

Laudatio for Robert Seiringer

By Jakob Yngvason

It is my privilege to present to you Robert Seiringer who receives the Henri Poincaré Prize for his ‘major contributions to the mathematical analysis of low temperature condensed matter systems’. Triggered by experimental advances, cold quantum gases and Bose-Einstein condensation came into the focus of attention of physicists about 15 years ago and this topic is currently one of the most active research areas in physics. Mathematical physics has here an important role to play because due to the complexity of the quantum many-body problem some of the most fundamental questions can only be settled by penetrating mathematical analysis. It is here that Robert has been a key player and contributed deep insights.

It is now ten years since Robert came to me inquiring about a subject for a Master thesis. It so happened that Elliott Lieb and I had a little earlier written a paper on the ground state energy of an interacting, dilute Bose gas and were thinking about the extension to inhomogeneous systems. Physicists usually describe such systems by the Gross-Pitaevskii equation that is a non-linear Schrödinger equation expected to provide a good approximation to the many-body problem in the case of dilute gases. But no rigorous derivation starting from the full many-body

problem existed yet. As a first task I suggested to Robert to work on the energetic upper bound. This was on a Friday. A normal, good student would probably have started by studying lots of references and come back after three weeks, asking for further guidance. But I realized that Robert was someone quite out of the ordinary when he presented to me next Monday a beautiful LaTeX document with the complete solution of the problem. Thus Robert was from the very outset a full-fledged research partner rather than a student or junior partner and this first cooperation between Elliott, Robert and me very soon resulted in a joint publication as well as Robert's Master degree. His PhD followed a year later, his thesis containing besides further contributions to the theory of Bose gases some beautiful results about quite a different topic, namely atoms in strong magnetic field, partly obtained in collaboration with Bernhard Baumgartner.

At the age of 33 Robert is at present author or co-author of more than 40 original publications in prestigious journals. These publications are devoted to many different topics but I shall here focus on a few that are particularly linked to the citation for the Henri Poincaré prize.

A notoriously difficult problem in quantum many-body theory is to prove the existence of Bose-Einstein condensation (BEC) for a system of interacting Bosons in the thermodynamic limit. This phase transition is accompanied by spontaneous breaking of a continuous symmetry (gauge

symmetry), which explains at least partly the difficulty in proving it. In fact, for a continuous system with realistic interactions this is still an open problem. Experiments with cold atoms are, however, carried out in traps where the gas has a finite extension and a simpler, but still far from an easy problem is to prove Bose-Einstein condensation in an appropriate limit, called the Gross-Pitaevskii (GP) limit, for such systems. In fact, this had been an open problem for quite some time when Robert entered the scene. Robert, who in the meantime had moved from Vienna to Princeton with a scholarship of the Austrian Science Fund, realized that the energy estimates used in the derivation of the Gross-Pitaevskii equation could be generalized to prove that the kinetic energy of the ground state wave function is essentially localized in small regions in configuration space and that this would be the key to a proof of BEC in the GP limit. An additional ingredient, namely a generalization of classical Poincaré inequalities, was also needed, and all this was put together in a beautiful joint publication of Robert and Elliott Lieb in 2002. This was the first derivation of BEC for an interacting system of direct experimental relevance.

The theory of BEC in the GP limit was subsequently generalized to prove also superfluidity in this limit and somewhat later a further outstanding problem, posed six years earlier by Lev Pitaevskii, could be solved, namely the derivation of the GP equation and the proof of BEC for gases in a rotating container. This problem is much more complex

than for the non-rotating case due to the occurrence of quantized vortices that can break rotational symmetry. Here the solution was obtained in several steps with two papers of Robert playing a key role. One is his analysis of the Gross-Pitaevskii equation for a rotating system, containing the first general proof of rotational symmetry breaking for this equation. Most work on the GP equation with rotation is concerned with asymptotic analysis with respect to a small parameter, the ratio of the healing length to the size of the system, but Robert's paper is one of the few that has no such limitations. In a second paper Robert linked the symmetry breaking to another important difference between the rotating and non-rotating case: while in the non-rotating case the absolute ground state, i.e., the ground state without symmetry restrictions, is automatically the Bose ground state, this is in general no longer true for rotating systems. He was nevertheless able to prove that the GP equation provides an upper bound to the bosonic ground state, also in the case of symmetry breaking. The derivation of the GP equation in the rotating case was finally completed when the lower bound was proved in an impressive joint paper with Elliott Lieb. Further work of Robert on rotating Bose gases includes quite recent papers with Mathieu Lewin, and with Elliott and myself on gases that are in such rapid rotation that they become effectively two-dimensional and their state confined to the lowest Landau level.

Besides his work on the ground states, Robert has also been a pioneer in the rigorous study of quantum gases at nonzero temperatures. His work here includes the proof of the next to leading order term in the pressure of a dilute Fermi gas, generalizing previous joint work with Lieb and Solovej on the ground state, and a highly sophisticated derivation of a lower bound to the free energy of a Bose gas, containing next to leading order contributions. A beautiful recent paper with Daniel Ueltschi contains rigorous upper bounds for the transition temperatures for superfluidity in two-dimensions and to BEC in a three dimensions.

As already mentioned, Robert has worked on many topics besides cold quantum gases, including Coulomb systems, quantum electrodynamics and BCS theory, partly in collaboration with another former student of mine, Christian Hainzl. All his work is characterized by depth of the mathematical analysis and elegance and clarity of the presentation. His accomplishments make him already at this early stage of his career a most deserving recipient of the Henri Poincaré Prize and point to a bright future. I congratulate him heartily on this occasion.

Jakob Yngvason