mathematical and professional maturity was recognized with his very first academic appointment as a full Professor at Caltech in 2003.

Following on earlier work of Kerov, Olshanski and Vershik, the key observation of Borodin and Olshanski in the representation theory of "big" groups, such as the infinite symmetric group and the infinite unitary group, is that the characters for the group are naturally associated with stochastic point processes. In terms of the harmonic analysis on groups, this works out as follows: Given a representation of the group, the character of the representation can be expressed as an integral of the irreducible characters of the group with respect to some measure. In the case of "big"groups, this measure turns out to be a measure on point processes. In this way, representations of the group give rise to stochastic point processes. At the technical level, the challenge is then to compute the correlation functions for these processes, and one of the main results of the theory, which was new and unexpected, is an explicit determinantal formula for the correlation functions associated with the so-called generalized regular representation of the infinite symmetric group (Borodin) and similarly of the infinite unitary group (Borodin and Olshanski). The fundamental insight here, due to Borodin, is that the representation theory of "big" groups should be viewed and analyzed as statistical mechanical systems.

Recall that Young diagrams are pictorial representations of the partitions of numbers. For example, $6 = 3 + 2 + 1$ is represented by a Young diagram with 3 boxes in the first row, 2 boxes in the second row, and 1 in the third. For the partition $4 + 2$, one has 4 boxes in the first row and 2 in the second row, and so on. The Young diagrams of size N carry a natural probability measure, viz., Plancherel measure.

A stunning consequence of the work of Borodin and his collaborators is one of the first proofs of a general conjecture in combinatorics that the lengths of the first k rows of a Plancherel-random Young diagram of size N behaves statistically as N goes to infinity like the k largest eigenvalues of a Gaussian Unitary Ensemble (GUE) matrix. Here you see Alexei taking his first steps into the circle of ideas mentioned above:

(1) Young diagrams parametrize the *irreducible representations* of the infinite symmetric group

(2) Young diagrams are intimately related via the Robinson-Schensted algorithm to the classical longest increasing subsequence problem of Ulam in *combinatorics*

(3) The lengths of the rows of Young diagrams of size N are interpreted as the positions particles on the line. So for $6 = 3+2+1$, one has 1 particle at position 3, one at 2, and one at 1. As the Young diagrams are random, one obtains a *random particle system.*

(4) By the conjecture, as $N \to \infty$, the leading particles behave like the largest eigenvalues of a *random GUE matrix*, from which we also see that the particle system is *integrable* as the distribution of the eigenvalues of a GUE matrix involves a solution of the Painlevé II equation, and as we know, the Painlevé equations are classical integrable Hamiltonian systems.

Bringing this all together regarding the above conjecture on the lengths of the rows of Young diagrams, it is, in the end, Borodin's explicit determinantal formulae for the correlation functions that makes the analysis possible.

In his career Alexei has made profound contributions to all four components of the circle—to the representation of "big" groups, to combinatorics, to integrable random particle systems, and to random matrix theory. The guiding motivation for Alexei has been to find an overall, umbrella integrable system from which all the many known systems in the circle can be obtained by specialization, and their integrability so explained. Alexei's recent work with Ivan Corwin on MacDonald processes is a remarkable example of this philosophy.

Alexei has received many honors: Here are some of them:

- In 1992 he represented the Ukraine at the International Math Olympiad, earning a silver medal
- In 2001 he was awarded a prestigious long-term Clay Research Fellowship
- In 2003 he was awarded the Prize of the Moscow Mathematical Society

- In 2008 he was awarded the Prize of the European Mathematical Society, and
- In 2010 he gave a Plenary Lecture at the International Congress of Mathematicians in Hyderabad.

I offer Alexei my warmest congratulations on his outstanding achievement in winning the Poincaré Prize for 2015.

Percy Deift
Santiago, July 27, 2015